

# Fluids – Lecture 18 Notes

1. Prediction of Lift
2. Vortex Sheets

Reading: Anderson 3.17

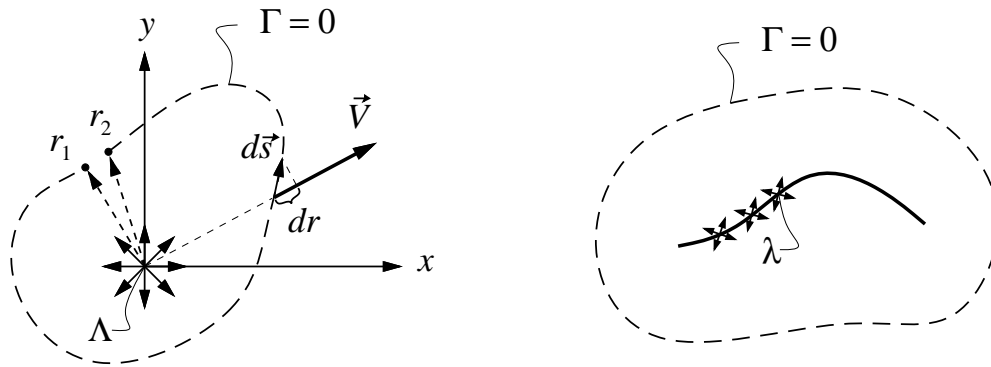
## Prediction of Lift

### Limitations of Source Sheets

A point source has zero circulation about any circuit. Evaluating  $\Gamma$  using its definition we have

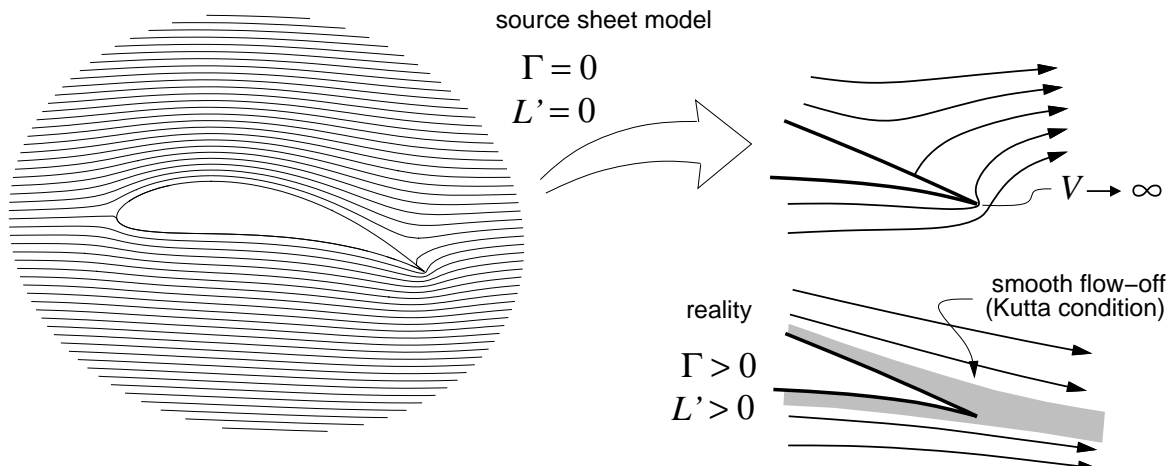
$$\Gamma \equiv - \oint \vec{V} \cdot d\vec{s} = - \oint V_r dr = - \int_{r_1}^{r_2} \frac{\Lambda}{2\pi r} dr = - \frac{\Lambda}{2\pi} (\ln r_2 - \ln r_1) = 0$$

which gives zero simply because  $r_1 = r_2$  for any closed circuit, whether the origin is enclosed or not. A source sheet, which effectively consists of infinitesimal sources, must have zero circulation as well.



