a) \( \vec{V}(t) = 1 \hat{i} + kt \hat{j} \) \( \rightarrow \) \( \vec{U}(t) = 1 \) (constant), \( \vec{V}(t) = kt \hat{j} \)

\[
\begin{align*}
x(t) &= x_0 + \int_0^t u \, dt' = x_0 + ut = t \\
y(t) &= y_0 + \int_0^t kt' \, dt' = y_0 + \frac{1}{2} kt^2 = \frac{1}{2} kt^2
\end{align*}
\]

b) Using \( x = t \), \( y(t) = \frac{1}{2} k t^2 \), parabolas...

c) \( \vec{V} \) is spatially constant, so pathline & streakline are 180° images.

\( t = \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad \text{etc.} \)

Overlay:

\begin{align*}
\text{streakline at } t &= 1 \\
\text{k=2 pathline}
\end{align*}
a) Mass Eqn: \[ \int \rho \mathbf{V} \cdot d\mathbf{A} = 0 \]
\[-\rho V_1 A + \rho V_2 A = 0\]
\[V_1 = V_2\]

b) x-momentum eqn: \[ \int \rho (\mathbf{V} \cdot \mathbf{n}) u \, dA + \int \rho p u_x \, dA + D' = 0 \]
\[\mathbf{R'} = D' + L'\]
\[\rho (-V_1) V_1 A + \rho V_2 V_2 A - p_1 A + p_2 A + D' = 0 \]
\[\text{no 3, 4 contributions}\]
\[(p_2 - p_1) A + D' = 0\]
\[p_2 - p_1 = -\frac{D'}{A}\]

c) y-momentum eqn: \[ \int \rho (\mathbf{V} \cdot \mathbf{n}) v \, dA + \int \rho p u_y \, dA + L = 0 \]
\[\rho (-V_1) A \cdot O A + \rho V_2 A \cdot O A + \int_{x_1}^{x_2} -p_3 dx + \int_{x_1}^{x_2} p_4 dx + L' = 0 \]
divide by \(x_2 - x_1\):
\[\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} p_4 \, dx - \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} p_3 \, dx + \frac{L'}{x_2 - x_1} = 0\]
\[\bar{p}_4 - \bar{p}_3 = -\frac{L}{x_2 - x_1}\]

Top wall pressure is less than bottom due to pressure field of lifting wing.
a) Assume constant total pressure along nozzle.

\[ (p + \frac{1}{2} \rho V^2)_{o} = (p + \frac{1}{2} \rho V^2)_{l} \]

\[ p(o) = p(l) + \frac{1}{2} \rho V^2 \]

\[ \Delta p = p(o) - p(l) = \frac{1}{2} \rho V^2 \]

\[ V_{1} = \sqrt{\frac{2 \Delta p}{\rho}} = 28.28 \text{ m/s} \]

b) Mass conservation along nozzle:

\[ \rho V(x) \cdot A(x) = \text{constant} = \rho V_{1} A_{1} \]

\[ V(x) = \frac{V_{1} A_{1}}{A(x)} = -\frac{V_{1} A_{1}}{A_{o} + (A_{1} - A_{o}) \frac{x}{L}} \]

c) \( a(x) = \frac{dV}{dt} = \frac{dV}{dx} + V \frac{d}{dx} \frac{dV}{dx} = V \frac{d^2V}{dx^2} \)

\[ \frac{dV}{dx} = -\frac{V}{A(x)} \quad \frac{dA}{dx} = -\frac{V}{A(x)} \frac{A_{1} - A_{o}}{L} \]

\[ a(x) = -\frac{V^2}{A(x)} \frac{A_{1} - A_{o}}{L} = \frac{V_{1} A_{1}}{[A_{o} + (A_{1} - A_{o}) \frac{x}{L}]^3} \frac{A_{o} - A_{1}}{L} \]

\[ a \approx 80000 \text{ m/s}^2 \]

\[ a_{max} \approx 76000 \text{ m/s}^2 \]

d) Gravity \( g = 9.8 \text{ m/s}^2 \) is negligible compared to max acceleration.

\[ a^{max} = 76000 \text{ m/s}^2 \]
M4.1

(a) Draw the Free Body Diagram

(b) Determine the reaction forces by applying the equilibrium equations

\[ \Sigma F_1 = 0 \implies 5kN + H_A = 0 \]
\[ \implies H_A = -5000 \text{ N} \]

\[ \Sigma F_2 = 0 \ \theta^+ \implies 20kN + V_A + V_H = 0 \]

\[ \Sigma M = 0 \ \text{(about Point A)} \implies -5000N(5m) + 20000N(5m) + V_H(20m) = 0 \]
\[
\Rightarrow \quad V_H = -15,000 \text{N} (5\text{m}) / 20\text{m} \\
\Rightarrow \quad V_H = -3,750 \text{N} \\
\]

Using this in the second equation:
\[
\Rightarrow \quad V_A = -20,000 \text{N} + 3,750 \text{N} \\
\Rightarrow \quad V_A = -16,250 \text{N} \\
\]

**Summarizing:**

\[
\begin{align*}
V_A &= -16,250 \text{N} \\
F_A &= -5,000 \text{N} \\
V_H &= -3,750 \text{N} \\
\end{align*}
\]

(c) **YES!** Consider point D and point G, and vertical equilibrium at that point.

