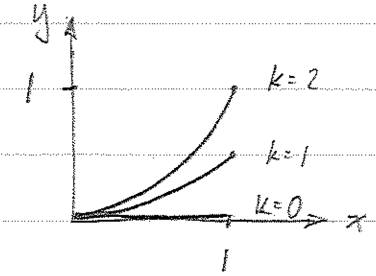
UE Fluids Problem 8 Solution Fall '06

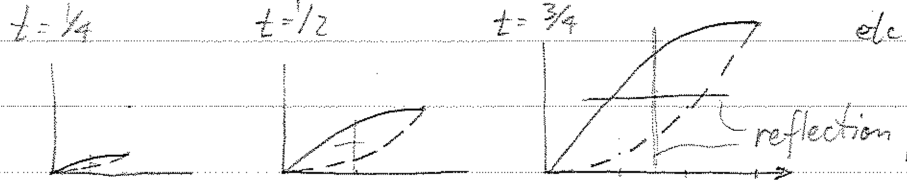
a) $\vec{V}(t) = 1\,\hat{i} + kt\,\hat{j} \;\rightarrow\; u(t) = \underline{1}$ (constant), $v(t) = kt$

$$\boxed{\begin{aligned} x(t) &= x_0 + \int_0^t u\,dt' = x_0 + ut = t \\ y(t) &= y_0 + \int_0^t kt'\,dt' = y_0 + \tfrac{1}{2}kt^2 = \tfrac{1}{2}kt^2 \end{aligned}}$$

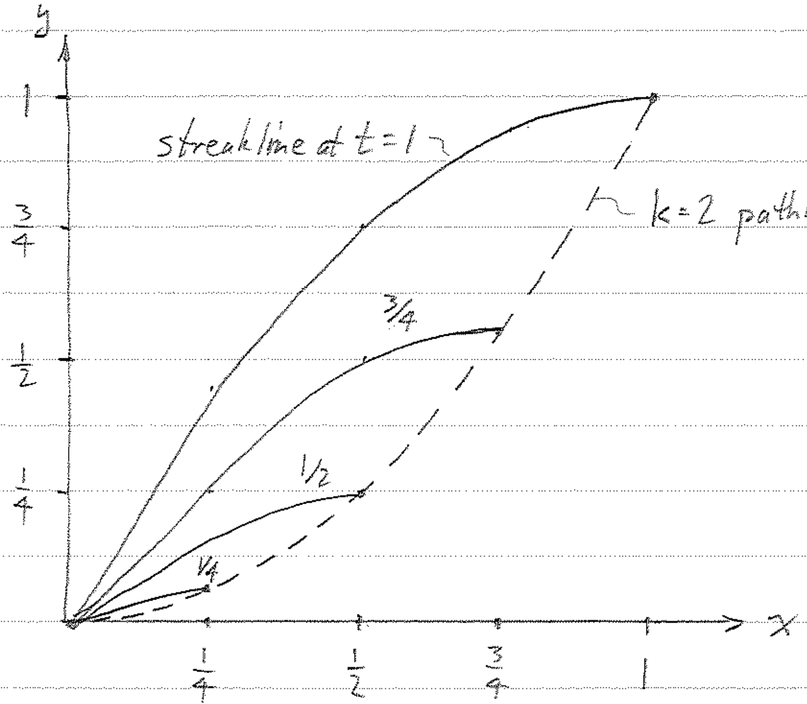
b) Using $x = t$, $y(x) = \frac{1}{2}kx^2$, parabolas...



c) $\vec{V}$ is spatially constant, so pathline & streakline are 180° images.

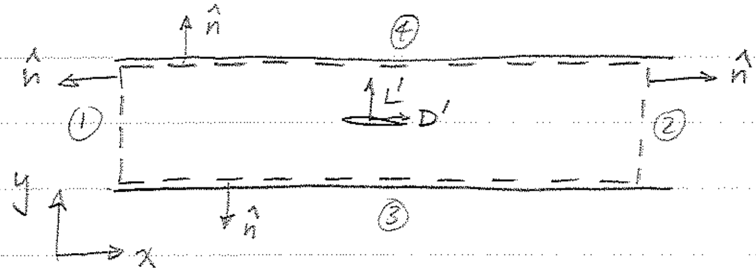


Overlay:



UE Fluids Problem 9 Solution Fall '06

a) Mass Eq'n: $\oint \rho \vec{V}\cdot\hat{n}\, dA = 0$



$-\rho V_1 A + \rho V_2 A = 0$

$$\boxed{V_1 = V_2}$$

b) x-momentum eq'n: $\oint \rho(\vec{V}\cdot\hat{n})\, u\, dA + \oint p\, n_x\, dA + D' = 0$, $\vec{R}' = D'\hat{i} + L'\hat{j}$

$$\rho(-V_1)V_1 A + \rho V_2 V_2 A - p_1 A + p_2 A + D' = 0$$ no ③, ④ contributions

$$(p_2 - p_1)A + D' = 0$$

$$\boxed{p_2 - p_1 = -D'/A}$$

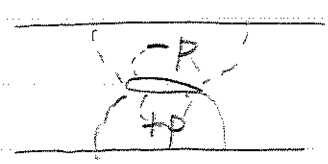
c) y-momentum eq'n: $\oint \rho(\vec{V}\cdot\hat{n})\, v\, dA + \oint p\, n_y\, dA + L = 0$

$$\rho(-V_1)\cdot 0\cdot A + \rho V_2\cdot 0\, A + \int_{x_1}^{x_2} -p_3\, dx + \int_{x_1}^{x_2} p_4\, dx + L' = 0$$

divide by $x_2 - x_1$: $$\frac{1}{x_2-x_1}\int_{x_1}^{x_2} p_4\, dx - \frac{1}{x_2-x_1}\int_{x_1}^{x_2} p_3\, dx + \frac{L'}{x_2-x_1} = 0$$

$$\boxed{p_{4avg} - p_{3avg} = -\frac{L}{x_2-x_1}}$$

top wall pressure is less than bottom,
due to pressure field of lifting wing



a) Assume constant total pressure along nozzle.

