Fluids – Lecture 3 Notes

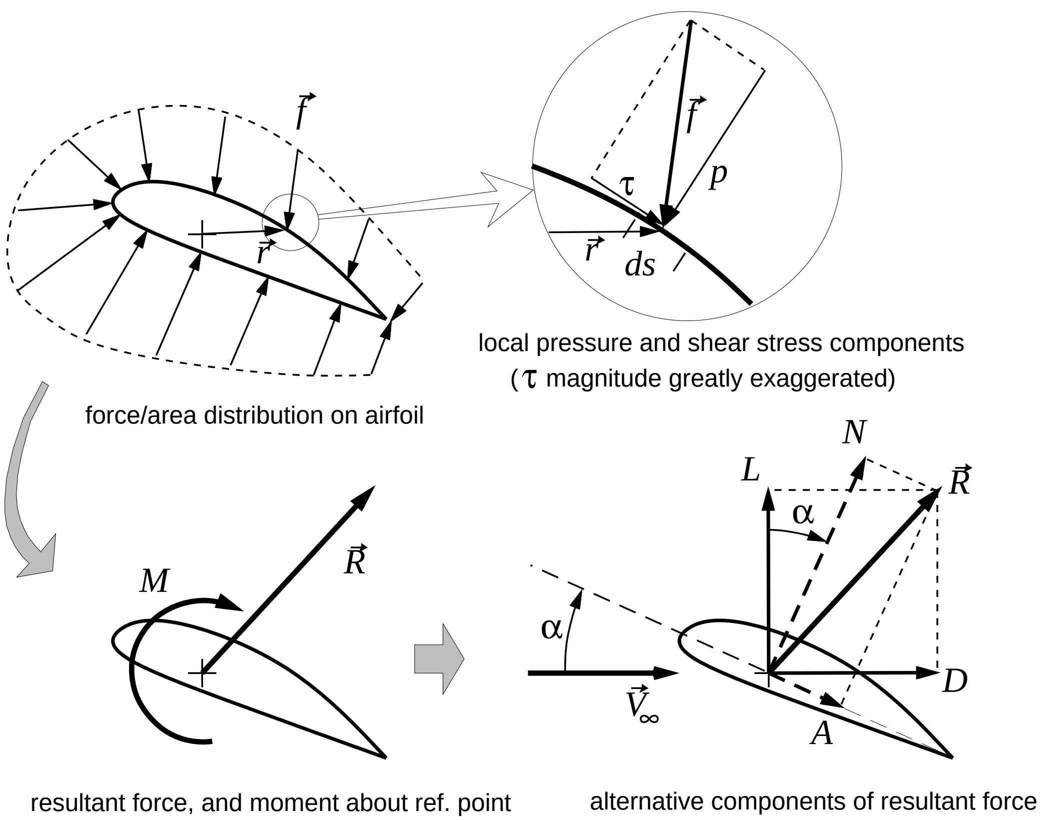
1. 2-D Aerodynamic Forces and Moments
2. Center of Pressure
3. Nondimensional Coefficients

Reading: Anderson 1.5 – 1.6

Aerodynamics Forces and Moments

Surface force distribution

The fluid flowing about a body exerts a local force/area (or stress) $\vec{f}$ on each point of the body. Its normal and tangential components are the pressure p and the shear stress τ.



The figure above greatly exaggerates the magnitude of the τ stress component just to make it visible. In typical aerodynamic situations, the pressure p (or even the relative pressure $p - p_\infty$) is typically greater than τ by at least two orders of magnitude, and so $\vec{f}$ is very nearly perpendicular to the surface. But the small τ often significantly contributes to drag, so it cannot be neglected entirely.

The stress distribution $\vec{f}$ integrated over the surface produces a resultant force $\vec{R}$, and also a moment M about some chosen moment-reference point. In 2-D cases, the sign convention for M is positive nose up, as shown in the figure.

Force components

The resultant force $\vec{R}$ has perpendicular components along any chosen axes. These axes are arbitrary, but two particular choices are most useful in practice.

Freestream Axes: The $\vec{R}$ components are the drag D and the lift L, parallel and perpendicular to $\vec{V}_\infty$.

Body Axes: The $\vec{R}$ components are the axial force A and normal force N, parallel and perpendicular to the airfoil chord line.

