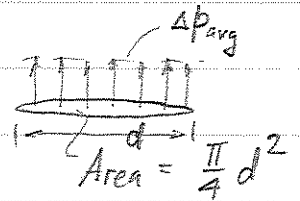


$$a) \text{ weight} = mg = 0.175 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 1.715 \text{ N}$$

$$\text{weight} = \text{lift} = \Delta p_{\text{avg}} \cdot \frac{\pi}{4} d^2 = mg$$

$$\Rightarrow \boxed{\Delta p_{\text{avg}} = \frac{mg}{\frac{\pi}{4} d^2} = \frac{1.715 \text{ N}}{\frac{\pi}{4} \cdot (0.27 \text{ m})^2} = 29.95 \text{ Pa}}$$



$$d = 0.27 \text{ m}$$

$$b) \text{ Assume } \frac{1}{2} \rho V_{\infty \text{ min}}^2 \approx \Delta p_{\text{avg}}$$

$$\boxed{V_{\infty \text{ min}} = \sqrt{\frac{2 \Delta p_{\text{avg}}}{\rho}} = \sqrt{\frac{2 \cdot 29.95 \text{ Pa}}{1.22 \text{ kg/m}^3}} = 7.0 \text{ m/s}}$$

$$\boxed{= 15.6 \text{ mph}}$$

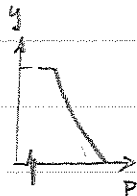
a) Hydrostatic Eq'n:  $dp/dy = -\rho g = -[\rho_0 + (\rho_1 - \rho_0) \frac{y}{h}] g$

$$dp = -[\rho_0 + (\rho_1 - \rho_0) \frac{y}{h}] g dy$$

$$P = -[\rho_0 y + (\rho_1 - \rho_0) \frac{1}{2} \frac{y^2}{h}] g + C$$

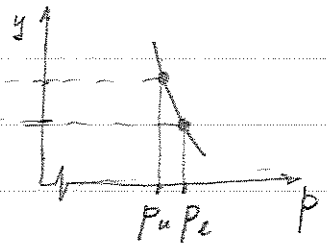
Evaluate C: At  $y=h$ :  $P_1 = -[\rho_0 h + (\rho_1 - \rho_0) \frac{1}{2} h] g + C = -[\frac{1}{2}(\rho_1 + \rho_0) h] g + C$

$$P(y) = P_1 + \frac{1}{2}(\rho_1 + \rho_0) h g - [\rho_0 y + (\rho_1 - \rho_0) \frac{1}{2} \frac{y^2}{h}] g \quad \text{quadratic in } y$$



b) Method 1:  $F = (P_c - P_u) A$ ,  $P_c = P(y=4.5)$ ,  $P_u = P(y=5.5)$ ,  $A = 1 \text{ cm}^2$

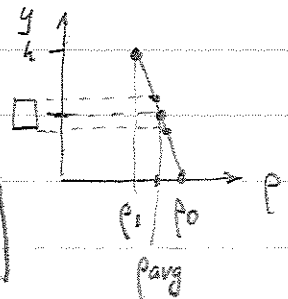
The expression gets a bit messy...



Method 2: Use Archimedes' Principle

$$F = \text{weight of displaced fluid} = \rho_{\text{avg}} \cdot V \cdot g$$

For cube centered on  $y = 5 \text{ cm} = h/2$ ,  $\rho_{\text{avg}} = \frac{1}{2}(\rho_0 + \rho_1) = 1.1 \text{ g/cm}^3$

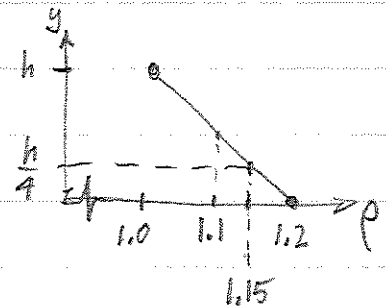


$$F = 1.1 \text{ g/cm}^3 \cdot 1 \text{ cm}^3 \cdot 980 \text{ cm/s}^2 = 1078 \text{ dyn} = 0.01078 \text{ N}$$

c) Look for  $y$  position where  $F = \text{weight} = mg = 1.15 \text{ g} \cdot 980 \text{ cm/s}^2 = 1127 \text{ dyn}$   
 Or equivalently, where  $\rho(y) = \rho_{\text{avg}} = \rho_{\text{cube}} = 1.15 \text{ g/cm}^3$

By inspection,  $\rho = 1.15 \text{ g/cm}^3$  occurs at  $y = \frac{1}{4} h = 2.5 \text{ cm}$

