Flight Power and Thrust Relations

7 Feb 08

Lab 1 Lecture Notes

Nomenclature

D	aircraft drag	T	propeller thrust
L	aircraft lift	T_c	thrust coefficient
W	total aircraft weight	P_{prop}	propulsive thrust power ($\equiv TV$)
W_0	empty aircraft weight	P_{shaft}	motor shaft power
W_p	payload weight	R	propeller radius
V	flight speed	η_{prop}	propeller efficiency
S	reference area (wing area)	η_{v}	profile efficiency (viscous loss)
b	wing span	η_{i}	Froude efficiency (inviscid loss)
AR	wing aspect ratio	Re	chord Reynolds number
C_L	aircraft lift coefficient	c_ℓ	wing-airfoil profile lift coefficient
C_D	aircraft drag coefficient	c_d	wing-airfoil profile drag coefficient
CDA_0	drag area of non-wing components	ρ	air density

Thrust Power

Generation of thrust in flight requires the expenditure of power. For a propeller or jet-engine fan, the shaft power and the thrust are related by the definition of propeller efficiency.

$$\frac{TV}{P_{\mathrm{shaft}}} \;\equiv\; \frac{P_{\mathrm{prop}}}{P_{\mathrm{shaft}}} \;\equiv\; \eta_{\mathrm{prop}} \tag{1}$$

The η_{prop} is the product of a viscous profile efficiency η_{v} which accounts for the viscous profile drag on the blades, and an inviscid Froude efficiency η_{i} which accounts for the kinetic energy lost in the accelerated propwash.

$$\eta_{\mathrm{prop}} \;=\; \eta_{\mathrm{v}}\,\eta_{\mathrm{i}} \tag{2}$$

An upper limit and estimate of η_{i} is

$$\eta_{\mathrm{i}} \;\le\; \frac{2}{1+\sqrt{1+2T/\rho V^2 \pi R^2}} \tag{3}$$

which strongly depends on the thrust and flight speed. In contrast, η_{v} does not vary much and is often considered a constant. Combining all of the above relations gives

$$P_{\mathrm{shaft}} \;=\; T\,\frac{1}{2}\left(V+\sqrt{V^2+\frac{2T}{\rho\,\pi R^2}}\right)\frac{1}{\eta_{\mathrm{v}}} \tag{4}$$

$$\;=\; T\,V\,\frac{1+\sqrt{1+T_c}}{2}\,\frac{1}{\eta_{\mathrm{v}}} \tag{5}$$

$$T_c \;\equiv\; \frac{T}{\frac{1}{2}\rho V^2\,\pi R^2} \tag{6}$$

where we define the convenient dimensionless Thrust Coefficient. Limiting cases are

$$T_c \gg 1 \;:\; P_{\mathrm{shaft}} \;\simeq\; \frac{T^{3/2}}{(2\pi\,\rho)^{1/2}}\,\frac{1}{R}\,\frac{1}{\eta_{\mathrm{v}}} \quad , \qquad \text{(Heavy loading, low-speed takeoff)} \tag{7}$$

$$T_c \ll 1 \;:\; P_{\mathrm{shaft}} \;\simeq\; T\,V\,\frac{1}{\eta_{\mathrm{v}}} \quad , \qquad \text{(Light loading, high-speed cruise)} \tag{8}$$

Level-Flight Relations

In level flight we have $W = L$, which gives the velocity in terms of aircraft parameters.

$$W \;=\; L \;=\; \frac{1}{2}\rho V^2 S\, C_L \tag{9}$$

$$V \;=\; \left(\frac{2\,W}{\rho\, S\, C_L}\right)^{1/2} \tag{10}$$

In steady level flight we also have $T = D$, in which case the propulsive thrust power can then be given as follows.

$$P_{\rm prop} \;=\; TV \;=\; DV \;=\; \frac{1}{2}\rho V^3 S\, C_D \;=\; \left(\frac{2W^3}{\rho\, S}\right)^{1/2} \frac{C_D}{C_L^{3/2}} \tag{11}$$

In the level-flight case we can also express the thrust coefficient and hence the prop Froude efficiency in a convenient manner.

$$T_c \;=\; \frac{T}{\frac{1}{2}\rho V^2\, \pi R^2} \;=\; \frac{D}{\frac{1}{2}\rho V^2\, \pi R^2} \;=\; \frac{S}{\pi R^2}\, C_D \tag{12}$$

Drag Breakdown

To obtain the propulsive power via (11), we need the overall aircraft drag coefficient C_D, which is broken down into three basic components.

$$C_D \;=\; \frac{CDA_0}{S} \;+\; c_d(C_L, Re) \;+\; \frac{C_L^2}{\pi\, AR} \tag{13}$$

The first term gives the combined drag of all the non-wing components, such as the fuselage, tail, landing gear, etc. The second term is the wing profile drag, estimated from the wing's 2D airfoil $c_d(c_\ell, Re)$ data, and by assuming that the typical wing airfoil operates at $c_\ell \simeq C_L$. The last term is the induced drag coefficient C_{D_i}, which depends on C_L and the aspect ratio of the wing.

$$AR \;=\; \frac{b^2}{S} \tag{14}$$

Figure 1 shows the three C_D components versus C_L for a typical 1.5 m span light RC sport aircraft.