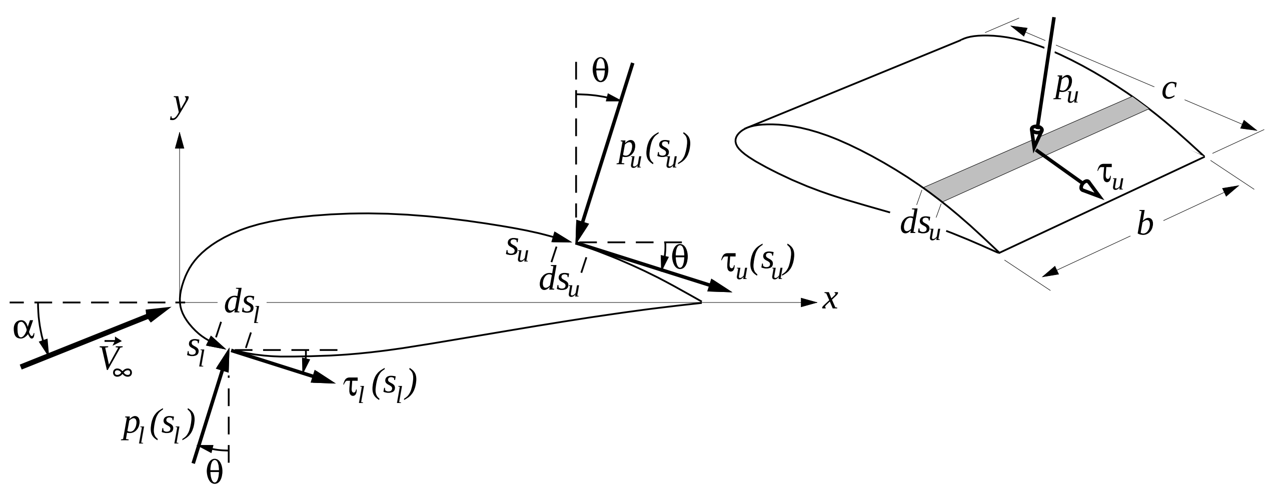
If one set of components is computed, the other set can then be obtained by a simple axis transformation using the angle of attack α. Specifically, L and D are obtained from N and A as follows.

$$\begin{aligned} L &= N\cos\alpha \,-\, A\sin\alpha \\ D &= N\sin\alpha \,+\, A\cos\alpha \end{aligned}$$

Force and moment calculation

A cylindrical wing section of chord c and span b has force components A and N, and moment M. In 2-D it's more convenient to work with the unit-span quantities, with the span dimension divided out.

$$A' \equiv A/b \qquad N' \equiv N/b \qquad M' \equiv M/b$$



On the upper surface, the unit-span force components acting on an elemental area of width ds_u are

$$\begin{aligned} dN'_u &= (-p_u \cos\theta \,-\, \tau_u \sin\theta)\; ds_u \\ dA'_u &= (-p_u \sin\theta \,+\, \tau_u \cos\theta)\; ds_u \end{aligned}$$

And on the lower surface they are

$$\begin{aligned} dN'_\ell &= (p_\ell \cos\theta \,-\, \tau_\ell \sin\theta)\; ds_\ell \\ dA'_\ell &= (p_\ell \sin\theta \,+\, \tau_\ell \cos\theta)\; ds_\ell \end{aligned}$$

Integration from the leading edge to the trailing edge points produces the total unit-span forces.

$$\begin{aligned} N' &= \int_{\mathrm{LE}}^{\mathrm{TE}} dN'_u \,+\, \int_{\mathrm{LE}}^{\mathrm{TE}} dN'_\ell \\ A' &= \int_{\mathrm{LE}}^{\mathrm{TE}} dA'_u \,+\, \int_{\mathrm{LE}}^{\mathrm{TE}} dA'_\ell \end{aligned}$$

The moment about the origin (leading edge in this case) is the integral of these forces, weighted by their moment arms x and y, with appropriate signs.

$$M'_{\rm LE} \;=\; \int_{\rm LE}^{\rm TE} -x\, dN'_u \;+\; \int_{\rm LE}^{\rm TE} -x\, dN'_\ell \;+\; \int_{\rm LE}^{\rm TE} y\, dA'_u \;+\; \int_{\rm LE}^{\rm TE} y\, dA'_\ell$$

From the geometry, we have

$$ds\, \cos\theta \;=\; dx \qquad\qquad ds\, \sin\theta \;=\; -dy \;=\; -\frac{dy}{dx}\, dx$$

which allows all the above integrals to be performed in x, using the upper and lower shapes of the airfoil $y_u(x)$ and $y_\ell(x)$. Anderson 1.5 has the complete expressions.