$$\left(p + \cancel{\tfrac{1}{2}\rho V^2}^{\text{neglect}}\right)_0 = \left(p + \tfrac{1}{2}\rho V^2\right)_\ell$$

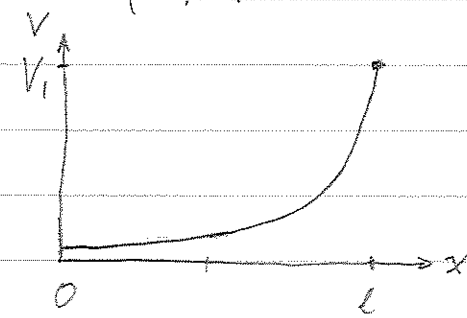
$$p(0) = p(\ell) + \tfrac{1}{2}\rho V_1^2$$

$$\Delta p \equiv p(0) - p(\ell) = \tfrac{1}{2}\rho V_1^2$$

$$\boxed{V_1 = \sqrt{\frac{2\Delta p}{\rho}} = 28.28 \text{ m/s}}$$

b) mass conservation along nozzle: $\rho V(x) \cdot A(x) = \text{constant} = \rho V_1 A_1$

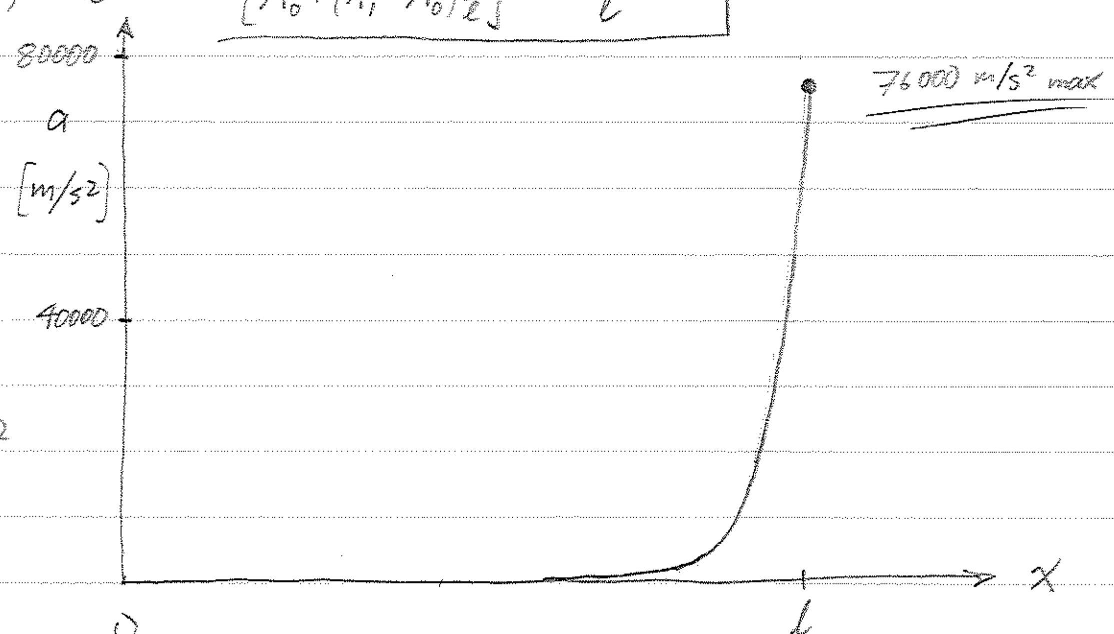
$$V(x) = \frac{V_1 A_1}{A(x)} = \frac{V_1 A_1}{A_0 + (A_1 - A_0)\frac{x}{\ell}}$$



c) $$a(x) = \frac{DV}{Dt} = \cancel{\frac{\partial V}{\partial t}}^{0} + V\frac{\partial V}{\partial x} = V\frac{dV}{dx}$$

$$\frac{dV}{dx} = -\frac{V}{A(x)} \cdot \frac{dA}{dx} = -\frac{V}{A(x)} \cdot \frac{A_1 - A_0}{\ell}$$

$$\boxed{a(x) = -\frac{V^2}{A(x)}\frac{A_1 - A_0}{\ell} = \frac{V_1 A_1}{\left[A_0 + (A_1 - A_0)\frac{x}{\ell}\right]^3}\frac{A_0 - A_1}{\ell}}$$



d) Gravity $g = 9.8$ m/s² is negligible compared to max acceleration of $a_{max} \approx 76\,000$ m/s²