For the case of D:

\[
\begin{array}{cc}
\downarrow & \\
\downarrow & \\
F_{DB} & F_{DF} \quad \rightarrow \\
& F_{DE} \\
\end{array}
\]

Since only the bar DE has a vertical direction, only it can carry a vertical load (x_2-direction). There is no load applied at point D, so this gives \( F_{DE} = 0 \) by
equilibrium in the vertical ($x_2$) direction. The same argument can be made for bar $FG$ at point $G$.

$$F_{FG} = 0$$

(1) Using the method of joints, choose an end from which to start.

Choose $A$ here:

$\begin{aligned}
A & \\
F_{AB} & \rightarrow F_{AB} \\
F_{AC} & \rightarrow F_{AC} \\
H_A &= -500 \text{ N} \\
V_A &= -16,250 \text{ N}
\end{aligned}$

Note that the diagonal bars make 45° angles to the $x_1$ and $x_2$ axes since the base unit is a square (5 m x 5 m).

Thus, the $x_1$ and $x_2$ force components of the diagonal bars are:

$x_1: F_{Bar} \cos 45^\circ = F_{Bar} (0.707)$

$x_2: F_{Bar} \sin 45^\circ = F_{Bar} (0.707)$

Now use equilibrium:

$$\Sigma F_1 = 0 \quad \Rightarrow \quad -5,000 \text{ N} + 0.707 F_{AB} + F_{AC} = 0$$

$$\Sigma F_2 = 0 \quad \Rightarrow \quad -16,250 \text{ N} + 0.707 F_{AB} = 0$$
the second equation gives
\[ \mathbf{F}_{AB} = 23,000 \, \text{N} \]
using this in the \( \Sigma F_i \) equation:
\[ -5000 \, \text{N} + 16,250 \, \text{N} + F_{AC} = 0 \]
\[ \Rightarrow F_{AC} = -11,250 \, \text{N} \]

Proceed to Joint C

\[ \Sigma F_x = 0 \quad \Rightarrow \quad F_{CA} + F_{CE} = 0 \]
using \( F_{AC} = F_{CA} = -11,250 \, \text{N} \)
\[ \Rightarrow F_{CE} = 11,250 \, \text{N} \]

\[ \Sigma F_y = 0 \quad \Rightarrow \quad 20,000 \, \text{N} + F_{CB} = 0 \]
\[ \Rightarrow F_{CB} = -20,000 \, \text{N} \]

Now Joint B

\[ \Sigma F_x = 0 \quad \Rightarrow \quad F_{BA_1} - F_{BA_2} = 0 \]
\[ \Sigma F_y = 0 \quad \Rightarrow \quad F_{BA} - F_{BA_2} - F_{BC} - F_{BE} = 0 \]
\[ \sum F_i = 0 \quad \Rightarrow \quad 5000N - 0.707F_{BA} + F_{BD} + 0.707F_{BF} = 0 \]

Using \( F_{BA} = 23,000N \) gives:

\[ F_{BD} + 0.707F_{BF} = 11,250N \]

Now:

\[ \sum F_2 = 0 \quad \Rightarrow -F_{BC} - 0.707F_{BA} - 0.707F_{BF} = 0 \]

Using \( F_{BC} = -20,000N, F_{BA} = 23,000N \) gives:

\[ 20,000N - 16,250N - 0.707F_{BF} = 0 \]

\[ \Rightarrow F_{BF} = \frac{3,750N}{0.707} \]

\[ \Rightarrow F_{BF} = 5,300N \]

Using this in the resulting \( \sum F_1 \) equation above gives:

\[ F_{BD} + 3750N = 11,250N \]

\[ \Rightarrow F_{BD} = 7500N \]

To Joint D

\[ \begin{array}{c}
F_{DB} \leftarrow D \quad \Rightarrow \quad F_{DF} \\
\rightarrow x_1 \quad \Rightarrow \quad x_2
\end{array} \]

\[ \sum F_1 = 0 \quad \Rightarrow \quad -F_{DB} + F_{DF} = 0 \]

with \( F_{DB} = 7500N \Rightarrow F_{DF} = 7500N \)

\[ \sum F_2 = 0 \quad \Rightarrow \quad F_{DE} = 0 \quad \text{(as in part (c))} \]
Proceed to Joint E

\[ \Sigma F_1 = 0 \Rightarrow -F_{EC} - 0.707F_{EB} + 0.707F_{EP} + F_{EG} = 0 \]