This zero-circulation property of source sheets has severe consequences for flow representation. Any aerodynamic model consisting only of a freestream and superimposed source sheets will have  $\Gamma = 0$ , and hence  $L' = 0$  as well. Hence, lifting flows cannot be represented by source sheets alone.

This limitation is illustrated if we use source panels to model a flow expected to produce lift, such as that on an airfoil at an angle of attack. Examination of the streamlines reveals that the rear dividing streamline leaves the airfoil off one surface as shown in the figure. The model also predicts an infinite velocity going around the sharp trailing edge.

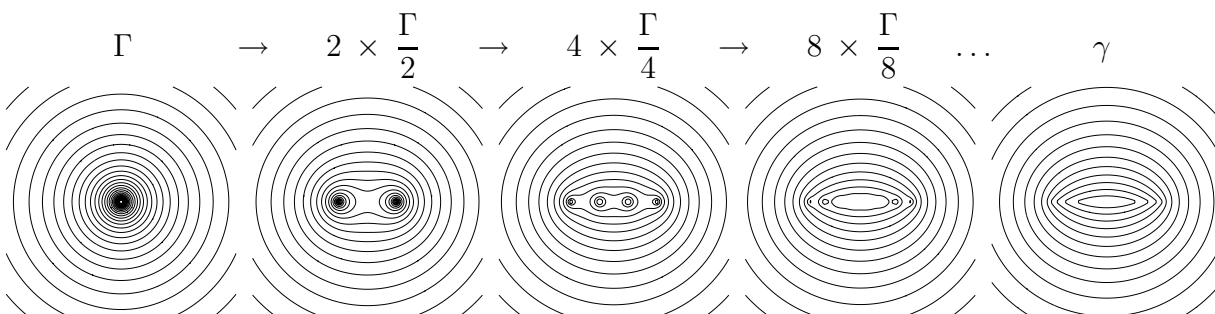


On real airfoils the flow always flows smoothly off the sharp trailing edge, with no large local velocities. This smooth flow-off is known as the *Kutta condition*, and it must be faithfully duplicated in any flow model which seeks to predict the lift correctly. Changing the streamline pattern to force the flow smoothly off the trailing edge requires the addition of circulation, which implies that vortices must be included in the flow representation in some manner.

## Vortex Sheets

### Definition

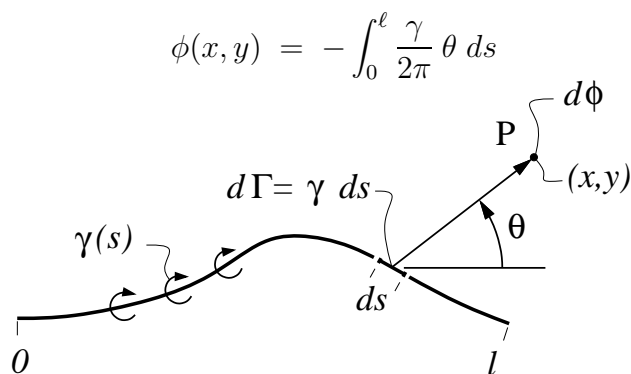
Consider a sequence of flows where a single vortex of strength  $\Gamma$  is repeatedly subdivided into smaller vortices which are evenly distributed along a line segment of length  $\ell$ . The limit of this subdivision process is a *vortex sheet* of strength  $\gamma = \Gamma/\ell$ .



Like with the source sheet strength  $\lambda$ , the units of  $\gamma$  are length/time (or velocity).