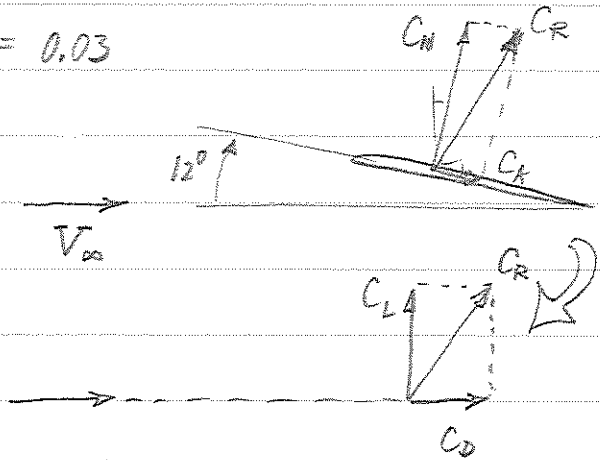
$$\text{Cube will come to rest centered on } y = 2.5 \text{ cm}$$



a) Anderson 1.5:  $\alpha = 12^\circ$ ,  $C_N = 1.2$ ,  $C_A = 0.03$

$$C_L = C_N \cos \alpha - C_A \sin \alpha = 1.1675$$

$$C_D = C_N \sin \alpha + C_A \cos \alpha = 0.2788$$



b) Anderson 1.6: Using moment-translation formula:  $M'_{c/4} = M'_{LE} + \frac{c}{4} L'$   $\frac{1}{\rho \infty c^2}$

or  $C_{m/c4} = C_{m/LE} + \frac{1}{4} C_L$

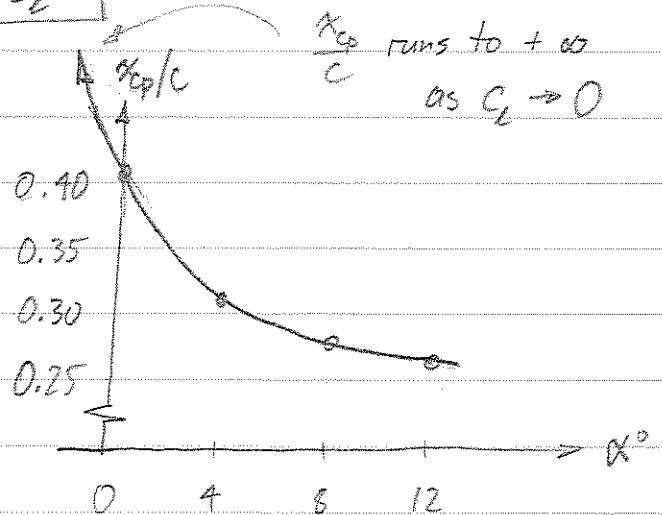
$\therefore C_{m/LE} = C_{m/c4} - \frac{1}{4} C_L$

Equation for  $x_{cp}$ :  $\left\{ x_{cp} = -M'_{LE} / L' \right\} \frac{1}{c}$

or  $x_{cp}/c = -C_{m/LE} / C_L = -\left[ C_{m/c4} - \frac{1}{4} C_L \right] \frac{1}{C_L}$

or  $\frac{x_{cp}}{c} = \frac{1}{4} - \frac{C_{m/c4}}{C_L}$

$\alpha$	$C_L$	$C_{m/c4}$	$\frac{x_{cp}}{c}$
$0^\circ$	0.25	-0.040	0.410
$4^\circ$	0.64	-0.036	0.306
$8^\circ$	1.08	-0.036	0.283
$12^\circ$	1.43	-0.030	0.271



Parameter	Units
$D$	$m/l^2$
$\alpha$	—
$\rho$	$m/l^3$
$V$	$l/t$
$a$	$l/t$
$\mu$	$m/l \cdot t$
$b$	$l$
$c$	$l$
$\delta$	$3$

$N - K = 8 - 3 = 5$  Pi groups expected.

$$\Pi_1 = \frac{D}{\frac{1}{2} \rho V^2 b c} \equiv C_D$$

$$\Pi_2 = \alpha$$

$$\Pi_3 = \frac{\rho V c}{\mu} \equiv Re \quad \text{Reynolds \#}$$

$$\Pi_4 = \frac{V}{a} \equiv M_\infty \quad \text{Mach \#}$$

$$\Pi_5 = \frac{b}{c} \equiv R \quad \text{aspect ratio}$$

Many other alternatives are possible, e.g.

$$\Pi_3 = \frac{\rho a c}{\mu} = \frac{Re}{M_\infty} \quad \text{weird, but OK}$$

$$\text{or } \Pi_3 = \frac{\rho V b}{\mu} = Re \cdot R \quad \text{even more weird}$$

$$\Pi_5 = \frac{c}{b} = \frac{1}{R} \quad \text{unconventional}$$

etc.

**Unified Engineering Problem Set  
Week 2 Fall, 2006**

SOLUTIONS

**M2.1** There are multiple design considerations for all structures and these inevitably involve tradeoffs. The important point here is to identify those considerations which are the most important (i.e. key) to the particular design and thus purpose of that structure. In this vein, there are no "right" answers here since the specific purposes of the structure were not defined. Thus, the first part to answering this question is to define, in your mind, what purpose(s) the structure serves.