Using previous results of \( F_{EB} = 5300 \text{ N} \)
and \( F_{EC} = -11,250 \text{ N} \) gives:
\[ 11,250 \text{ N} - 3750 \text{ N} + 0.707F_{EF} + F_{EG} = 0 \]

Then also consider
\[ \Sigma F_2 = 0 \iff 0.707F_{EB} + F_{ED} + 0.707F_{EF} = 0 \]

already have \( F_{ED} = 0 \), with \( F_{EB} = 5300 \text{ N} \)
\[ \Rightarrow 3750 \text{ N} + 0.707F_{EF} = 0 \]
\[ \Rightarrow F_{EF} = -5300 \text{ N} \]

Use this in the results from \( \Sigma F_1 \) to get:
\[ 7500 \text{ N} - 3750 \text{ N} + F_{EG} = 0 \]
\[ \Rightarrow F_{EG} = -3750 \text{ N} \]

Next to Joint G

\[ X_2 \xrightarrow{a} X_1 \]

\[ F_{GF} \]

\[ F_{GF} \xleftarrow{a} \]
\[ \Sigma F_1 = 0 \implies -F_{Gx} + F_{GH} = 0 \]

with \( F_{Gx} = -3,750 \text{ N} \) gives:
\[ F_{GH} = -3,750 \text{ N} \]

\[ \Sigma F_2 = 0 \implies F_{GF} = 0 \quad (\text{as in part (c)}) \]

Then to Joint F

\[ \Sigma F_i = 0 \implies -F_{FD} - 0.707 F_{FE} + 0.707 F_{FH} = 0 \]

with \( F_{FD} = 7500 \text{ N} \) and \( F_{FE} = -5300 \text{ N} \) given:
\[ -7500 \text{ N} + 3750 \text{ N} + 0.707 F_{FH} = 0 \]
\[ \implies F_{FH} = 5300 \text{ N} \]

\[ \Sigma F_2 = 0 \implies -0.707 F_{FE} + F_{FG} - 0.707 F_{FH} = 0 \]

with \( F_{FG} = 0 \) and \( F_{FE} = -5300 \text{ N} \) get:
\[ 3750 \text{ N} - 0.707 F_{FH} = 0 \]
\[ \implies F_{FH} = 5300 \text{ N} \checkmark \]
Then to Joint $H$ to do final check:

$$
\sum F_x = 0 \quad \Rightarrow \quad -F_{AH} - 0.707 F_{HF} = 0
$$

with $F_{AH} = -3750 \text{ N}$, $F_{HF} = 5300 \text{ N}$

$$
\Rightarrow \quad +3750 \text{ N} - 3750 \text{ N} = 0 \quad \checkmark \quad \text{checks}
$$

$$
\sum F_y = 0 \quad \Rightarrow \quad 0.707 F_{HF} + V_H = 0
$$

$$
\Rightarrow \quad 3750 \text{ N} - 3750 \text{ N} = 0 \quad \checkmark \quad \text{checks}
$$

Summarize by drawing the truss and placing the bar load above each bar with (+) tension and (-) compression.
(e) To determine the load in the bar FG by the method of sections, a "sectional cut" must be made that goes through this bar. There must be only three bars of unknown load so that the three equilibrium equations yield a determinate solution.

Choose the following cut from the end of point H (the same could be done for point A - less external loads/reactions are involved here):

\[
\begin{align*}
&Q \times 2 \\
&\times 1
\end{align*}
\]

Apply the equations of equilibrium:

\[\Sigma F_1 = 0 \quad \Rightarrow \quad -F_{GE} - 0.707F_{FE} - F_{FD} = 0\]

\[\Sigma F_2 = 0 \quad \Rightarrow \quad V_H - 0.707F_{FE} = 0\]

with \(V_H = -3750\, N\) \(\Rightarrow F_{FE} = -5300\, N\)

\[\Sigma M = 0 \quad (\text{about point } H) \quad \Rightarrow \quad V_H (5m) - F_{GE} (5m) = 0\]

\[\Rightarrow -3750\, N - F_{GE} = 0\]

\[\Rightarrow F_{GE} = -3750\, N \quad \checkmark\]
Using these results in the EF equation gives:

\[
3750 \text{ N} - 0.707 F_{FD} - F_{FD} = 0
\]

\[
\Rightarrow 3750 \text{ N} + 3750 \text{ N} - F_{FD} = 0
\]

\[
\Rightarrow F_{FD} = 7500 \text{ N} \checkmark \text{check}
\]

\[
\Rightarrow \text{all 3 check with } F_{Eq} = -3750 \text{ N}
\]

as requested
M 4.2

(a) The first step is to draw the **free body diagram**. One could draw the entire structure or do as shown in the problem statement showing the applied load applied downward with a magnitude of 5000 lbs every 20 feet.