Simplifications

In practice, the shear stress τ has negligible contributions to the lift and moment, giving the following simplified forms.

$$L' \;=\; \cos\alpha \int_0^c (p_\ell - p_u)\, dx \;+\; \sin\alpha \int_0^c \left(p_\ell \frac{dy_\ell}{dx} \;-\; p_u \frac{dy_u}{dx} \right) dx$$

$$M'_{\rm LE} \;=\; \int_0^c \left[p_u \left(x + \frac{dy_u}{dx} y_u \right) \;-\; p_\ell \left(x + \frac{dy_\ell}{dx} y_\ell \right) \right] dx$$

A somewhat less accurate but still common simplification is to neglect the $\sin\alpha$ term in L', and the dy/dx terms in M'.

$$L' \;\simeq\; \int_0^c (p_\ell - p_u)\, dx$$

$$M'_{\rm LE} \;\simeq\; \int_0^c -(p_\ell - p_u)\, x\, dx$$

The shear stress τ <u>cannot</u> be neglected when computing the drag D' on streamline bodies such as airfoils. This is because for such bodies the integrated contributions of p toward D' tend to mostly cancel, leaving the small contribution of τ quite significant.

Center of Pressure

Definition

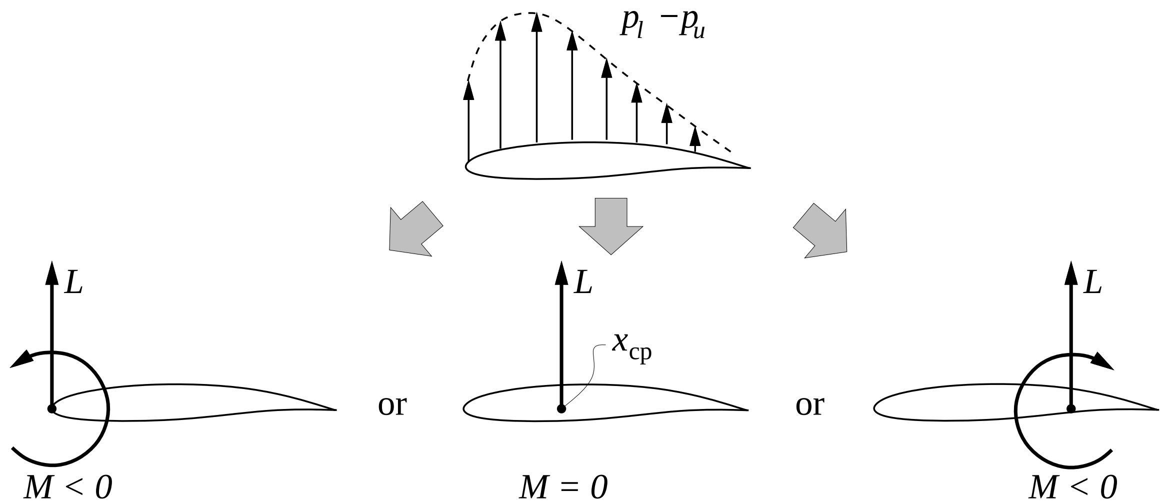
The value of the moment M' depends on the choice of reference point. Using the simplified form of the $M_{\rm LE}$ integral, the moment $M_{\rm ref}$ for an arbitrary reference point $x_{\rm ref}$ is

$$\begin{aligned} M'_{\rm ref} &= \int_0^c -(p_\ell - p_u)\,(x - x_{\rm ref})\, dx \\ &= M'_{\rm LE} \;+\; L' x_{\rm ref} \end{aligned}$$

This can be positive, zero, or negative, depending on where $x_{\rm ref}$ is chosen, as illustrated in the figure.

At one particular reference location $x_{\rm cp}$, called the *center of pressure*, the moment is defined to be zero.

$$\begin{aligned} M'_{\rm cp} &= M'_{\rm LE} \;+\; L' x_{\rm cp} \;\equiv\; 0 \\ x_{\rm cp} &= -M'_{\rm LE}/L' \end{aligned}$$



The center of pressure asymptotes to $+\infty$ or $-\infty$ as the lift tends to zero. This awkward situation can easily occur in practice, so the center of pressure is rarely used in aerodynamics work.