It is also important to realize that true tradeoffs (i.e. *relative* importance) can only be done quantitatively when a clear objective is in mind and the factors associated with the different design factors can be quantified. Otherwise, only general statements about tradeoffs can be made. This should become clearer in the "answers" below.

- (a) Space station: The structures of a space station serve a number of important purposes. The habitat structure must protect the inhabitants and thus be resistant to meteorite impact. The station will be in operation for many years and thus must have longevity. In orbit, this means resistance to uv, atomic oxygen, fatigue, and other long-term considerations. In orbit, a structure goes in and out of earth's shadow and thus can undergo very large thermal gradients. Thus, dimensional stability is important. Weight and cost are always factors, but it is hard to evaluate these without knowing the scheme for getting the parts of the space station to orbit and also knowing how important the space station is to the country and thus its priority in the budget process.
- (b) Commercial transport aircraft: With safety as generally the number one consideration, a key item is strength -- ability to carry loads without failure. This also means that there are deformation resistance considerations at various parts of the aircraft. Clearly the wing must hold its shape to a certain margin, otherwise there will be a potential loss in the aerodynamic characteristics and thus the forces on the wing, known as lift and drag. The airplane is designed to last a number of years, so corrosion resistance is important as are considerations of fatigue and general durability. Weight is a clear consideration since this involves a tradeoff with additional payload/fuel/range. Finally the buyers of the aircraft must be able to afford the aircraft, so cost of the airplane is a key (including items such as operating cost), if not *the* key consideration.
- (c) Glider: A glider is also an airplane, but it looks quite different from a commercial transport. Glider wings have a very large "aspect ratio" (length of the wing/"width" of the wing). Due to this great length, glider wings must be very stiff, so deformation resistance is a key consideration with strength being an important design consideration but not as great a consideration as the stiffness/rigidity. Gliders are unpowered and rely on a tow to get to altitude and then on "thermals" to soar higher (Ever watch a hawk or eagle? They do the same thing!). Thus, weight is a

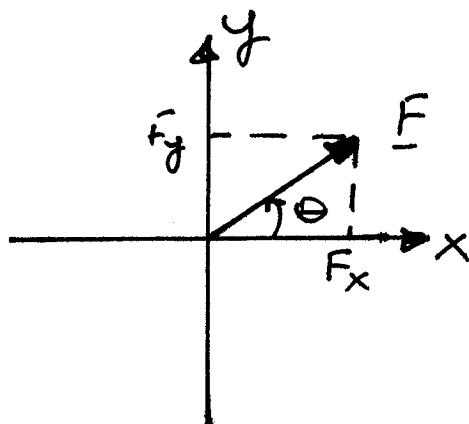
much more critical factor with regard to gliders. How about cost? Clearly, cost is a consideration in any consumer product. However, in the sports industry, many people with the “means” are willing to pay extra for extra performance. so cost becomes quite a different factor here since performance becomes a much greater consideration for which people are willing to pay. Thus, cost/performance becomes a key tradeoff.

- (d) Automobile: The main considerations in an automobile are safety and cost with cost probably being the most important. This is truly a consumer product and the general consumer is much less likely to care about technical innovations and performance capability. They want a product that will work, will get them there, won't break down, and will last. Thus, the next key design consideration is longevity. With care, especially car bodies, this is inherently linked to corrosion resistance. Another key design consideration is energy absorption in impacts (also known as “crashes”). Much of the body and structure of a car is designed to absorb the energy in such an incident in order to protect the occupant(s).
- (e) Bridge: A bridge is a pretty basic structure and the design considerations are also pretty straightforward. The structure must be strong enough to carry the loads without failure. However, the rigidity of the bridge can be an even more important consideration since deflection/ deformation must be resisted (No one wants a “floppy” bridge). Bridges are exposed to the elements, so corrosion resistance is also an important consideration. And this is linked to the final item which is longevity. Bridges are made and used for decades and thus must not fatigue, corrode, etc. This ends up being a tradeoff with the other major consideration -- cost. It is a question of the up-front cost of the structure versus the cost “down the road” to maintain and repair. This should include consideration of the inconvenience caused to commuters, etc. when bridges are closed or traffic restricted when major repairs are made (You should have been around when they did this to the Harvard Bridge a few years ago!).
- (f) Step ladder: As a consumer product, the most important consideration is cost. However, safety must be right behind this, so this brings in considerations of strength, rigidity, and longevity. Longevity is less important here since it is relatively easy enough to replace this product and one can tell when the material starts to corrode, degrade, etc. Stiffness and strength, however, are clearly important. There are also other considerations. One of the main ones is electrical conductivity. Wood ladders were used for many years and then metal ones were introduced. The problem with metal (generally aluminum) ladders is that if they touch a wire, the person touching the ladder can be electrocuted (or at least shocked). Plastic/glass-reinforced ladders have been introduced because of this. They keep down the weight, which is important because people have to be able to carry these things, but they are not conductive and provide that extra electrical safety.

