Lecture F10 Mud: Momentum Theorem Applications
(37 respondents)

1. **What's the difference between $D()/Dt$ and $d()/dt$?** (1 student)
What we're after is the time rate of change of some flowfield quantity like $p(x, y, z, t)$, as seen by a sensor moving as a fluid element. Just writing dp/dt is very sloppy math, since p depends on four variables x, y, z, t, not just on t. So we must work with the four partial derivatives of p. The correct expression for the time rate of change seen by the sensor is
$$\frac{\partial p}{\partial t} \; + \; u\frac{\partial p}{\partial x} \; + \; v\frac{\partial p}{\partial y} \; + \; w\frac{\partial p}{\partial z}$$
but this is long and tedious to write. So we use the shorthand Dp/Dt for this whole expression.

2. **Not clear how $D\rho/Dt$ differs from $\partial\rho/\partial t$.** (2 students)
The partial derivative $\partial\rho/\partial t$ merely gives the rate of change of density seen by fixed sensor sitting at one particular x, y, z location. But we're after the rate seen by a sensor which <u>moves</u> with the flow velocity, so the sensor's x, y, z are continually changing. Using the chain rule then brings in the three spatial partial derivatives $\partial\rho/\partial x, y, z$ whose contributions are added to the $\partial\rho/\partial t$ term. See point 1 above.

3. **Why does time dependence depend on where you're at?** (1 student)
Look at the first figure in the F10 notes and try to see why the $p_1(t)$ signal has a different shape than the $p_2(t)$ signal. In particular, we're after the t derivative of a signal, which are different for the two sensors, even when the sensors are instantaneously at the same point.

4. **What is the substantial derivative used for?** (3 students)
A physicist might use it for formulating equations which describe some physical process happening to a fluid element (combustion, precipitate formation, heat conduction, . . .). An experimentalist might use it to determine fluid accelerations from the $\vec{V}(x, y, z, t)$ field measured in the lab frame. A computational aerodynamicist might use it to formulate the equations of motion to allow numerical solution, etc, etc, etc.

5. **What are some example applications?** (2 students)
In the current problem set you will use D/Dt to deduce the pressure field from a given velocity field. We will come back to it occasionally later in the course.

6. **What is $\vec{F}_{\text{viscous}}$? How do you calculate it?** (2 students)
$\vec{F}_{\text{viscous}}$ is the force applied to the control volume boundary by viscous stresses, the main one being the viscous shear stress τ. The full expression for $\vec{F}_{\text{viscous}}$ using the velocities and viscosity is long and tedious, and not really important at this point. We will look at it in detail later in the course when dealing with viscous flows.

7. **How do you know that $-\partial p/\partial x + \rho g_x + (F_x)_{\text{viscous}}$ is force/volume?** (1 student)
First of all, the units of each term is N/m^3, which is one clue. Second, the pressure gradient also appears in the primary hydrostatic buoyancy equation:
$$(F_x)_{\text{buoyancy}} \;=\; -(\partial p/\partial x) \times \text{volume}$$

which clearly shows that $-\partial p/\partial x$ is equal to the applied force/volume.

8. **Clarify the interpretations of Control-Volume vs. Substantial Derivative forms of continuity.** (3 students)
   Look at the last figure in the F10 notes to help visualization. See also mud point 2 above. The control-volume (CV) form is

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \vec{V}\right)$$

   or ... "The rate of change of mass inside a fixed CV is equal to the mass flow out of the CV, all per unit volume"

   The substantial derivative form is

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla \cdot \vec{V}$$

   or ... "Fractional rate of change of density of a moving fluid element is equal and opposite to the expansion rate of the fluid element"

9. **When asked to find the derivative (.e.g acceleration), will it be specified what the sensor motion should be?** (1 student)
   The substantial derivative (sensor moves with local $\vec{V}$) is the only physically-meaningful way to get the acceleration of the fluid, if you want to apply $F = ma$ for instance. In practice, physical considerations dictate how the derivative should be defined.

10. **Can the substantial derivative be used to get 2nd derivatives?** (1 student)
    Yes, but it's not necessary in practice, since second time derivatives do not appear in physically-derived equations of fluid motion. Second derivatives like $\partial^2()/\partial t^2$ might appear, but only after some elaborate mathematical manipulations of the equations. In 8.01, for example, $F = ma$ could have been written as $dF/dt = m\, da/dt = m\, d^2u/dt^2$, but the d^2u/dt^2 here is completely artificial.

11. **Many sections in Chapter 3 of Anderson apply to Pset questions. Can you add these to the reading list?** (1 student)
    Chapter 2 is for "tool building", while Chapters 3 onward are more concerned with applications of these tools. If you find it helpful to read ahead, then by all means do so. It's not practical for me to assign reading not only for the basic material, but also all later applications which appear in the book. The reading list for each lecture would be huge, with lots of material extraneous to what we're trying to focus on.

12. **No mud** (15 students)