

Definition of a streamline:

$$w \, dy - v \, dz = 0 \qquad u \, dz - w \, dx = 0 \qquad v \, dx - u \, dy = 0$$

Rate of change seen by drifting sensor of any field quantity $f(t, x, y, z)$:

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + \vec{V} \cdot \nabla f = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z}$$

Convective form of mass equation:

$$\begin{aligned} \frac{D\rho}{Dt} + \rho \nabla \cdot \vec{V} &= 0 \\ \nabla \cdot \vec{V} &= 0 \quad (\text{low speed flow case, } \rho = \text{constant}) \end{aligned}$$

Convective form of x , y -momentum equations:

$$\begin{aligned} \rho \frac{Du}{Dt} &= -\frac{\partial p}{\partial x} + \rho g_x + (F_x)_{\text{viscous}} \\ \rho \frac{Dv}{Dt} &= -\frac{\partial p}{\partial y} + \rho g_y + (F_y)_{\text{viscous}} \end{aligned}$$

Definition of Vorticity:

$$\begin{aligned} \vec{\xi} &\equiv \nabla \times \vec{V} \\ \xi &\equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (\text{in 2-D}) \end{aligned}$$

Definition of Circulation around a specified circuit:

$$\begin{aligned} \Gamma &\equiv - \oint \vec{V} \cdot d\vec{s} \quad (\text{always valid, by definition}) \\ &= - \iint \xi \, dA \quad (\text{in 2-D, only for reducible circuits}) \end{aligned}$$

Helmholtz Equation for inviscid flow:

$$\begin{aligned} \frac{D\xi}{Dt} &= 0 \\ \xi &= 0 \quad (\text{for flow which is uniform upstream}) \end{aligned}$$

Bernoulli Equation for inviscid, low speed flow:

$$p + \frac{1}{2}\rho V^2 = p_o \quad (\text{constant along a streamline})$$

Definition of Pressure Coefficient:

$$\begin{aligned} C_p &\equiv \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \quad (\text{always valid, by definition}) \\ &= 1 - \frac{V^2}{V_\infty^2} \quad (\text{only if } p_o \text{ is constant for all streamlines}) \end{aligned}$$

Velocities given via potential function $\phi(x, y)$:

$$u = \frac{\partial \phi}{\partial x} \quad , \quad v = \frac{\partial \phi}{\partial y} \quad \text{or} \quad \vec{V} = \nabla \phi$$

Velocities given via stream function $\psi(x, y)$:

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad \text{or} \quad \vec{V} = (\nabla \psi) \times \hat{k}$$

Governing equations for incompressible (low speed), irrotational flows:

Using ϕ	Using ψ	
$\nabla^2 \phi = 0$	$\nabla^2 \psi = 0$	(everywhere)
$\partial \phi / \partial n = 0$	$\psi = \text{constant}$	(on solid body)
$\partial \phi / \partial x = V_\infty$	$\partial \psi / \partial y = V_\infty$	(far away)

Simple 2-D incompressible, irrotational flows:

	ϕ	ψ	u	v	V_r	V_θ
Uniform flow :	$V_\infty x$	$V_\infty y$	V_∞	0	$V_\infty \cos \theta$	$-V_\infty \sin \theta$
Point source :	$\frac{\Lambda}{2\pi} \ln r$	$\frac{\Lambda}{2\pi} \theta$	$\frac{\Lambda}{2\pi} \frac{x}{r^2}$	$\frac{\Lambda}{2\pi} \frac{y}{r^2}$	$\frac{\Lambda}{2\pi r}$	0
Point vortex :	$-\frac{\Gamma}{2\pi} \theta$	$\frac{\Gamma}{2\pi} \ln r$	$\frac{\Gamma}{2\pi} \frac{y}{r^2}$	$-\frac{\Gamma}{2\pi} \frac{x}{r^2}$	0	$-\frac{\Gamma}{2\pi r}$
Doublet :	$\frac{\kappa \cos \theta}{2\pi r}$	$-\frac{\kappa \sin \theta}{2\pi r}$	$\frac{\kappa}{2\pi} \frac{y^2 - x^2}{r^4}$	$-\frac{\kappa}{2\pi} \frac{2xy}{r^4}$	$-\frac{\kappa \cos \theta}{2\pi r^2}$	$-\frac{\kappa \sin \theta}{2\pi r^2}$

Potential of source, vortex sheets:

$$\phi(x,y) = \int_0^\ell \frac{\lambda(s)}{2\pi} \ln r(s,x,y) ds \qquad \phi(x,y) = \int_0^\ell -\frac{\gamma(s)}{2\pi} \theta(s,x,y) ds$$

Jump conditions:

$$\begin{aligned} \text{Source sheet:} \quad & \Delta \vec{V} \cdot \hat{n} = \lambda & \Delta \vec{V} \cdot \hat{s} = 0 \\ \text{Vortex sheet:} \quad & \Delta \vec{V} \cdot \hat{n} = 0 & \Delta \vec{V} \cdot \hat{s} = \gamma \end{aligned}$$

Kutta-Joukowski Theorem, for any 2-D lifting object:

$$L' = \rho V_\infty \Gamma$$

Normal-momentum equation:

$$\frac{\partial p}{\partial n} = -\rho V^2 \kappa \qquad (\kappa = \text{streamline curvature})$$