PAL  
9/9/06Unified EngineeringHomework - SOLUTIONS  
week # 2

M 2.2

(a) Resolve each force into  $x$  and  $y$  components with the use of  $\hat{i}$  and  $\hat{j}$  vectors in order to simplify plotting and for describing each as a vector

Note:

components are determined via:

$$F_x = |F| \cos \theta$$

$$F_y = |F| \sin \theta$$

So:  $F_i(x_i, y_i)$   $\xrightarrow{\quad}$   $x$  and  $y$  location in unit of [meters]  
 $\uparrow$  force number

\*NOTE: At times unit vectors are noted via "hats" ( $\hat{\quad}$ ) rather than an underline (or overbar) for a general vector.

$$\underline{F}_1(-2, -1) = (4N) \{ \hat{i} \cos(73.5^\circ) + \hat{j} \sin(73.5^\circ) \}$$

$$= (1.14N) \hat{i} + (3.84N) \hat{j}$$

$$\underline{F}_2(5, -5) = (5N) \{ \hat{i} \cos(135^\circ) + \hat{j} \sin(135^\circ) \}$$

$$= (-3.54N) \hat{i} + (3.54N) \hat{j}$$

$$\underline{F}_3(-3, -4) = (2N) \{ \hat{i} \cos(-31.5^\circ) + \hat{j} \sin(-31.5^\circ) \}$$

$$= (1.42N) \hat{i} + (1.42N) \hat{j}$$

$$\underline{F}_4(1, 1) = (4N) \{ \hat{i} \cos(-106.5^\circ) + \hat{j} \sin(-106.5^\circ) \}$$

$$= (-1.14N) \hat{i} + (-3.84N) \hat{j}$$

$$\underline{F}_5(3, 2) = (3N) \{ \hat{i} \cos(0^\circ) + \hat{j} \sin(0^\circ) \}$$

$$= (3.0N) \hat{i}$$

$$\underline{F}_6(-4, 3) = (3N) \{ \hat{i} \cos(-25^\circ) + \hat{j} \sin(-25^\circ) \}$$

$$= (2.72N) \hat{i} + (-1.27N) \hat{j}$$

So the vector description is given by the magnitudes of the forces in each direction times the unit vectors, with indication of the (x, y) location from which the force vector acts. Summarizing:

$$\underline{F}_1(-2, -1) = (1.14N) \hat{i} + (3.84N) \hat{j}$$

$$\underline{F}_2(5, -5) = (-3.54N) \hat{i} + (3.54N) \hat{j}$$

$$\underline{F}_3(-3, -4) = (1.42N) \hat{i} + (1.42N) \hat{j}$$

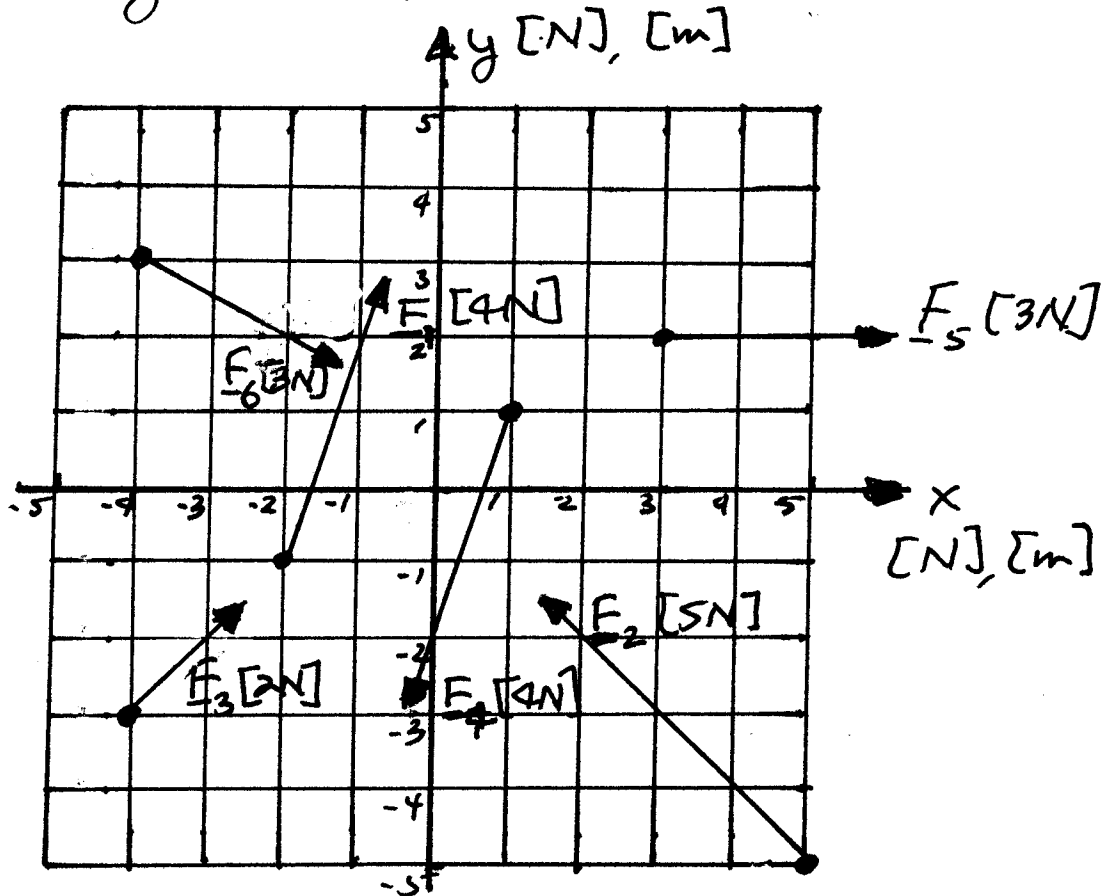
$$\underline{F}_4(1, 1) = (-1.14N) \hat{i} - (3.84N) \hat{j}$$

