Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 6

Name: _____

Due Date: 10/17/2006

	Time Spent (min)
F15	
M6.1	
M6.2	
M6.3	
Study Time	

Announcements:

The potential and streamfunction of a freestream + source flow are given by

$$\begin{aligned} \phi(x,y) &= V_\infty x + \frac{\Lambda}{2\pi} \ln\sqrt{x^2+y^2} \\ \psi(x,y) &= V_\infty y + \frac{\Lambda}{2\pi} \arctan(y/x) \end{aligned}$$

or alternatively,

$$\begin{aligned} \phi(r,\theta) &= V_\infty r \cos\theta + \frac{\Lambda}{2\pi} \ln r \\ \psi(r,\theta) &= V_\infty r \sin\theta + \frac{\Lambda}{2\pi} \theta \end{aligned}$$

These assume the source is at the origin. The corresponding streamline pattern is shown on page 3 of the F15 notes. Assume that $V_\infty = 1$, $\Lambda = 1$, for questions a) ... d) below.

a) Determine the x, y location of the stagnation point.

b) For the dividing streamline passing through the stagnation point, determine the numerical value of the streamfunction ψ as given by one of the above expressions.

c) Determine the y location of the stagnation streamline at $x = 0$ (directly above the source).

d) At the x, y location found in c), determine the surface slope dy/dx of the stagnation streamline.

e) Answer a) ... d) again for the case of a doubled source strength, $V_\infty = 1$, $\Lambda = 2$. It's OK to just write down the answers, by inspection of your previous work.

Unified Engineering Problem Set **Lectures: M12, M13**
Week 6 Fall, 2006 **Units: M2.1, U-A (part)**

M6.1 ***(5 points)*** Write out the succinct tensor equation that describes the following notation:

$$\begin{bmatrix} S_{111} & 2S_{112} & S_{122} \\ S_{211} & 2S_{212} & S_{222} \\ S_{311} & 2S_{312} & S_{322} \end{bmatrix} \begin{Bmatrix} F_{11} \\ F_{12} \\ F_{22} \end{Bmatrix} = \begin{Bmatrix} \square_1 \\ \square_2 \\ \square_3 \end{Bmatrix}$$

M6.2 ***(5 points)*** For the following sets of equations, write the tensorial equation that represents the equations. Be sure to explain all steps:

(a) $a_{11}\, q_1 q_1 + a_{21}\, q_2 q_1 + a_{12}\, q_1 q_2 + a_{22}\, q_2 q_2 = f_1\, g_1 + f_2\, g_2$

(b) $R_1 = b_{11} \sin \pi x$
$R_2 = b_{22} \sin 2\pi x$
$R_3 = b_{33} \sin 3\pi x$

M6.3 ***(10 points)*** A second axis system is defined by rotating the x_2- x_3 plane by an angle $\square$ of 30° about the x_1 axis. A force vector, $\underline{F}$, is described in the original axis system as (NOTE: The vector is unitless):

$$\underline{F} = 8\,\underline{i}_1 - 5\,\underline{i}_2 + 3\,\underline{i}_3$$

(a) Determine the expression for the vector in the rotated coordinate system.

(b) Prove, by some means, that these two expressions are equivalent.