Fluids – Lecture 17 Notes

1. Flowfield prediction
2. Source Sheets

Reading: Anderson 3.17

Flowfield Prediction

Problem definition

The flowfield examples used so far were used to demonstrate the basic ideas behind the method of superposition. We chose some combination of elementary flows (uniform flow, sources, vortices, etc.), and then determined the resulting flowfield. The corresponding body shape was determined from the shape of the dividing streamline. However, such an approach is not practical for engineering applications, where we want to specify the body shape, rather than have it as an outcome. The problem can therefore be stated as follows.

Given: Body shape $Y(x)$, Freestream velocity $\vec{V}_\infty$

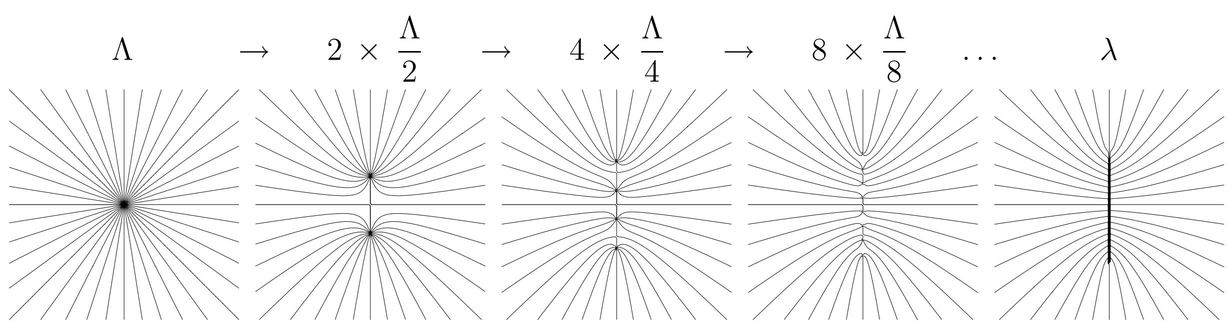
Determine: Superposition of suitable elementary flows which produce the velocity field $\vec{V}(x,y)$ about the body.

It turns out that sources, vortices, and doublets are not ideally suited to this task because of their strong singularities. The constraint that these singularities must be inside the body is difficult to meet, especially if the body is very slender. For this reason we now define slightly more elaborate elementary flows which are smoother, and therefore better suited to representing smooth bodies.

Source Sheets

Definition

Consider a sequence of flows where a single source of strength Λ is repeatedly subdivided into smaller sources which are evenly distributed along a line segment of length ℓ. The limit of this subdivision process is a *source sheet* of strength $\lambda = \Lambda/\ell$.



The units of Λ are length^2/time, while the units of λ are length/time (or velocity). Note that the total source strength is not changed in this process.

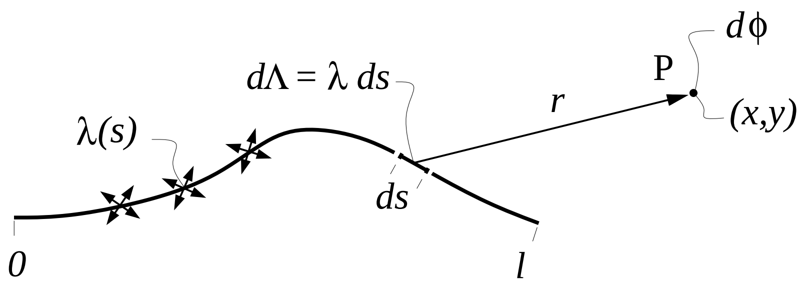
The limiting process shown above has assumed that the sheet is straight, and that the sources are uniformly subdivided and uniformly distributed along the sheet. Neither of these assumptions are required. The subdivided sources can be distributed along any chosen curve, in any chosen density. Hence, the source sheet can be curved, and its strength λ can vary along the sheet.

Properties

Consider an infinitesimal length ds of the sheet. The infinitesimal source strength of that piece is $d\Lambda = \lambda\, ds$, and the corresponding potential at some field point P at (x, y) is

$$d\phi \;=\; \frac{d\Lambda}{2\pi} \ln r \;=\; \frac{\lambda}{2\pi} \ln r \; ds$$

where r is the distance between point (x, y) and the point on the sheet.