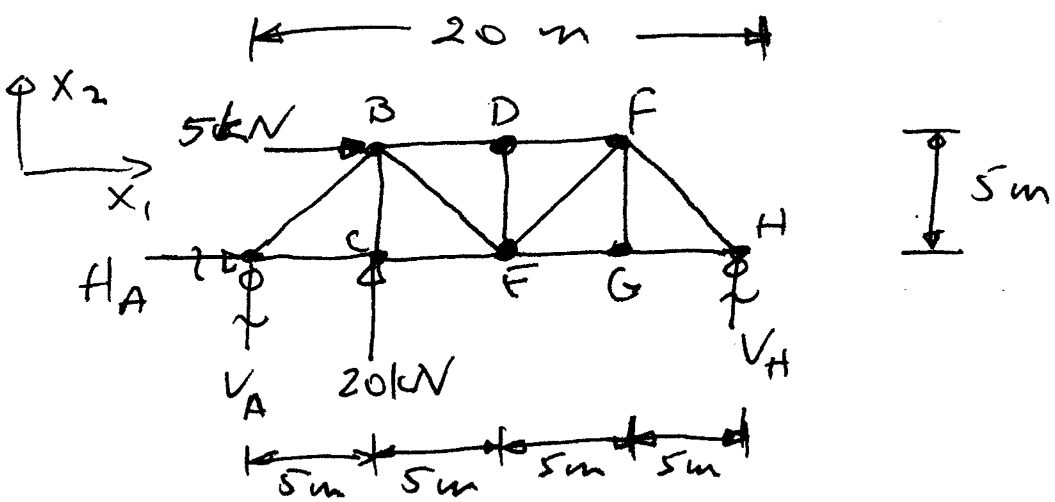
PAL
9/23/06

Unified Engineering Problem Set
Week 4 Fall, 2006

SOLUTIONS

M4.1

(a) Draw the Free Body Diagram



(b) Determine the reaction forces by applying the equilibrium equations

$$\sum F_1 = 0 \quad \overset{+}{\rightarrow} \quad \Rightarrow 5kN + H_A = 0$$

$$\Rightarrow H_A = -5000\ N$$

$$\sum F_2 = 0 \quad \uparrow + \quad \Rightarrow 20kN + V_A + V_H = 0$$

$$\sum M = 0 \quad \circlearrowleft + \quad \Rightarrow -5{,}000N(5m) + 20{,}000\ N(5m) + V_H(20m) = 0$$

(about point A)

$$\Rightarrow V_H = -15{,}000\,N\,(5m)/20m$$

$$\Rightarrow V_H = -3{,}750\,N$$

using this in the second equation:

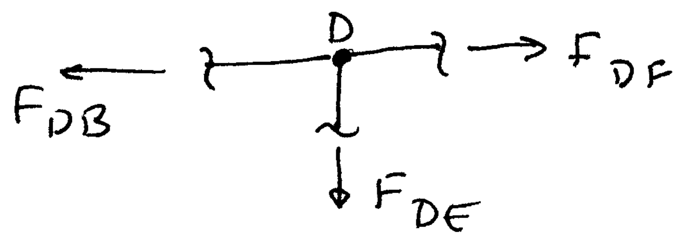
$$\Rightarrow V_A = -20{,}000\,N + 3{,}750\,N$$

$$\Rightarrow V_A = -16{,}250\,N$$

Summarizing:

$$\boxed{\begin{aligned} V_A &= -16{,}250\,N \\ H_A &= -5{,}000\,N \\ V_H &= -3{,}750\,N \end{aligned}}$$

(c) <u>YES</u>! Consider point D and point G and vertical equilibrium at that point. For the case of D:

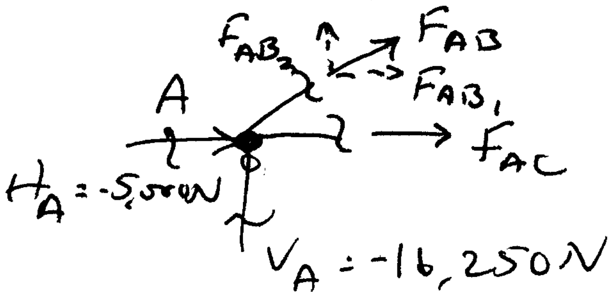


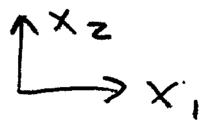
Since only the bar DE has a vertical direction, only it can carry a vertical load (x_2-direction). There is no load applied at point D, so this gives $\boxed{F_{DE} = 0}$ by

equilibrium in the vertical (x_2) direction.
The same argument can be made for bar FG at point G $\Rightarrow$ $\boxed{F_{FG} = 0}$

(d) Using the method of Joints, choose an end from which to start.