$$\underline{F}_5(3, 2) = (3.0N) \hat{i}$$

$$\underline{F}_6(-4, 3) = (2.72N) \hat{i} - (1.27N) \hat{j}$$



Draw a grid with the vectors on it:



$$(b) \quad \underline{F}_{\text{total}} = \sum \underline{F}_i$$

easiest to sum  $\hat{i}$  and  $\hat{j}$  components (i.e. force components in x and y directions)

$$\Rightarrow \underline{F}_{\text{total}} = \hat{i} (1.14N - 3.54N + 1.42N - 1.14N + 3.0N + 2.72N) + \hat{j} (3.84N + 3.54N + 1.42N - 3.84N - 1.27N)$$

$$\Rightarrow \underline{F}_{\text{total (net)}} = (3.6N) \hat{i} + (3.69N) \hat{j}$$

the overall magnitude is

$$|F| = \sqrt{(F_x)^2 + (F_y)^2}$$

$$\Rightarrow |F_{\text{total}}| = \sqrt{(3.6\text{ N})^2 + (3.6\text{ N})^2}$$

$$|F_{\text{total}}| = 5.16\text{ N}$$

(c) The definition of a couple is it results from two parallel (coplanar) forces of equal magnitude and opposite directions such that there is a moment but no net force

Notice on the grid that  $\underline{F}_1$  and  $\underline{F}_4$  are equal in magnitude and opposite in direction and thus satisfy this definition.

So, the answer is:  $\boxed{\text{YES -- } \underline{F}_1 \text{ and } \underline{F}_4}$

Now use the expression for the couple:

$$\underline{C} = \underline{r} \times \underline{F}$$

where:

$\underline{F}$  is one of the vectors

$\underline{r}$  is the position vector from one to the other

to get  $\underline{r}$ , subtract one (x, y) position from the other and use the unit vectors for each component: [from  $\underline{F}_1$  to  $\underline{F}_4$ ]

$$\Rightarrow \underline{r} = (1 - (-2))\hat{i} + (1 - (-1))\hat{j}$$

$$= 3\hat{i} + 2\hat{j} \quad \text{units in [m]}$$

Thus:

$$\underline{C} = (3\hat{i} + 2\hat{j}) \times (-1.14\hat{i} + 3.84\hat{j}) \quad [\text{N}\cdot\text{m}]$$

Recall:

$$\hat{i} \times \hat{j} = \hat{k} \quad (\text{z-direction})$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{i} = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\Rightarrow \underline{C} = \left\{ -11.52\hat{k} + (-2.28)(-\hat{k}) \right\} \quad [\text{N}\cdot\text{m}]$$

$$\Rightarrow \underline{C} = -9.24 \text{ N}\cdot\text{m} \hat{k}$$

(d) Determine the moment about the origin of each force ( $\underline{F}_i$ )  $\Rightarrow \underline{M}_{O_i}$  and sum them up to determine the net moment.

$$\underline{M} = \underline{r} \times \underline{F}$$

So:

$$\underline{M}_{O_i} = (x_i \hat{i} + y_i \hat{j}) \times (F_{x_i} \hat{i} + F_{y_i} \hat{j})$$

$\uparrow$  location vector for force  $\underline{F}_i$        $\uparrow$  force vector  $\underline{F}_i$

$$\begin{aligned}\underline{M}_{01} &= (-2\hat{i} - \hat{j})[m] \times (1.14\hat{i} + 3.84\hat{j})[N] \\ &= (-7.68\hat{k} + (-1.14)(-\hat{k}))[\text{N}\cdot\text{m}] = -6.54\text{N}\cdot\text{m}\hat{k}\end{aligned}$$

$$\begin{aligned}\underline{M}_{02} &= (5\hat{i} - 5\hat{j})[m] \times (-3.54\hat{i} + 3.54\hat{j})[N] \\ &= (-17.7\hat{k} + 17.7(-\hat{k}))[\text{N}\cdot\text{m}] = \underline{0}\end{aligned}$$

