16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 2

<table>
<thead>
<tr>
<th></th>
<th>Time Spent (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5</td>
<td></td>
</tr>
<tr>
<td>F6</td>
<td></td>
</tr>
<tr>
<td>F7</td>
<td></td>
</tr>
<tr>
<td>M3.1</td>
<td></td>
</tr>
<tr>
<td>M3.2</td>
<td></td>
</tr>
<tr>
<td>Study Time</td>
<td></td>
</tr>
</tbody>
</table>

Name: ______________________

Due Date: 9/26/2006

Announcements:
Lab 1 used a 47:1 scale model of the Boeing Blended Wing Body (BWB). The actual BWB is designed to operate at 10km altitude, where the air properties compare to the sea level values as follows:

\[
\begin{align*}
\rho &= 0.35 \rho_{\text{SL}} \\
\alpha &= 0.94 \alpha_{\text{SL}} \\
\mu &= 0.94 \mu_{\text{SL}}
\end{align*}
\]

The model was operated at \( V_\infty = 18 \text{ m/s} \), while the actual BWB is designed to operate at \( V_\infty = 250 \text{ m/s} \) at the 10km altitude.

a) Using the centerline chord as the reference length (\( c_o = 37.7 \text{ in} \)), determine \( Re \) and \( M_\infty \) for the BWB model in the wind tunnel, and the full-size BWB at altitude. Assume the tunnel is at standard atmospheric conditions at sea level.

b) If the model and full-size BWB are both operating at \( \alpha = 5^\circ \), can the model's measured \( C_L \) and \( C_D \) value be used to compute the lift and drag on the full-size BWB? Explain why or why not.

c) By varying the tunnel speed, is it possible to match both \( Re \) and \( M_\infty \) between the model and full-size BWB? Explain why or why not.
Consider the 2-D velocity field $\vec{V} = u\hat{i} + v\hat{j}$

$$u = y \quad v = x$$

whose hyperbolic-shape streamlines are sketched below. The density $\rho$ is everywhere constant.

a) Evaluate the mass-flow integral

$$I = \oint \rho (\vec{V} \cdot \hat{n}) \, dA$$

for the pie-slice control volume of radius $R$ shown in the figure.

Hint: First express $I$ as three separate integrals $I_1$, $I_2$, $I_3$ over each of the three segments of the C.V. boundary. Also, $I_3$ is most easily evaluated by first writing $\vec{V}$, $\hat{n}$, and $dA$ as functions of the polar coordinates $r$, $\theta$. Note that $dA$ is a length in 2-D.

b) Does this flow satisfy the mass conservation law? Explain.
A pipe of cross-sectional area $A$ has air flowing in it at a known velocity $V_1$. The pipe then suddenly increases to an area of $4A$, at station 1 where the pressure is some $p_1$. After undergoing mixing in the larger pipe, the velocity becomes uniform again at station 2, where the velocity is $V_2$, and the pressure is $p_2$. Assume the density $\rho$ is constant everywhere (low speed flow). Be sure to clearly draw the control volume you will be using.

a) Determine the velocity $V_2$.

b) Determine the pressure difference $p_2 - p_1$. 

\[ \begin{align*}
\text{(Diagram showing the pipe with areas and pressures at stations 1 and 2.)}
\end{align*} \]
M3.1 (15 points) In this problem, we consider a Boeing 777 and learn about the modeling of the lift on a wing in order to consider the loads acting on the structure. The 777 has approximate values for the gross takeoff weight of 506,000 pounds and for the wing half-span of 100 feet. The overall weight can be considered to act at the center of the fuselage. The half-span of the wing is the distance from the root (where the wing connects to the fuselage) to the tip. (See the simple diagram of the geometry below.)

For structural purposes, an airplane wing can be modeled in two dimensions as a linear structural member of length \( s/2 \) (the half-span) emanating from the root. Consider the fuselage to be a point for this initial structural consideration (not too comfortable for the passengers!). The lift (pressure differential between the top and bottom surfaces) on the wing can be represented as a lineload and thus has dimensions of [force/length]. Let’s consider different possible models for the lift distribution: (1) constant along the span of the wing; (2) linear variation along the span of the wing with its maximum value at the root and with a value of zero at the tip; (3) quadratic variation along the span of the wing such that \( \text{Lift} = b - ax^2 \) with its maximum value at the root and with a value of zero at the tip. For each case, assume that in steady flight, each wing must provide sufficient lift to support half the total weight.

![Diagram](image.png)

We work to see how the model for lift changes the following results. For each of the three models:

(a) Draw this configuration showing all loads.

(b) Determine the maximum magnitude of the lineload of lift and where it occurs.

(c) For one wing, determine the equipollent force system at the root for the lift.
M3.2 (15 points) A 10-meter long beam is attached via a roller at one end and is pinned at the other end. This configuration is known as simply-supported. A 100 kg person walks back and forth across this beam. In addition to the person, the beam has a cable attached to it 4 meters from the roller end. This cable makes a 70° angle with the beam and goes through a pulley, with a radius of r, attached by a pin to a ceiling 8 meters above the beam. At the other end of the cable is a mass of 50 kg. The overall situation is illustrated in the accompanying figure.

(a) Draw the free body diagram for this situation (choose any location for the person).

(b) If possible, determine the reaction forces as a function of the point on the beam at which the person is located.

(c) With the person at any location, determine if the mass hanging from the cable can be changed in such a way that the reaction(s) at the roller is(are) zero. Clearly explain your reasoning.

(d) The end with the pin is now clamped. Draw the free body diagram for this case and then determine the reaction forces or, if the reaction forces cannot be exactly determined, clearly explain the reasons for that.