Choose A here:





Note that the diagonal bars make 45° angles to the x_1 and x_2 axes since the truss unit is a square ($5m \times 5m$)

Thus, the x_1 and x_2 force components of the diagonal bars are:

x_1: $F_{Bar} \cos 45^\circ = F_{Bar}(0.707)$

x_2: $F_{Bar} \sin 45^\circ = F_{Bar}(0.707)$

Now use equilibrium:

$\Sigma F_1 = 0 \quad \xrightarrow{+} \quad \Rightarrow -5{,}000N + 0.707 F_{AB} + F_{AC} = 0$

$\Sigma F_2 = 0 \quad \uparrow + \quad \Rightarrow -16{,}250N + 0.707 F_{AB} = 0$

the second equation gives

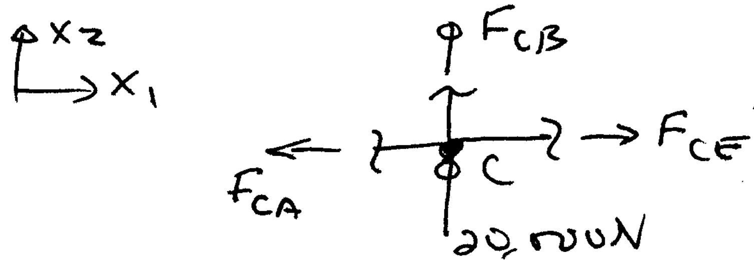
$$F_{AB} = 23{,}000\ N$$

using this in the ΣF_1 equation:

$$-5000\,N + 16{,}250\,N + F_{AC} = 0$$

$$\Rightarrow F_{AC} = -11{,}250\ N$$

Proceed to Joint C



$$\Sigma F_1 = 0 \quad \overset{+}{\rightarrow} \quad \Rightarrow -F_{CA} + F_{CE} = 0$$

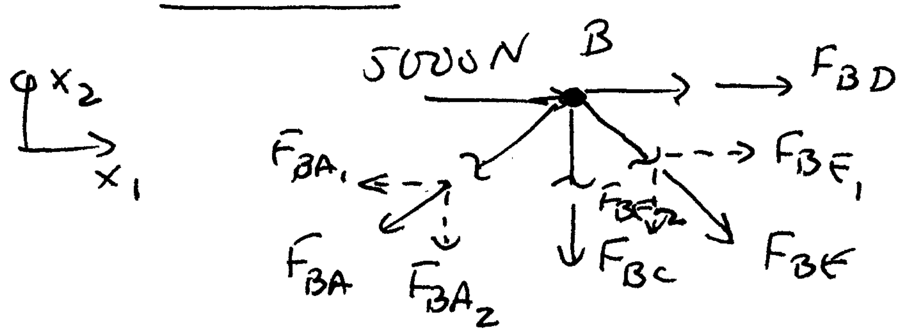
using $F_{AC} = F_{CA} = -11{,}250\ N$

$$\Rightarrow F_{CE} = -11{,}250\,N$$

$$\Sigma F_2 = 0 \quad \uparrow + \quad \Rightarrow 20{,}000\,N + F_{CB} = 0$$

$$\Rightarrow F_{CB} = -20{,}000\ N$$

Now Joint B



$$\sum F_1 = 0 \overset{+}{\rightarrow} \Rightarrow 5000\,N - 0.707\,F_{BA} + F_{BD} + 0.707\,F_{BE} = 0$$

using $F_{BA} = 23{,}000\,N$ gives

$$F_{BD} + 0.707\,F_{BE} = 11{,}250\,N$$

now:

$$\sum F_2 = 0 \uparrow + \Rightarrow -F_{BC} - 0.707\,F_{BA} - 0.707\,F_{BE} = 0$$

using $F_{BC} = -20{,}000\,N$, $F_{BA} = 23{,}000\,N$ gives:

$$20{,}000\,N - 16{,}250\,N - 0.707\,F_{BE} = 0$$

$$\Rightarrow F_{BE} = 3{,}750\,N/(0.707)$$

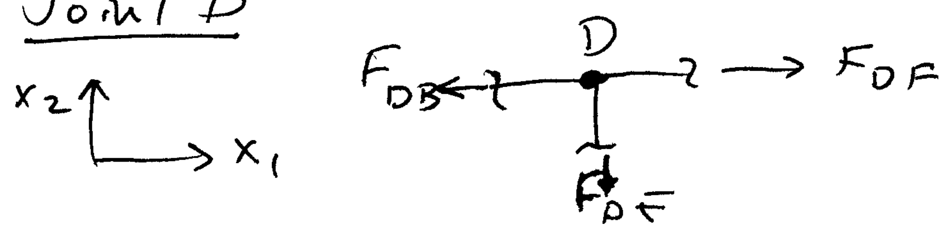
$$\Rightarrow F_{BE} = 5300\,N$$

using this in the resulting $\sum F_1$ equation above gives:

$$F_{BD} + 3750\,N = 11{,}250\,N$$

$$\Rightarrow F_{BD} = 7500\,N$$

To Joint D

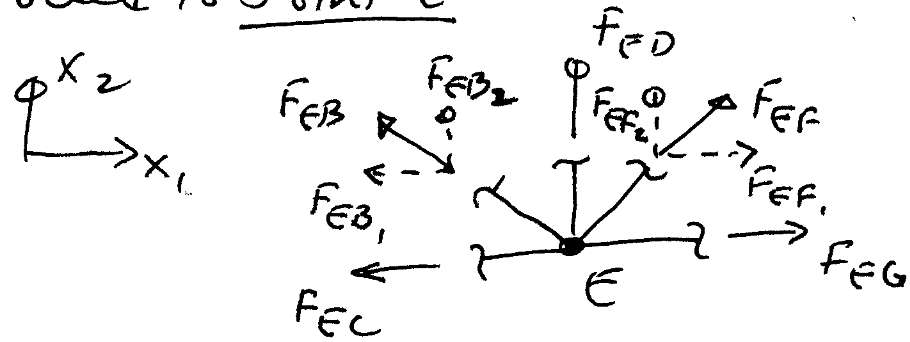


$$\sum F_1 = 0 \overset{+}{\rightarrow} \Rightarrow -F_{DB} + F_{DF} = 0$$

with $F_{DB} = 7500\,N \Rightarrow F_{DF} = 7500\,N$

$$\sum F_2 = 0 \uparrow + \Rightarrow F_{DE} = 0$$ (as in part (c))