### Properties

The analysis of the vortex sheet closely follows that of the source sheets. The potential of the vortex sheet at point P is

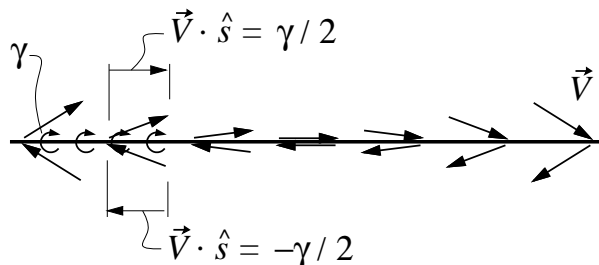


For a straight vortex sheet extending from  $(-\ell/2, 0)$  to  $(\ell/2, 0)$ , with a constant strength  $\gamma$ , the potential and the velocity components at point P are given by

$$\begin{aligned} \phi(x, y) &= \frac{\gamma}{2\pi} \int_{-\ell/2}^{\ell/2} -\arctan \frac{y}{x-s} ds \\ u(x, y) &= \frac{\partial \phi}{\partial x} = \frac{\gamma}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{\partial}{\partial x} \left[ -\arctan \frac{y}{x-s} \right] ds = \frac{\gamma}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{y}{(x-s)^2 + y^2} ds \\ v(x, y) &= \frac{\partial \phi}{\partial y} = \frac{\gamma}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{\partial}{\partial y} \left[ -\arctan \frac{y}{x-s} \right] ds = \frac{\gamma}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{-x}{(x-s)^2 + y^2} ds \end{aligned}$$

As with the earlier source sheets, these integrals are cumbersome to evaluate in general. But if we evaluate very close to the sheet, either just above at  $y = 0^+$ , or just below at  $y = 0^-$ , the tangential velocity becomes very simple.

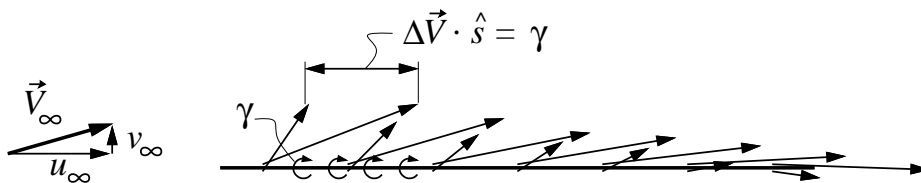
$$u(x, 0^+) = \frac{\gamma}{2} \quad , \quad u(x, 0^-) = -\frac{\gamma}{2} \quad (\text{flat, isolated vortex sheet})$$



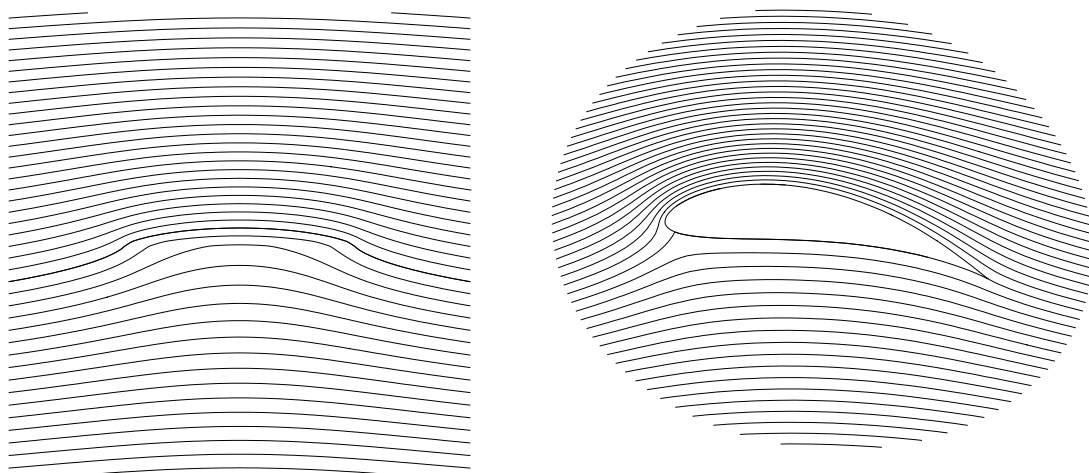
The tangential velocity is then simply a constant  $\gamma/2$  directed clockwise around the sheet. By taking the difference between the upper and lower points at some  $x$  location, we obtain a very general tangential-velocity jump condition for any vortex sheet in any situation.

$$u(x, 0^+) - u(x, 0^-) \equiv \Delta \vec{V} \cdot \hat{s} = \gamma \quad (1)$$

The figure shows the vortex sheet with a freestream superimposed. The surface velocity vector pattern is very complicated, but the tangential velocity jump across the sheet is a constant equal to  $\gamma$  at all points.