For a typical operating point at $C_L = 1.0$ (low speed) and $C_L = 0.3$ (high speed), indicated by the symbols in Figure 1, the three components contribute roughly the following percentages to the total drag:

C_L	CDA_0/S	c_d	$C_L^2/\pi AR$	C_D
1.0	0.0167	0.0335	0.0406	0.0909
0.3	0.0167	0.0220	0.0037	0.0424
1.0	18 %	37 %	45 %	100 %
0.3	39 %	52 %	9 %	100 %

The corresponding propulsive power is shown in Figure 2.

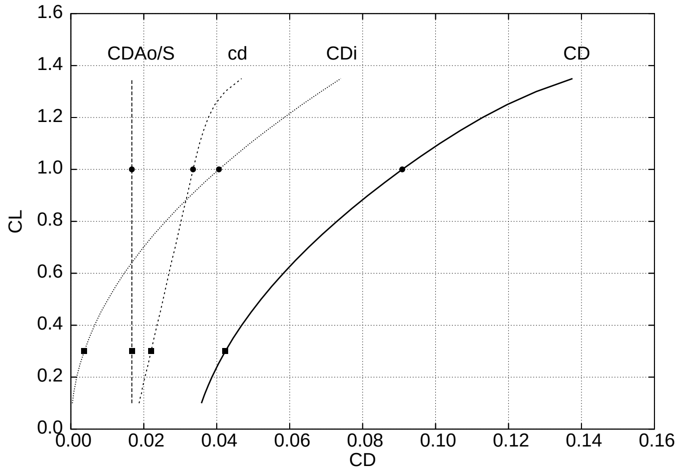

Figure 1: Drag polar and drag polar components for electric sport aircraft. $AR = 9.0$

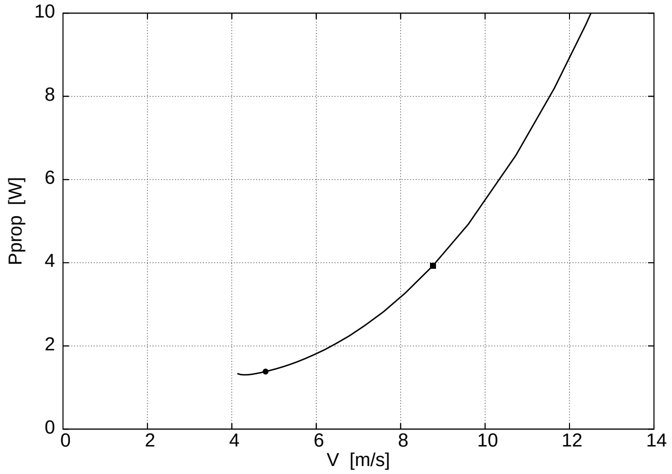

Figure 2: Propulsive thrust power $P_{\text{prop}} = DV = TV$ for electric sport aircraft.

Flight Power for Maximum Payload

We now consider how much payload weight the aircraft can lift and still maintain flight. The overall weight will now consist of the "empty" aircraft weight W_0, plus the payload weight W_p.

$$W \;\; = \;\; W_0 + W_p \tag{15}$$

We will also assume that the aircraft will have a motor with some maximum power which cannot be exceeded.

$$P_{\text{shaft}} \;\; \leq \;\; (P_{\text{shaft}})_{\max} \tag{16}$$

Combining the relations and assumptions above, all the aircraft variables and parameters must satisfy the following equality in barely-sustained level flight at full engine power.

$$\boxed{(P_{\text{shaft}})_{\max} \;=\; \frac{1}{\eta_{\text{v}}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{S}{\pi R^2}\,C_D}\right)\left(\frac{2(W_0+W_p)^3}{\rho\, S}\right)^{1/2}\frac{C_D}{C_L^{3/2}}} \qquad (17)$$

$$\text{where} \qquad \boxed{C_D \;=\; \frac{CDA_0}{S} \;+\; c_d(C_L, Re) \;+\; \frac{C_L^2}{\pi}\frac{S}{b^2}} \qquad (18)$$

The auxilliary C_D expression (18) is the same as (13), restated here for convenience.

Parameter Coupling and Design Optimization

Any change in the aircraft variables which permits a larger W_p in equation (17), while still maintaining the equality, will enable a greater payload to be carried. However, it's essential to realize that most of the variables and parameters in equations (17) and (18) are coupled in an actual design application, so the effect of changing one will have multiple side effects, with the net effect being nonobvious.

One example which might appear if one attempts to enable a larger W_p by an increasing the wing area S. The immediately-apparent benefits of a larger S are:

- Pro: Direct $1/S^{1/2}$ reduction in (17)
- Pro: Reduction of the CDA_0 term in (18)

There are also obvious drawbacks:

- Con: Increase in the first propeller loss term in (17)
- Con: Increase in the last induced drag term in (18)

Furthermore, there will likely be additional drawbacks which are not explicitly apparent:

- Con: Increase in the empty weight W_0 because of more wing material, etc.

Other Pros and Cons may be present in addition to those listed above, depending on the situation.

Much of the activity which occurs during aircraft design and sizing consists of identifying and quantifying such couplings. Knowing the couplings then allows suitable tradeoffs to be performed, in order to find the best set of design parameters to maximize the design objective. Once a good or optimum design has been reached, all its competing tradeoffs are in balance, so that there are no more "easy" design changes which can be made without adversely affecting something else.