Proceed to <u>Joint E</u>



$$\Sigma F_1 = 0 \ \overset{+}{\rightarrow} \ \Rightarrow -F_{EC} - 0.707 F_{EB} + 0.707 F_{EF} + F_{EG} = 0$$

using previous results of $F_{EB} = 5300\,N$
and $F_{EC} = -11,250\,N$ gives:

$$11,250\,N - 3750\,N + 0.707 F_{EF} + F_{EG} = 0$$

then also consider

$$\Sigma F_2 = 0 \ \uparrow + \ \Rightarrow 0.707 F_{EB} + F_{ED} + 0.707 F_{EF} = 0$$

already have $F_{ED} = 0$, with $F_{EB} = 5300\,N$

$$\Rightarrow 3750\,N + 0.707 F_{EF} = 0$$

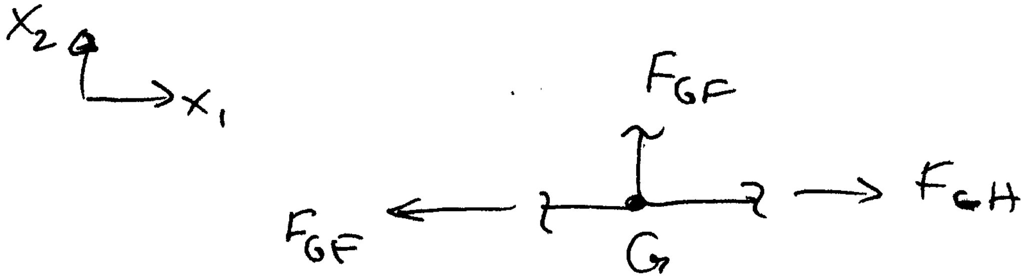
$$\Rightarrow F_{EF} = -5300\,N$$

use this in the results from ΣF_1 to get:

$$7500\,N - 3750\,N + F_{EG} = 0$$

$$\Rightarrow F_{EG} = -3,750\,N$$

Next to <u>Joint G</u>



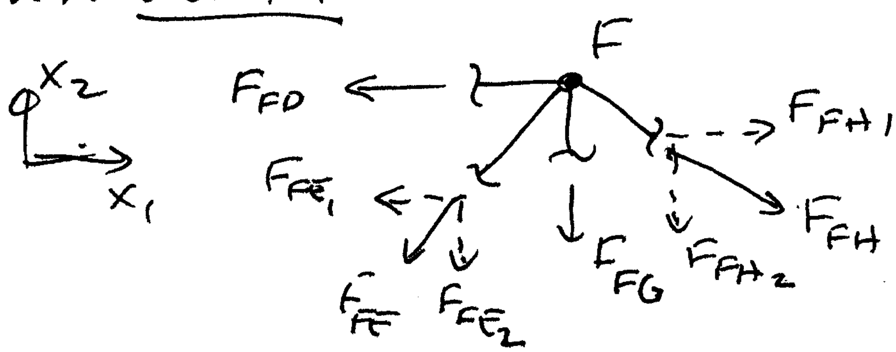
$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{GF} + F_{GH} = 0$

with $F_{GF} = -3{,}750\,N$ gives:

$F_{GH} = -3{,}750\,N$

$\sum F_2 = 0 \uparrow + \Rightarrow F_{GF} = 0$ (as in part (c))

then to <u>Joint F</u>



$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{FD} - 0.707\,F_{FE} + 0.707\,F_{FH} = 0$

with $F_{FD} = 7500\,N$ and $F_{FE} = -5300\,N$ gives:

$-7500\,N + 3750\,N + 0.707\,F_{FH} = 0$

$\Rightarrow F_{FH} = 5300\,N$

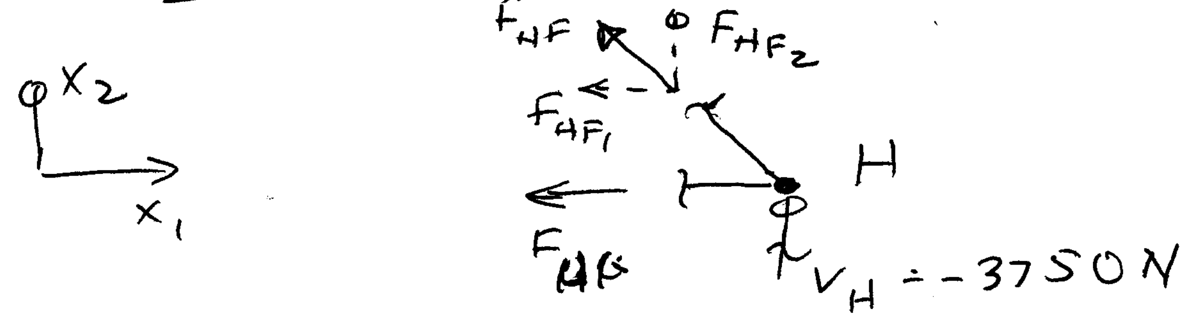
$\sum F_2 = 0 \uparrow + \Rightarrow -0.707\,F_{FE} + F_{FG} - 0.707\,F_{FH} = 0$

with $F_{FG} = 0$ and $F_{FE} = -5300\,N$ to get:

$3750\,N - 0.707\,F_{FH} = 0$

$\Rightarrow F_{FH} = 5300\,N$ ✓ <u>checks</u>

Then to Joint H to do final checks:



$$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -F_{HG} - 0.707 F_{HF} = 0$$

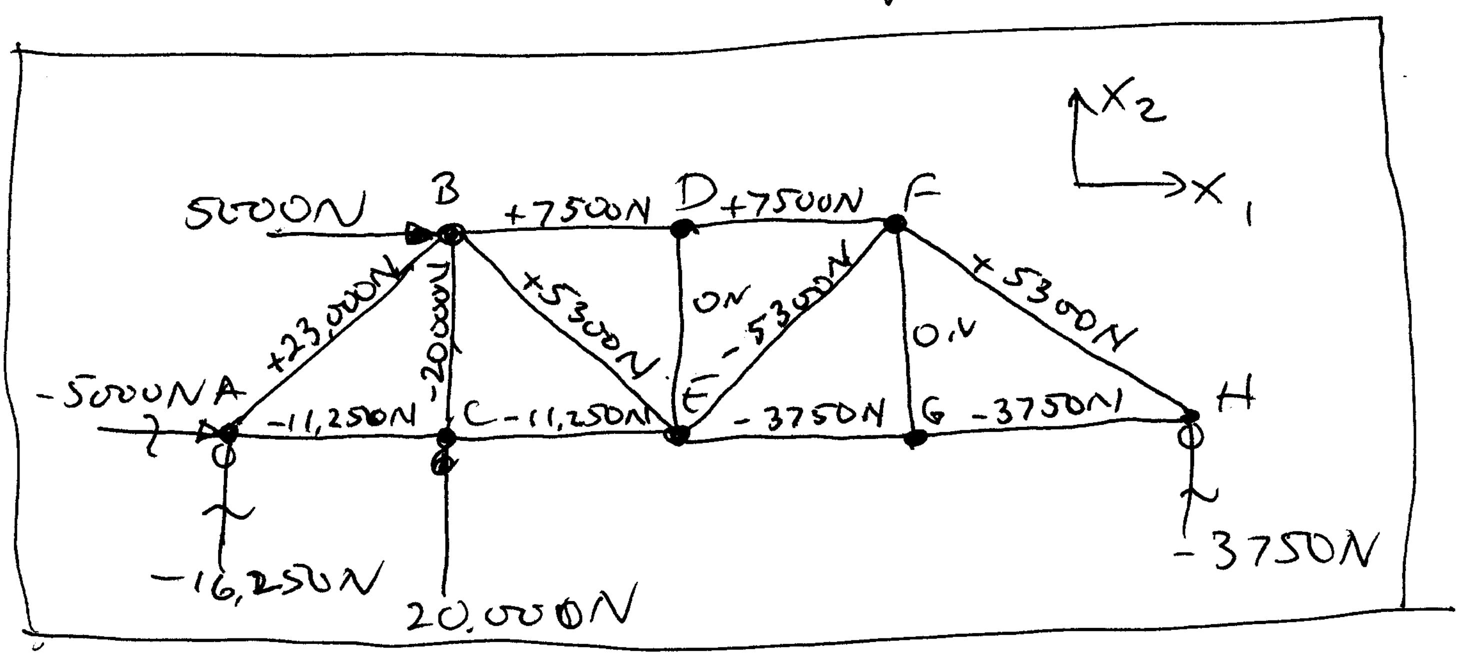
with $F_{HG} = -3{,}750\,N$ $\quad F_{HF} = 5300N$

$$\Rightarrow +3750N - 3750N = 0 \checkmark \text{ checks}$$

$$\Sigma F_2 = 0 \uparrow + \Rightarrow 0.707 F_{HF} + V_H = 0$$

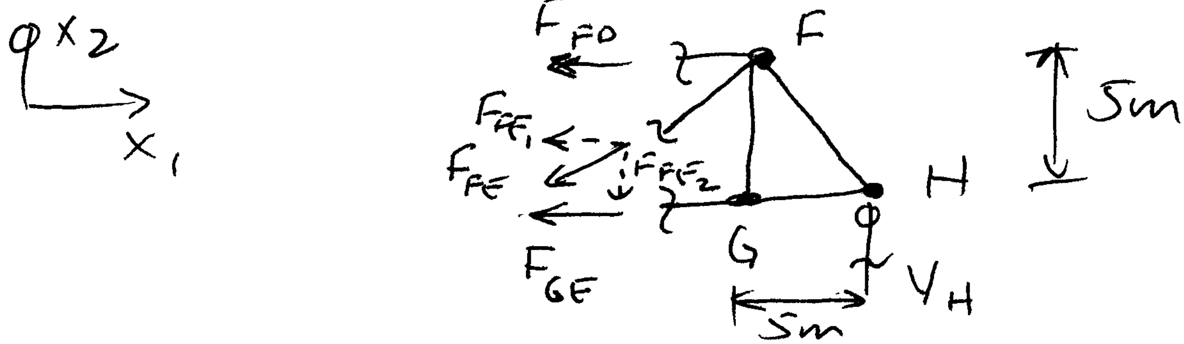
$$\Rightarrow 3750N - 3750N = 0 \checkmark \text{ checks}$$

Summarize by drawing the truss and placing the bar load above each bar with (+) tension and (−) compression



(e) To determine the load in the bar FG by the Method of Sections, a "sectional cut" must be made that goes through this bar. There must be only three bars of unknown load so that the three equilibrium equations yield a determinate solution.

