UE Fluids

Problem 1 Solution

1) By Kelvin’s Theorem, \( \Gamma_2 = \Gamma_1 \)

In general, \( L' = c V_w \Gamma = \frac{1}{2} c V_w^2 c_{L} \)

or \( \Gamma = \frac{1}{2} c V_w c_{L} \) (for airfoil)

\[ \therefore \Gamma_1 = \frac{1}{2} c V_w c_{L_1} = \frac{1}{2} \times 1 \times 30 \times 0.5 = 7.5 \text{ m/s} = \Gamma_2 \]

But we also know \( \Gamma_3 + \Gamma_4 = \Gamma_2 \) by geometry.

And since \( \Gamma_3 = \frac{1}{2} c V_w c_{L_2} = \frac{1}{2} \times 1 \times 30 \times 1.5 = 22.5 \text{ m/s} \)

\[ \therefore \Gamma_{\text{vortex}} = \Gamma_4 = \Gamma_2 - \Gamma_3 = -15 \text{ m/s} \] (counterclockwise)

2a) Observer sees:

- horizontal \[ U = V_w = 30 \text{ m/s} \]

- vertical \[ W = \frac{\Gamma_{\text{vortex}}}{2 \pi d} = -15 \text{ m/s} \]

\[ \vec{V}_{\text{apparent}} = U \hat{i} + W \hat{k} \]

2b) Airfoil thinks it’s in a freestream \( \vec{V}_{\text{apparent}} \):

Apparent Lift force is \( \perp \) to \( \vec{V}_{\text{apparent}} \):

\[ L' = \frac{1}{2} \rho \vec{V}_{\text{apparent}} \cdot c_{L} \approx \frac{1}{2} \rho V^2 c_{L} \]

Apparent Drag force is \( \parallel \) to \( \vec{V}_{\text{apparent}} = 0 \) (d’Alembert)

This lift force is tilted back relative to motion by angle \( \theta = \arctan \frac{-W}{U} = 0.456^{\circ} \)

True drag relative to \( V \) is

\[ D' = L' \sin \theta \]

or \[ c_d = c \sin \theta = 0.0119 \]