Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 5

Name: ______________________

Due Date: 10/11/2006

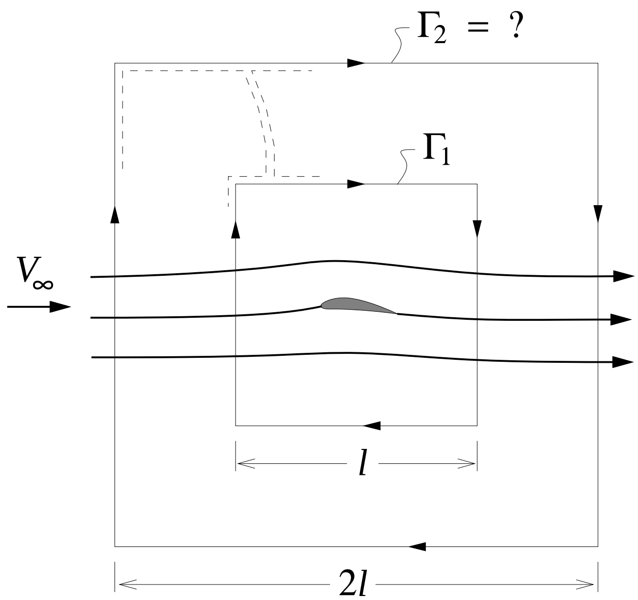
	Time Spent (min)
F11	
F12	
F13	
F14	
M5.1	
M5.2	
M5.3	
Study Time	

Announcements:

We will assume here that the boundary layers on an airfoil are neglibly thin, so that the velocity field about this airfoil is irrotational effectively everywhere (i.e. $\vec{\xi} = \nabla \times \vec{V} = 0$).

A square circuit of side length ℓ is drawn the airfoil as shown in the figure, and the circulation about this circuit is known to have some positive value Γ_1. A second square circuit of side length 2ℓ is also placed around the airfoil as shown. You are to determine Γ_2 in terms of the other quantities in the figure.

Hint: Consider the circulation about a third circuit, part of which is shown as a dotted line in the figure.



A flow is defined by the following velocity components:

$$u = x^2 \qquad\qquad v = -2xy$$

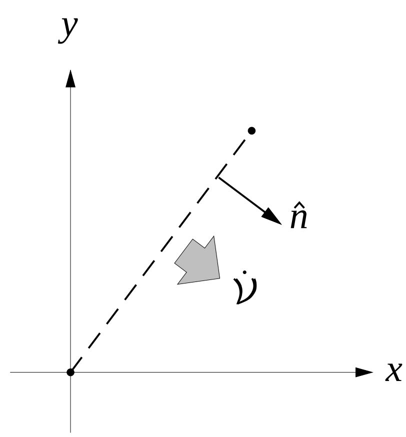
a) Determine the streamfunction $\psi(x, y)$ for this flow.

b) Sketch a few streamlines of this flow in the upper right quadrant.

c) Determine the volume flow rate

$$\dot{\mathcal{V}} \;=\; \int \vec{V} \cdot \hat{n}\, dA$$

passing through the diagonal line from $(x, y) = (0, 0)$ to $(2, 3)$.



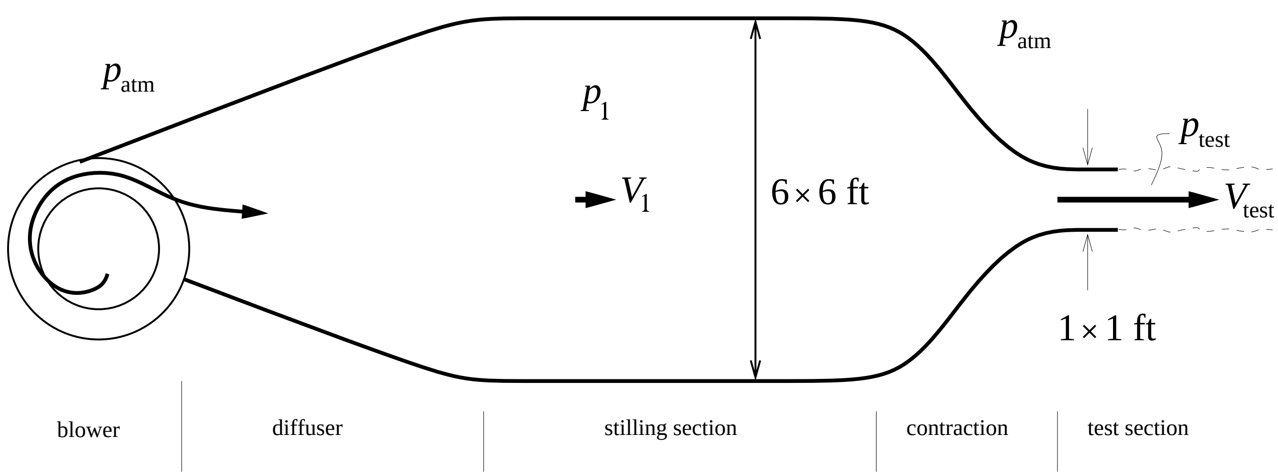
The tunnel in the Bldg 33 hangar has a centrifugal blower, followed by a conical diffuser, followed by a large constant-area stilling section roughly $6 \times 6\,\text{ft}$ in cross-section, followed by a contraction into the $1 \times 1\,\text{ft}$ open-jet test section. The ambient atmospheric pressure in the hangar is $p_{\text{atm}} = 10^5\,\text{Pa}$. The air density is approximately $\rho = 1.2\,\text{kg/m}^3$ everywhere.