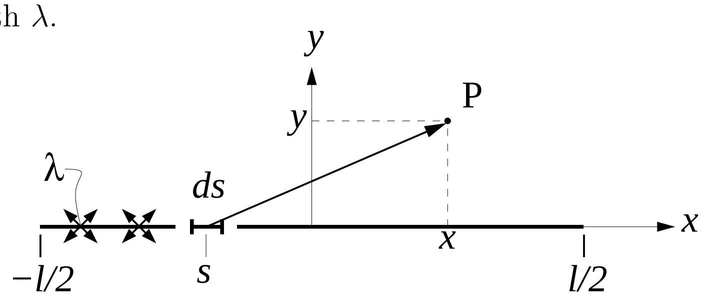


The potential of the entire sheet at point P is then obtained by integrating the infinitesimal contributions all along the sheet.

$$\phi(x, y) \;=\; \int_0^{\ell} \frac{\lambda}{2\pi} \ln r \; ds$$

The shape of the sheet and the $\lambda(s)$ distribution must be specified before this integral can be evaluated. The velocity of the sheet is then obtained by taking the gradient of the result. Note that to build up the entire flowfield, the integral must be evaluated for each point P in the xy plane. In practice this is not necessary, since for engineering purposes the velocity is required only at a small set of points, such as on the surface of a body to allow computation of the pressure and the resultant force.

Consider now a simpler straight source sheet extending from $(-\ell/2, 0)$ to $(\ell/2, 0)$, with a constant strength λ.



The potential and the velocity components at point P are given by

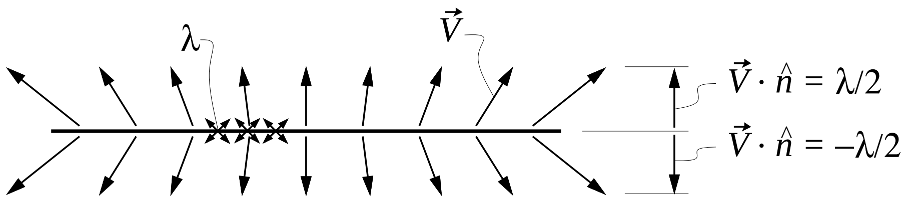
$$\phi(x, y) \;=\; \frac{\lambda}{2\pi} \int_{-\ell/2}^{\ell/2} \ln \sqrt{(x-s)^2 + y^2} \; ds \tag{1}$$

$$u(x, y) \;=\; \frac{\partial \phi}{\partial x} \;=\; \frac{\lambda}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{\partial}{\partial x} \left[\ln \sqrt{(x-s)^2 + y^2} \right] ds \;=\; \frac{\lambda}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{x - s}{(x-s)^2 + y^2} \, ds$$

$$v(x, y) \;=\; \frac{\partial \phi}{\partial y} \;=\; \frac{\lambda}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{\partial}{\partial y} \left[\ln \sqrt{(x-s)^2 + y^2} \right] ds \;=\; \frac{\lambda}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{y}{(x-s)^2 + y^2} \, ds$$

These integrals can be evaluated, although the resulting expressions are cumbersome, and not too important for our purposes here. The really interesting result is for the normal velocity $v(x,y)$ very close to the sheet, either just above at $y = 0^+$, or just below at $y = 0^-$. After the necessary integration, we find that

$$v(x, 0^+) \;=\; \frac{\lambda}{2} \quad , \qquad v(x, 0^-) \;=\; -\frac{\lambda}{2} \qquad\qquad \text{(flat, isolated source sheet)}$$

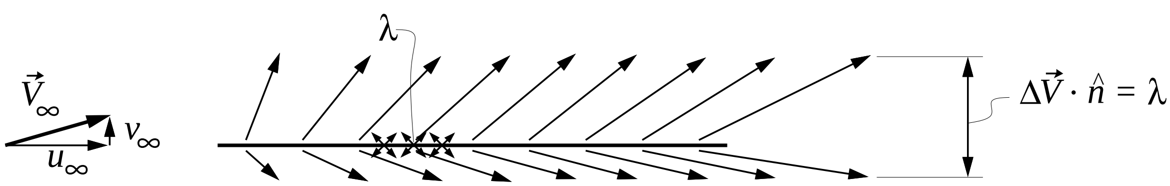


The normal velocity is then simply a constant $\lambda/2$ directed outward. But if any other singularity or freestream is present, this additional velocity will be superimposed on each side of the sheet. For example, if the sheet is immersed in a freestream, we will have

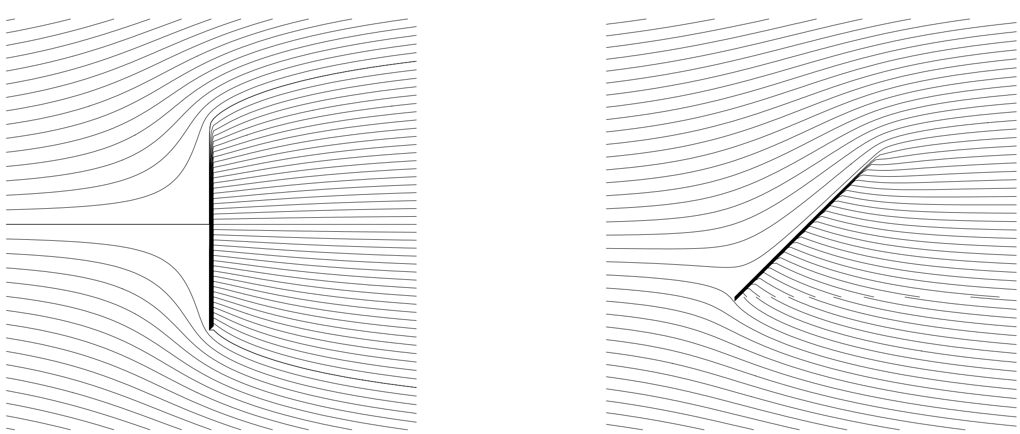
$$v(x, 0^+) \;=\; \frac{\lambda}{2} + v_\infty \qquad , \qquad v(x, 0^-) \;=\; -\frac{\lambda}{2} + v_\infty$$