Choose the following cut from the end of point H (the same could be done for point A -- less external loads/reactions are involved here):



Apply the equations of equilibrium:

$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{GE} - 0.707 F_{FE} - F_{FD} = 0$$

$$\sum F_2 = 0 \uparrow + \Rightarrow V_H - 0.707 F_{FE} = 0$$

with $V_H = -3750\ N \Rightarrow F_{FE} = -5300\ N$

$$\sum M = 0 \circlearrowleft + \Rightarrow V_H (5m) - F_{GE} (5m) = 0$$

(about point F)

$$\Rightarrow -3750\ N - F_{GE} = 0$$

$$\Rightarrow F_{GE} = -3750\ N$$ ✓ checks!

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Using these results in the ΣF_1 equation gives:

$$3750\,N - 0.707\,F_{FE} - F_{FD} = 0$$

$$\Rightarrow 3750\,N + 3750\,N - F_{FD} = 0$$

$$\Rightarrow F_{FD} = 7500\,N \quad \checkmark \text{ check}$$

$\Rightarrow$ all 3 check

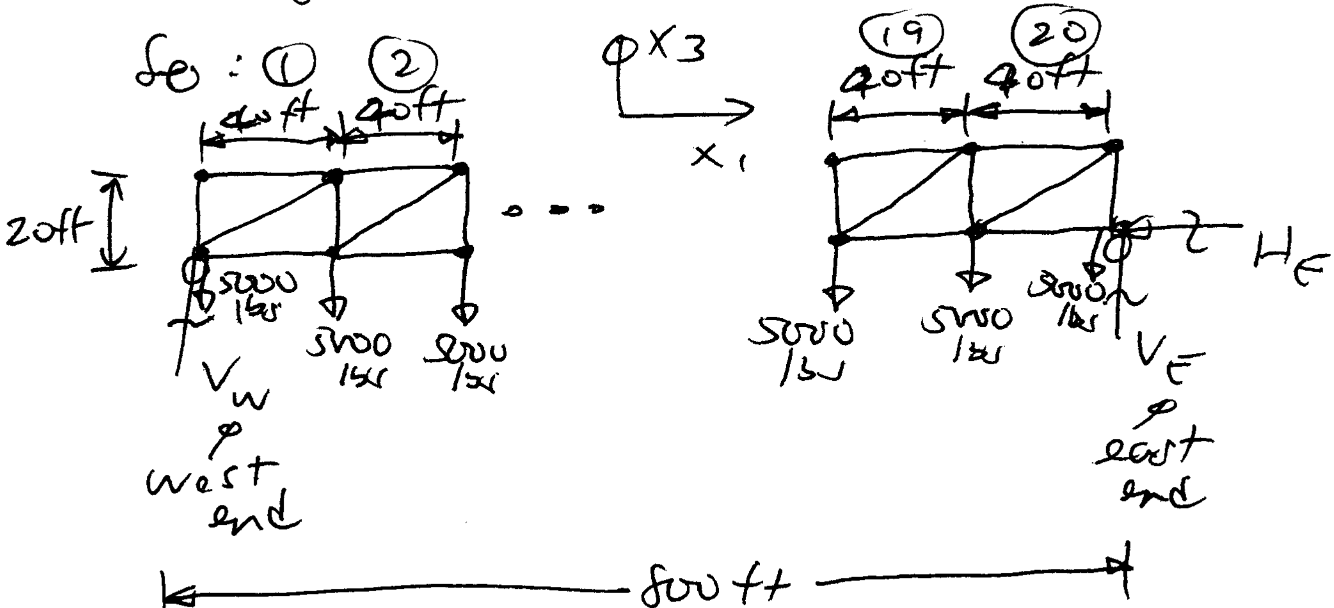
with $\boxed{F_{EG} = -3750\,N}$

as requested

M4.2

(a) The first step is to draw the free Body Diagram. One could draw the entire structure or do as done in the problem statement showing the applied load applied downward with a magnitude of 5000 lbs every 20 feet.

So:



apply equilibrium:

$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -H_E = 0$$

$$\sum F_3 = 0 \uparrow + \Rightarrow V_W + V_E - 5000\,\text{lbs}(21) = 0$$

there are 21 points of applied load. The first at the west end and one at the end of each bay.

$$\Sigma M = 0 \ \circlearrowleft^{+} \Rightarrow V_E(800\text{ ft}) - \sum_{i=1}^{20}(5000\text{ lbs})\underbrace{(i)(40\text{ft})} = 0$$

about west end

each load at the end of the bay is applied at (20ft)(i) from the west end

$$\sum_{i=1}^{20} i = 210$$

so:

$$V_E(800\text{ ft}) = (5000\text{ lbs})(210)(40\text{ ft})$$

$$\Rightarrow V_E = 52{,}500\text{ lbs}$$

using this in the ΣF_3 equation gives:

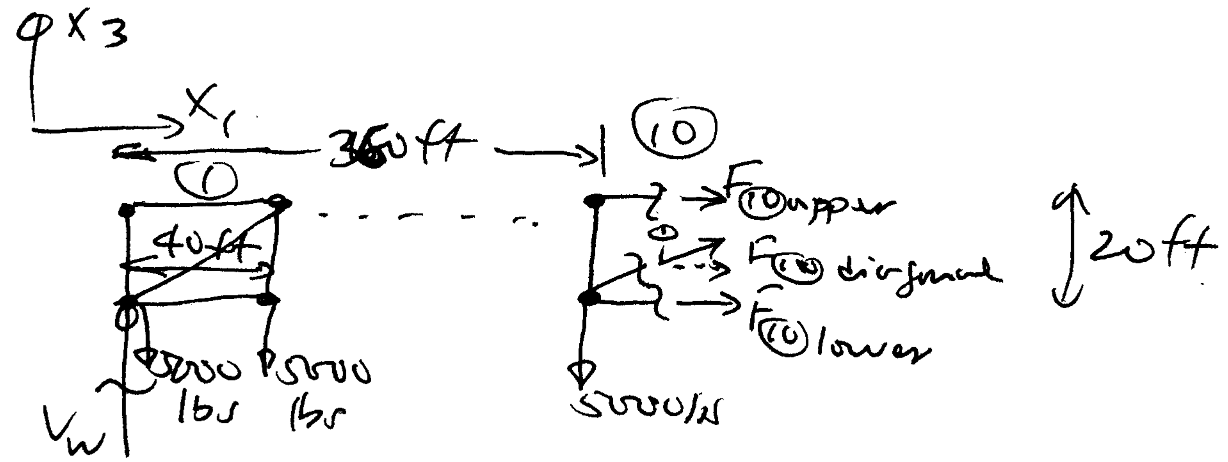
$$V_W + 52{,}500\text{ lbs} - 105{,}000\text{ lbs} = 0$$

$$\Rightarrow V_W = 52{,}500\text{ lbs}$$

Summarizing:

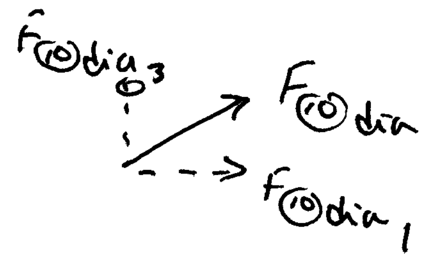
$$\boxed{\begin{aligned} V_W &= 52{,}500\text{ lbs} \\ H_E &= 0\text{ lbs.} \\ V_E &= 52{,}500\text{ lbs.} \end{aligned}}$$

(b) To get the bar loads in the bars in the 10th bay, one could use the Method of Joints or the Method of Sections. Using the Method of Joints requires working from a point with a known load and progressing to the desired point. This can involve significant points and equations. The Method of Sections is generally more efficient in such situations. Make a cut through the bars of the 10th bay (horizontal and diagonal as desired) and draw the resulting free body diagram:



Resolve $F_{⑩}$ diagonal into x_3 and x_1 components

Note via geometry the angle of the diagonal bar:



$$\theta = \tan^{-1} \frac{20\,ft}{40\,ft}$$

$$\Rightarrow \theta = 26.6^{\circ}$$

and $F_{(10)\,diagonal}\,(x_1) = F_{(10)\,dia} \cos\theta = 0.89\,F_{(10)\,dia}$

$F_{(10)\,diagonal}\,(x_3) = F_{(10)\,dia} \sin\theta = 0.45\,F_{(10)\,dia}$

Now apply equilibrium

$$\Sigma F_1 = 0 \ \xrightarrow{+} \ \Rightarrow F_{(10)\,lower} + F_{(10)\,upper} + 0.894\,F_{(10)\,dia} = 0$$

$$\Sigma F_3 = 0 \ \uparrow + \ \Rightarrow V_w - 5000\,lbs\,(10) + 0.447\,F_{(10)\,dia} = 0$$

one load at the beginning node of each bay

with $V_w = 52{,}500$ lbs gives:

$$2{,}500\,lbs + 0.447\,F_{(10)\,dia} = 0$$

$$\Rightarrow F_{(10)\,dia} = -5580\,lbs$$

$$\Sigma M = 0 \;\; (\circlearrowleft \Rightarrow -V_w(360\,ft) - F_{⑩\,upper}(20\,ft) + \sum_{i=1}^{9} 5000\,lbs\,\underbrace{(i)(40\,ft)}_{\text{same consideration as before}} = 0$$

(about bottom node of ⑩)

$$\sum_{i=1}^{9}(i) = 45$$

$$\Rightarrow (-52{,}500\,lbs)(360\,ft) - F_{⑩\,upper}(20\,ft) + 5000\,lbs(45)(40\,ft) = 0$$

$$\Rightarrow F_{⑩\,upper} = -945{,}000\,lbs + 450{,}000\,lbs$$

$$\Rightarrow F_{⑩\,upper} = -495{,}000\,lbs$$

using these results in the ΣF_1 equation gives:

$$F_{⑩\,lower} - 495{,}000\,lbs - 5000\,lbs = 0$$

$$\Rightarrow F_{⑩\,lower} = 500{,}000\,lbs.$$

summarizing:

$$\boxed{\begin{aligned} F_{⑩\,lower} &= 500{,}000\,lbs. \\ F_{⑩\,upper} &= -495{,}000\,lbs \\ F_{⑩\,diagonal} &= -5580\,lbs \end{aligned}}$$