a) What do expect the difference $p_{\text{test}} - p_{\text{atm}}$ to be? Explain your reasoning.

b) Determine the V_1 which must exist in the stilling section if the exit has some known V_{test}.

c) Determine the pressure p_1 that the blower must generate in the stilling section so that the exit velocity is $V_{\text{test}} = 50\,\text{m/s}$ (about 111 mph).

d) When operating, the tunnel is in effect acting like a jet engine. Determing the tunnel's thrust force along the flow axis, at the operating condition in c).
Note: The blower takes in air at right angles to the tunnel's flow axis, so this doesn't contribute to the axial thrust.



The boundary layer flow above a solid wall is approximated by a 2-piece quadratic + constant velocity distribution.

$$u = \begin{cases} C\left[2y/\delta - (y/\delta)^2\right] & , \text{ for } y < \delta \\ C & , \text{ for } y > \delta \end{cases}$$

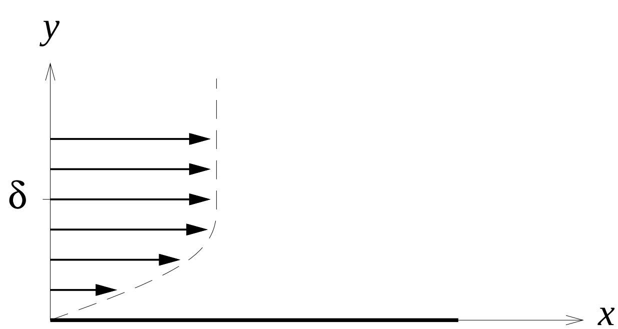
$$v = 0$$

where C is some constant.

a) Determine the vorticity $\xi(x, y)$ and stream function $\psi(x, y)$ for this flow. Sketch ξ versus y, and ψ versus y.

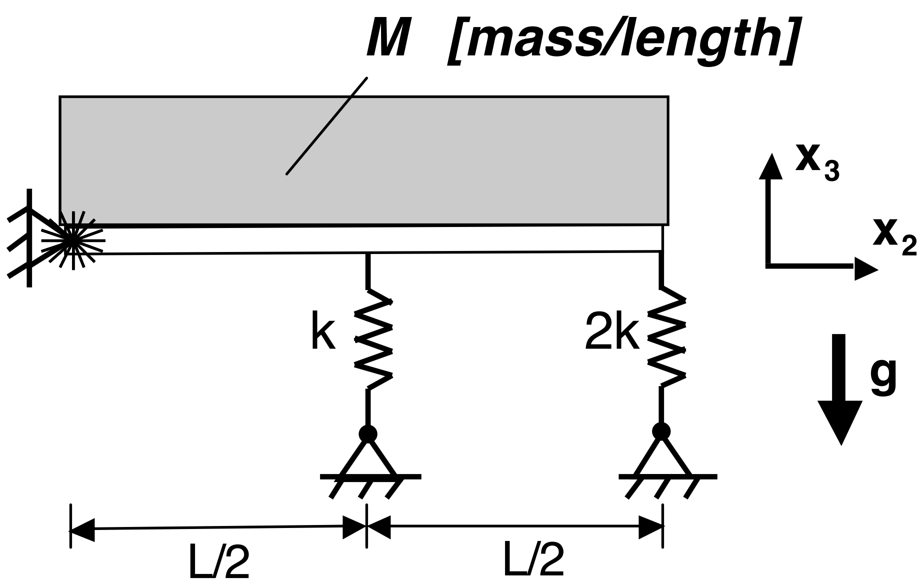
b) Can this entire flow be given in terms of a velocity potential $\phi(x, y)$? Explain.

c) Can a portion of this flow be given in terms of a velocity potential $\phi(x, y)$? Determine and sketch the isopotential lines in the part of the x-y plane where this is possible.