By taking the difference between the top and bottom points, any such additional velocity is removed, giving the very general *normal-velocity jump condition* for any source sheet in any situation.

$$v(x, 0^+) \;-\; v(x, 0^-) \;\equiv\; \Delta\vec{V} \cdot \hat{n} \;=\; \lambda \tag{2}$$



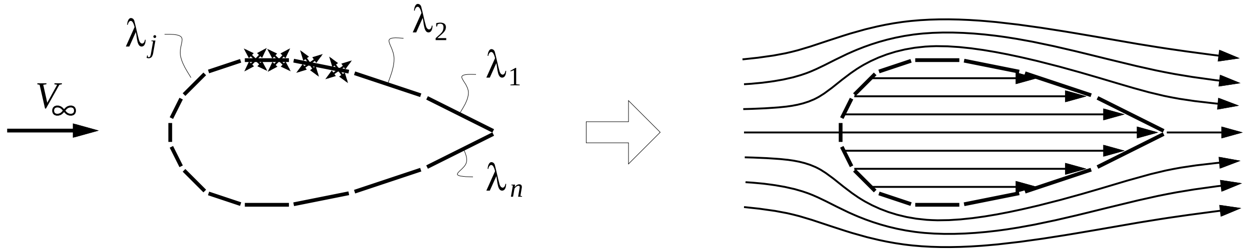
The advantage of using source sheets rather than sources to represent a flowfield is illustrated in the figure below, which shows source sheets superimposed on a uniform flow to the right. In each case the sheet's strength λ is set so as to cancel the freestream's component normal to the sheet, giving a net zero normal flow. Hence, the sheet is ideally suited for representing a solid surface of a body, since it can impose the physically necessary flow-tangency condition $\vec{V} \cdot \hat{n} = 0$ by suitably adjusting the sheet's strength λ.



Modeling approach

The fact that the velocity field of a source sheet is smooth, without the troublesome $1/r$

singularity of a point source, allows us to place some number of such sheets (or *panels*) end to end on the surface of the body. We then determine the strengths λ_j of all the panels $j = 1, 2, \ldots n$ such that the flow is tangent everywhere on the surface of the body. The superposition also incidentally produces some flow inside the body, but this is not physical and is simply ignored.



uniform flow + n source panels   resulting flowfield

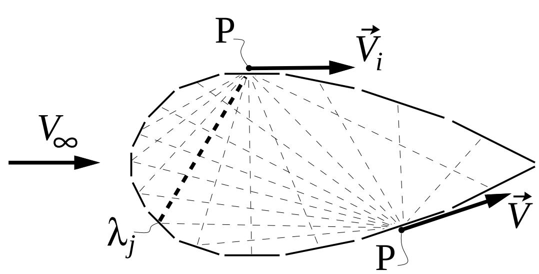
This use of source sheets in this manner to represent a flow is the basis of the *panel method*, which is widely used to compute the flow about aerodynamic bodies of arbitrary shape. The approach presented here is actually suitable only for non-lifting bodies such as fuselages. For airfoils, wings, and other lifting bodies, vortices must be added in some form to enable circulation to be represented. This modification will be treated later.

Solution technique

It is important to realize that each panel strength λ_j cannot be set independently of the others. With more than one panel present, the velocity $\vec{V}$ and hence the flow tangency condition $\vec{V} \cdot \hat{n} = 0$ at any point i on the surface is influenced not only by that panel's λ_i, but also by the strengths λ_j of all the other panels. In tensor notation this can be written as

$$\left(\vec{V} \cdot \hat{n}\right)_i \; = \; A_{ij}\, \lambda_j \; + \; \vec{V}_\infty \cdot \hat{n}_i$$

where A_{ij} is called the *aerodynamic influence matrix*, which can be computed once the geometry of all the panels is decided.



Requiring that $\vec{V} \cdot \hat{n} = 0$ for each of the n panel midpoints gives the following.

$$A_{ij}\, \lambda_j \; = \; -\vec{V}_\infty \cdot \hat{n}_i$$

This is a $n \times n$ linear system for the λ_j unknowns, which can be solved numerically using matrix solution methods such as Gaussian elimination. With the λ_j determined, the velocity and pressure (via Bernoulli) can then be computed at any point in the flowfield and on the surface of the body. Forces are then computed by integrating the surface pressures. This completes the aerodynamic analysis problem.