---

\[
\sum F_x = 0 \implies -H_E = 0
\]

\[
\sum F_y = 0 \implies V_w + V_E = -5000 \text{ lbs (at point 21) = 0}
\]

There are 21 points of applied load. The trot at the west end and one at the end of each bay.
\[ \sum M = 0 \quad \Rightarrow \quad V_e (800 \text{ ft}) - \sum_{i=1}^{20} (5000 \text{ lbs})(i)(40 \text{ ft}) = 0 \]

Each load at the end of the span is applied at (20ft) i from the west end.

\[ \sum_{i=1}^{20} i = 210 \]

\[ V_e (800 \text{ ft}) = (5000 \text{ lbs})(210)(40 \text{ ft}) \]

\[ \Rightarrow \quad V_e = 52,500 \text{ lbs} \]

Using this in the \( \sum F_3 \) equation gives:

\[ V_w + 52,500 \text{ lbs} - 105,000 \text{ lbs} = 0 \]

\[ \Rightarrow \quad V_w = 52,500 \text{ lbs} \]

**Summarizing:**

\[ V_w = 52,500 \text{ lbs} \]

\[ H_e = 0 \text{ lbs} \]

\[ V_e = 52,500 \text{ lbs} \]
(5) To get the bar loads in the 5th bay in the 10th bay, one could use the Method of Joints or the Method of Sections. Using the Method of Joints requires working from a point with a known load and proceeding to the desired point. This can involve significant points and equations. The Method of Sections is generally more efficient in such situations. Make a cut through the 5th bar of the 10th bay (horizontal and diagonal as desired) and draw the resulting free body diagram:

\[ \begin{align*}
& \text{Resolve } F_{\text{diagonal}} \text{ into } x_3 \text{ and } x_1 \text{ components.} \\
& \text{Note via geometry the angle of the diagonal bar:}
\end{align*} \]
\[ \theta = \tan^{-1} \frac{20ft}{40ft} \]
\[ \Rightarrow \theta = 26.6^\circ \]

and \[ F_{\text{diagonal}} (x_1) = F_{\text{dia}} \cos \theta = 0.89F_{\text{dia}} \]
\[ F_{\text{diagonal}} (x_3) = F_{\text{dia}} \sin \theta = 0.45F_{\text{dia}} \]

Now apply equilibrium.

\[ \Sigma F_i = 0 \quad \Rightarrow \quad F_{\text{lower}} + F_{\text{upper}} + 0.89F_{\text{dia}} = 0 \]

\[ \Sigma F_3 = 0 \quad \Rightarrow \quad V_w - 5000 \text{lbs} (10) + 0.45F_{\text{dia}} = 0 \quad \text{one load at each bag} \]

with \( V_w = 52,500 \text{ lbs} \) gives:

\[ 2,500 \text{ lbs} + 0.45F_{\text{dia}} = 0 \]
\[ \Rightarrow F_{\text{dia}} = -5,580 \text{ lbs} \]
\[ \Sigma M = 0 \quad \Rightarrow \quad -V_w(360\text{ ft}) - F_{\text{upper}}^{(20\text{ ft})} \]

(about bottom end of 10)

\[ + \sum_{i=1}^{9} 5000\text{ lb} \times (i) \times (40\text{ ft}) = 0 \]

Same consideration as before

\[ \sum_{i=1}^{9} (i) = 45 \]

\[ \Rightarrow -52,500 \text{ lb} \times (360 \text{ ft}) - F_{\text{upper}}^{(20\text{ ft})} \]

\[ + 5000 \text{ lb} \times 45 \times (40\text{ ft}) = 0 \]

\[ \Rightarrow F_{\text{upper}} = -945,000\text{ lb} + 450,000\text{ lb} \]

\[ \Rightarrow F_{\text{upper}} = -495,000\text{ lb} \]

Using these results in the \( \Sigma F \), equation gives:

\[ F_{\text{lower}} = 495,000\text{ lb} + 5000\text{ lb} = 0 \]

\[ \Rightarrow F_{\text{lower}} = 500,000\text{ lb}. \]

Summarizing:

\[ F_{\text{lower}} = 500,000\text{ lb}. \]

\[ F_{\text{upper}} = -495,000\text{ lb}. \]

\[ F_{\text{diagonal}} = -5580\text{ lb}. \]