Unified Engineering Problem Set **Lectures: M9, M10, M11**
Week 5 Fall, 2006 **Units: (M1.4), M1.5, M2.1 (part)**

M5.1 ***(10 points)*** A rigid bar of length L is supported by three springs as shown. The spring at the left end is of the torsional type and requires a moment of the same sense to the angle of rotation for displacement giving a constitutive relation of: M = k_T□, where k_T is the torsional spring constant. This is in addition to any reaction(s) due to the pin support at that point. The other two springs are of the linear type with the spring located at mid-span having a constant of k, and the spring at the right end having a constant of 2k. The bar is loaded due to the weight from gravity considerations caused by a distributed mass of constant height across the span of the bar of mass magnitude M [mass/length].



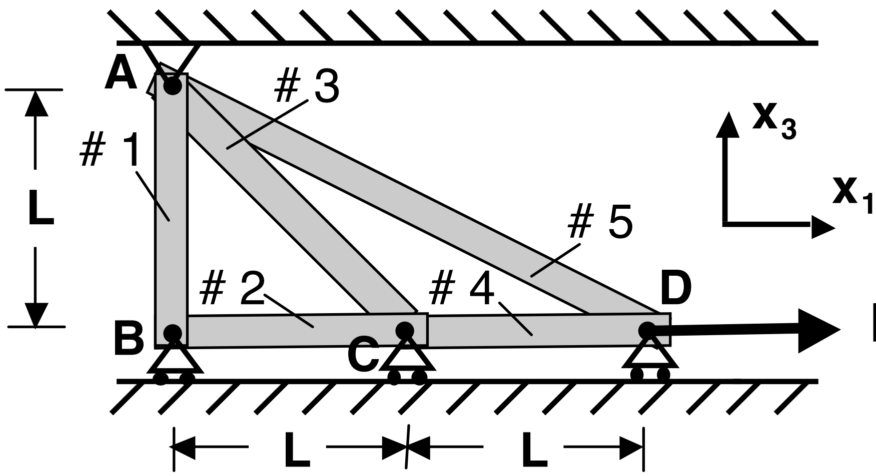
(a) Draw the free body diagram(s) for this situation (Consider the overall system and any appropriate subsystems).

(b) Determine whether this structural configuration is statically determinate or statically indeterminate and clearly explain your reasoning.

(c) Determine the manifestation of the Compatibility of Displacement for this configuration.

(d) Determine the reaction forces, the deflection of each spring, and the overall deflection of the bar.

M5.2 ***(15 points)*** A five-member truss arrangement between two floors in a building structure is used to resist a positive load applied horizontally as in the illustration. Three bars are attached to a common pin support at an overhead floor and each is attached to individual roller supports at the lower floor. Bars, of length L, are also attached between these roller supports. The floor height is also L. Each bar has the same cross-sectional area, A, and material modulus of elasticity, E, varying only in length and angle of orientation.



(a) Draw the free body diagram for this situation. (Consider the overall system and any appropriate subsystems.)

(b) Determine whether this structural configuration is statically determinate or statically indeterminate and clearly explain your reasoning.

(c) Determine the bar loads, the reaction forces, the deflection of each of the roller supports, and the associated deflections of each bar. Assume that all deflections are small and clearly explain any associated assumptions.

(d) If the deflections were to become "large", are there any adjustments which would need to be made in the analysis? If so, describe these adjustments and give your reasoning.

M5.3 ***(5 points)*** Write out the following tensor equations in full:

(**Note:** these equations do not necessarily have any real meaning)

(a) $\square_{13} = \ell_{1i'} \ell_{3k'} \square'_{i'k'}$

(b) $L_{\square\square} = \square_{ij} C_{\square i} M_{\square j}$

(c) $D_{pq} = S_{pqrs} z_r \square_s$ (for $p = 3$, $q = 2$)

(d) $E = 1/2 \, \square_{\square\square} \square_{\square\square}$

(e) $B_{ij} (\partial b / \partial x_j) + f_i = 0$