$$\begin{aligned}\underline{M}_{03} &= (-3\hat{i} - 4\hat{j})[m] \times (1.42\hat{i} + 1.42\hat{j})[N] \\ &= (-4.26\hat{k} + (-5.68)(-\hat{k}))[\text{N}\cdot\text{m}] = 1.42\text{N}\cdot\text{m}\hat{k}\end{aligned}$$

$$\begin{aligned}\underline{M}_{04} &= (\hat{i} + \hat{j})[m] \times (-1.14\hat{i} - 3.84\hat{j})[N] \\ &= (-3.84\hat{k} + (-1.14)(-\hat{k}))[\text{N}\cdot\text{m}] = -2.70\text{N}\cdot\text{m}\hat{k}\end{aligned}$$

$$\begin{aligned}\underline{M}_{05} &= (3\hat{i} + 2\hat{j})[m] \times (3.0\hat{i})[N] \\ &= -6\text{N}\cdot\text{m}\hat{k}\end{aligned}$$

$$\begin{aligned}\underline{M}_{06} &= (-4\hat{i} + 3\hat{j})[m] \times (2.72\hat{i} - 1.27\hat{j})[N] \\ &= (5.08\hat{k} + 8.16(-\hat{k}))[\text{N}\cdot\text{m}] = -3.08\text{N}\cdot\text{m}\hat{k}\end{aligned}$$

And then:  $\underline{M}_{0\text{net}} = \sum \underline{M}_{0i}$

$$\Rightarrow \underline{M}_{0\text{net}} = -16.9\text{N}\cdot\text{m}\hat{k}$$

(e) This is the same set of calculations as in part (d), except the position vector in each case,  $\underline{r}$ , is from  $(-5\text{m}, -5\text{m})$  to the vector (rather than the origin).

Thus:

$$\underline{r}_i = (x_i - (-5\text{m}))\hat{i} + (y_i - (-5\text{m}))\hat{j}$$

and again:

$$\underline{M}_i = \underline{r}_i \times \underline{F}_i$$

So:

$$\begin{aligned}\underline{M}_1 &= (3\hat{i} + 4\hat{j})[\text{m}] \times (1.14\hat{i} + 3.84\hat{j})[\text{N}] \\ &= (11.52\hat{k} + 4.56(-\hat{k}))[\text{N}\cdot\text{m}] = 6.96 \text{ N}\cdot\text{m} \hat{k}\end{aligned}$$

$$\underline{M}_2 = (10\hat{i})[\text{m}] \times (-3.54\hat{i} + 3.54\hat{j})[\text{N}] = 35.4 \text{ N}\cdot\text{m} \hat{k}$$

$$\begin{aligned}\underline{M}_3 &= (2\hat{i} + \hat{j})[\text{m}] \times (1.42\hat{i} + 1.42\hat{j})[\text{N}] \\ &= (2.84\hat{k} + 1.42(-\hat{k}))[\text{N}\cdot\text{m}] = 1.42 \text{ N}\cdot\text{m} \hat{k}\end{aligned}$$

$$\begin{aligned}\underline{M}_4 &= (6\hat{i} + 6\hat{j})[\text{m}] \times (-1.14\hat{i} - 3.84\hat{j})[\text{N}] \\ &= (-3.04\hat{k} + (-6.84)(-\hat{k}))[\text{N}\cdot\text{m}] = -16.2 \text{ N}\cdot\text{m} \hat{k}\end{aligned}$$

$$\underline{M}_5 = (8\hat{i} + 7\hat{j})[\text{m}] \times (3.0\hat{i})[\text{N}] = -21.0 \text{ N}\cdot\text{m} \hat{k}$$

$$\begin{aligned}\underline{M}_6 &= (\hat{i} + 8\hat{j})[\text{m}] \times (2.72\hat{i} - 1.27\hat{j})[\text{N}] \\ &= (-1.27\hat{k} + 21.76(-\hat{k}))[\text{N}\cdot\text{m}] = 23.03 \text{ N}\cdot\text{m} \hat{k}\end{aligned}$$

So with:  $\underline{M}_{\text{net}} = \sum M_i$

$$\Rightarrow \boxed{\underline{M}_{\text{net}} = 29.61 \text{ N}\cdot\text{m} \hat{k} \text{ about } (-5\text{m}, -5\text{m})}$$

(f) By examination, one can see there are no components acting about the y-axis or the x-axis since the only unit vector in the expression for the moment is  $\hat{k}$  (z-direction)

More generally, one can find:

$$\text{Moment about axis} = (\text{unit vector about axis}) \cdot (\underline{r} \times \underline{F})$$

So:  $M_x = \text{Moment component about x-axis}$

$$= \hat{i} \cdot \underline{M}_{\text{net}}$$

$$\text{with } \hat{i} \cdot \hat{k} = 0$$

$$\Rightarrow \boxed{M_x = 0}$$

Similarly:

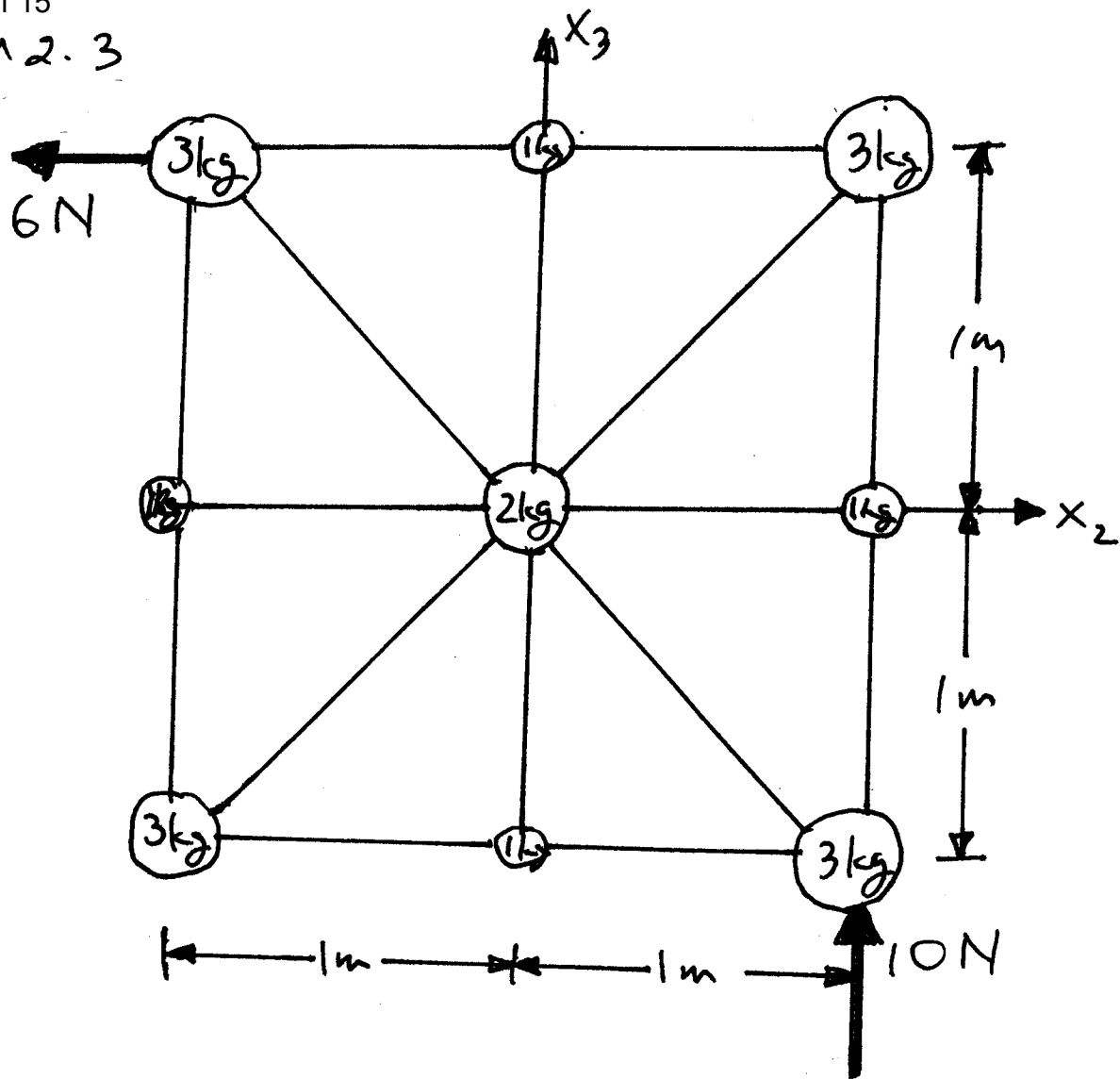
$M_y = \text{Moment component about y-axis}$

$$= \hat{j} \cdot \underline{M}_{\text{net}}$$

$$\text{with } \hat{j} \cdot \hat{k} = 0$$

$$\Rightarrow \boxed{M_y = 0}$$

M2.3



(a) This system is not in equilibrium.  
 One cause that both its net force and net moment are non-zero

Call the 10N force  $\underline{F}_1$  and the 6N force  $\underline{F}_2$

Express each as vectors:

$$\underline{F}_1 = 10\text{N } \hat{i}_3 \quad \text{at } (1\text{m}, -1\text{m})$$

$$\underline{F}_2 = -6\text{N } \hat{i}_2 \quad \text{at } (-1\text{m}, 1\text{m})$$

Thus, the net force,  $\underline{F}_N$ , is:

$$\underline{F}_N = \underline{F}_1 + \underline{F}_2 = 10\text{ N } \hat{i}_3 - 6\text{ N } \hat{i}_2$$

The acceleration of the rigidly connected system is:

$$\underline{a} = \frac{\underline{F}_{\text{net}}}{m}$$

where  $m$  is the total mass =  $(4 \times 3\text{ kg}) + (4 \times 1\text{ kg}) + 2\text{ kg}$   
 $= 18\text{ kg}$

$$\text{So: } \underline{a} = \frac{10\text{ N}}{18\text{ kg}} \hat{i}_3 - \frac{6\text{ N}}{18\text{ kg}} \hat{i}_2$$