For reasons which will become apparent when airfoil theory is studied, it is advantageous to define the "standard" location for the moment reference point of an airfoil to be at its quarter-chord location, or $x_{\mathrm{ref}} = c/4$. The corresponding standard moment is usually written without any subscripts.

$$M'_{c/4} \;\equiv\; M' \;=\; \int_0^c -(p_\ell - p_u)\,(x - c/4)\,dx$$

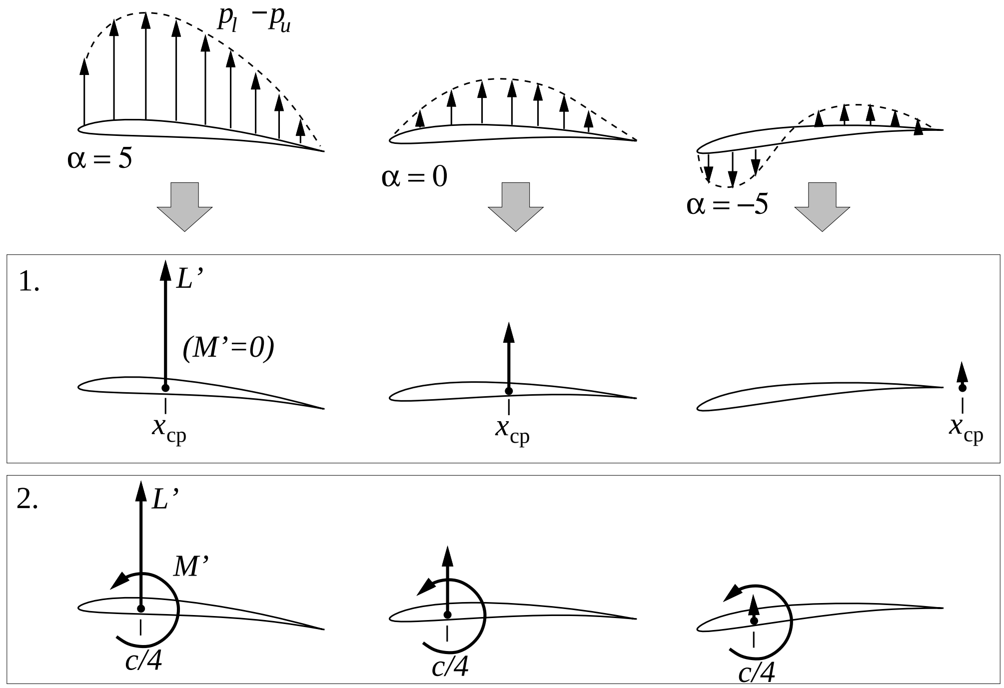
Aerodynamic Conventions

As implied above, the aerodynamicist has the option of picking any reference point for the moment. The lift and the moment then represent the integrated $p_l - p_u$ distribution. Consider two possible representations:

1. A resultant lift L' acts at the center of pressure $x = x_{\mathrm{cp}}$. The moment about this point is zero by definition: $M'_{\mathrm{cp}} = 0$. The x_{cp} location moves with angle of attack in a complicated manner.

2. A resultant lift L' acts at the fixed quarter-chord point $x = c/4$. The moment about this point is in general nonzero: $M'_{c/4} \neq 0$.

The figure shows how the L', M', and x_{cp} change with angle of attack for a typical cambered airfoil. Note that with representation 1, the x_{cp} location moves off the airfoil and tends to $+\infty$ as L' approaches zero. Fixing the moment reference point, as in representation 2, is a simpler and preferable approach. Choosing the quarter-chord location for this is especially attractive, since M' then shows little or no dependence on the angle of attack. This surprising fact will come from a more detailed airfoil analysis later in the course.



Nondimensional Coefficients

The forces and moment depend on a large number of geometric and flow parameters. It is often advantageous to work with nondimensionalized forces and moment, for which most of these parameter dependencies are scaled out. For this purpose we define the following reference parameters:

Reference area:	S
Reference length:	ℓ
Dynamic pressure:	$q_\infty = \frac{1}{2}\rho V_\infty^2$

The choices for S and ℓ are arbitrary, and depend on the type of body involved. For aircraft, traditional choices are the wing area for S, and the wing chord or wing span for ℓ. The nondimensional force and moment coefficients are then defined as follows:

Lift coefficient: $$C_L \equiv \frac{L}{q_\infty S}$$

Drag coefficient: $$C_D \equiv \frac{D}{q_\infty S}$$

Moment coefficient: $$C_M \equiv \frac{M}{q_\infty S \ell}$$

For 2-D bodies such as airfoils, the appropriate reference area/span is simply the chord c, and the reference length is the chord as well. The *local* coefficients are then defined as follows.

Local Lift coefficient: $$c_\ell \equiv \frac{L'}{q_\infty \, c}$$

Local Drag coefficient: $$c_d \equiv \frac{D'}{q_\infty \, c}$$

Local Moment coefficient: $$c_m \equiv \frac{M'}{q_\infty \, c^2}$$

These local coefficients are defined for each spanwise location on a wing, and may vary across the span. In contrast, the C_L, C_D, C_M are single numbers which apply to the whole wing.