The advantage of using vortex sheets rather than vortices to represent a flowfield is illustrated in the figure below. The left figure shows a vortex sheet superimposed on a uniform flow. The vortex sheets smoothly deforms the flowfield in the manner required to impose circulation and lift. The right figure shows the same airfoil as before, but now vortex panels have been used instead of source panels to represent the flow. The nature of the vortex panels permits the Kutta condition to be imposed, giving smooth flow off the trailing edge. The airfoil now has the expected amount of lift.



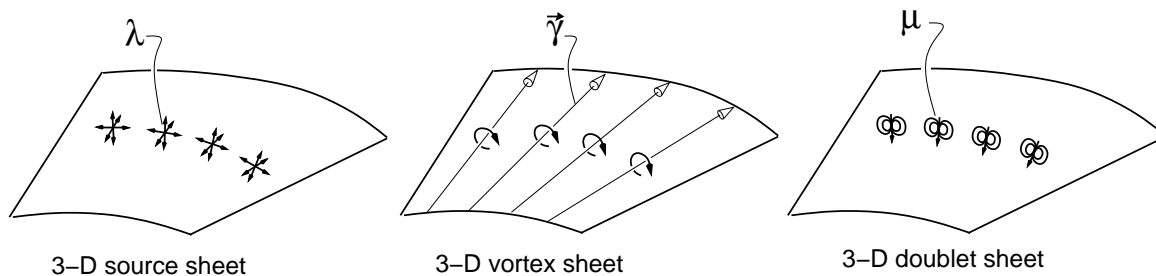
## Modeling approach and solution technique

As illustrated with the airfoil example, vortex panels provide an alternative way to model the flow about a body, both for lifting and non-lifting bodies. The solution approach is nearly the same as with source panels. The  $\vec{V} \cdot \hat{n} = 0$  flow tangency condition is imposed for each panel, but now the additional Kutta condition at the trailing edge is also imposed. The resulting linear system is then solved for all the panel strengths  $\gamma_j$ . The surface velocities, surface pressures, and overall forces can then be computed.

## Types of panels used in practice

Vortex panels are by far the most widely used for 2-D problems, such as the flow about an airfoil. Vortex panels can represent lifting or nonlifting flows equally well, so there is little reason to use the more restrictive source panels.

For 3-D problems, however, vortex panels run into serious difficulties. The main problem is that the sheet strength  $\vec{\gamma}$  is now a vector lying in the sheet. The associated tangential velocity of magnitude  $\gamma/2$  is also in the sheet, and perpendicular to  $\vec{\gamma}$ . In contrast, the source panel strength  $\lambda$  is still a scalar in 3-D.



Because  $\vec{\gamma}$  is now a vector, it is not well suited for solution in a panel method. Instead, 3-D panel methods employ *doublet sheets*, whose doublet strength  $\mu$  (unrelated to viscosity) is a scalar quantity, and can also represent lifting flows. The figure above conceptually shows a doublet sheet. The axis of each infinitesimal doublet is oriented normal to the sheet, rather than along the  $x$ -axis as in our previous examples.

Doublet sheets alone are sufficient to represent the flow about any lifting or nonlifting 3-D body. However, most modern 3-D panel methods actually employ a combination of source sheets and doublet sheets. Compared to using only doublet sheets, the source+doublet sheet combination turns out to give the best accuracy for a given computational time. The details of such combined methods are far beyond scope here.