(NOTE: watch units as  $\text{kg} = 10^3\text{ g}$ )  
 and  $1\text{ N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$

$$\text{So: } \underline{a} = \frac{5}{9} \text{ m/s}^2 \hat{i}_3 - \frac{1}{3} \text{ m/s}^2 \hat{i}_2$$

So the system has a linear acceleration of  $\frac{5}{9} \text{ m/s}^2$  in the  $+x_3$  direction and  $\frac{1}{3} \text{ m/s}^2$  in the  $-x_2$  direction

There will also be a counterclockwise rotation about the origin

This is due to the net moment:

$$\underline{M}_{\text{net}} = \sum (\underline{r}_i \times \underline{F}_i)$$

where:  $\underline{r}_i$  = position vector to force vector from origin



one can find:

$$\underline{M}_{\text{net}} = (1 \text{ m}) \hat{i}_2 \times (10 \text{ N}) \hat{i}_3 + (1 \text{ m}) \hat{i}_3 \times (-6 \text{ N}) \hat{i}_2$$

Also noting:

$$\hat{i}_2 \times \hat{i}_3 = \hat{i}_1$$

$$\hat{i}_3 \times \hat{i}_2 = -\hat{i}_1$$

$$\begin{aligned} \Rightarrow \underline{M}_{\text{net}} &= 10 \text{ N}\cdot\text{m} \hat{i}_1 + (-6 \text{ N}\cdot\text{m}) (-\hat{i}_1) \\ &= 16 \text{ N}\cdot\text{m} \hat{i}_1 \end{aligned}$$

(b) By applying other forces and moments, one can achieve equilibrium if the net force and moment are zero.

Thus, by applying a force, one can get the net force to be zero. The applied force must be equal in magnitude and opposite in direction to the current net force (as exists)

$$\Rightarrow \text{Apply } \underline{F} = -10 \text{ N} \hat{i}_3 + 6 \text{ N} \hat{i}_2$$

and the linear acceleration will be zero.

BUT, by applying a force at the origin, one cannot cause a moment about the origin (since  $\underline{r} = \underline{0}$ ), so the net moment cannot become zero.

$\Rightarrow$  NO

equilibrium  
cannot be  
achieved

(c) We have the opposite in this case. By applying a moment about the origin, we can get the net moment to be zero (apply one equal in magnitude and opposite in direction  $\Rightarrow \underline{M}_{\text{applied}} = -16 \text{ N}\cdot\text{m} \hat{i}_1$ ,

However, one cannot get the net force to be zero and thus there will be linear acceleration and motion

$\Rightarrow$  NO equilibrium cannot be achieved

(d) Go back to the reasoning of parts (b) and (c). Add these two (superposition applied) and equilibrium can be achieved

$\Rightarrow$  YES

Apply :

$$\underline{F} = -10 \text{ N} \hat{i}_3 + 6 \text{ N} \hat{i}_2$$

$$\underline{M} = -16 \text{ N}\cdot\text{m} \hat{i}_1$$

(e) The same reasoning applies as in the answer to part (c). A couple results in a net moment but no net force. So a couple can cause the rotation to not occur, but the total net force will still be non-zero and thus there will be linear acceleration and motion.

$\Rightarrow$  NO equilibrium cannot be achieved

(f) A force can achieve a desired moment by proper placement. Thus, using a similar reasoning as (d), a force properly placed gives the right opposite moment about the origin and the force must be as in (b) to achieve a net force of zero.

$$\text{So: } \underline{F} = -10 \text{ N } \hat{i}_3 + 6 \text{ N } \hat{i}_2$$

one must have

$$\underline{M} = \underline{r} \times \underline{F} = -16 \text{ N} \cdot \text{m } \hat{i}_1$$

$$\Rightarrow (x_3 \hat{i}_3 + x_2 \hat{i}_2) \times (-10 \text{ N } \hat{i}_3 + 6 \text{ N } \hat{i}_2) = -16 \text{ N} \cdot \text{m } \hat{i}_1$$

where  $x_3$  and  $x_2$  identify the location of the force vector

$$\text{using: } \hat{i}_2 \times \hat{i}_3 = \hat{i}_1, \\ \text{and } \hat{i}_3 \times \hat{i}_2 = -\hat{i}_1$$

$$\Rightarrow -6x_3 \text{ N } \hat{i}_1 - 10x_2 \text{ N } \hat{i}_1 = -16 \text{ N} \cdot \text{m } \hat{i}_1$$

$$\Rightarrow -6x_3 - 10x_2 = 16 \text{ m}$$

so this defines a line along which

$$\underline{F} = -10 \text{ N } \hat{i}_3 + 6 \text{ N } \hat{i}_2$$

can be placed to get

**YES**

equilibrium achieved