Massachusetts Institute of Technology
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16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 7

Name: ______________________

Due Date: 10/24/2006

	Time Spent (min)
F16	
F17	
M7.1	
M7.2	
M7.3	
Study Time	

Announcements:

As given in the F16 notes, the streamfunction of the flow about a circular cylinder is given by

$$\psi \;=\; V_\infty \, r \sin\theta \left(1 \;-\; \frac{R^2}{r^2}\right) \;+\; \frac{\Gamma}{2\pi} \ln r$$

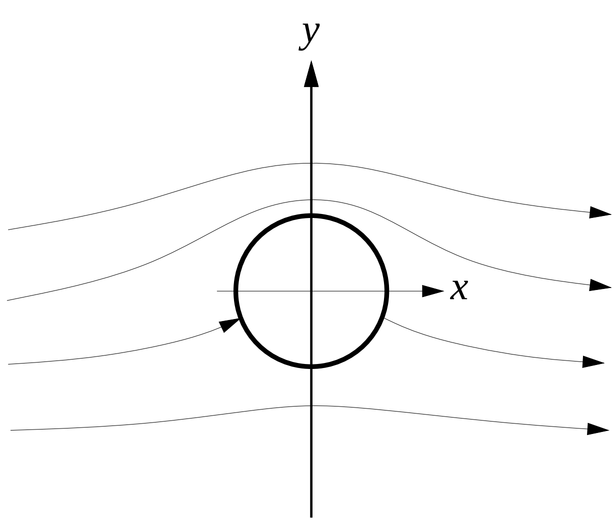
where Γ is some arbitrary circulation. For this exercise, assume a value of $\Gamma = 4\, V_\infty R$ which might be typical on a spinning cylinder.

a) Determine the horizontal velocity $u(y)$ along the y-axis, in the form of three terms $u(y) = u_{\text{freestream}}(y) + u_{\text{vortex}}(y) + u_{\text{doublet}}(y)$. Write out the three terms separately.

b) Plot the three separate u components as u/V_∞ versus y/R on the same plot. Make your plot go over the range $y/R = -10 \ldots 10$, but ignore the non-physical $u(y)$ inside the cylinder, over the center portion $y/R = -1 \ldots 1$.
Optional: You may also wish to plot the total u/V_∞ vs y/R to help answer c) and d) below.

c) Describe the relative importance of u_{vortex} and u_{doublet} to the total u for
i) small distance from the cylinder surface, and
ii) large distance from the cylinder surface.

d) Justify the statement *"To an observer far away, a lifting 2D object looks like a point vortex placed in the flow"*.



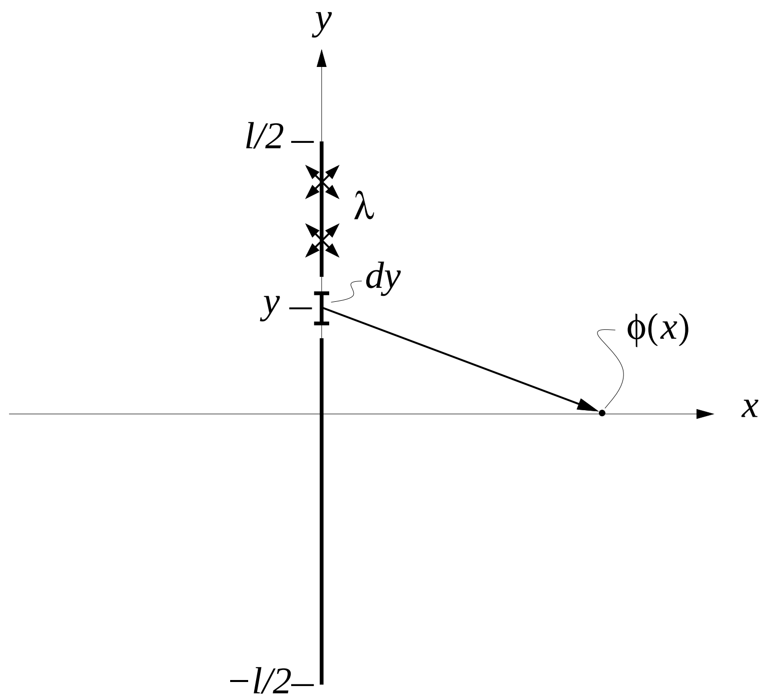
A vertical source sheet of constant strength λ is centered on the origin as shown. Following the notes, the potential function contribution of a small dy piece of the sheet at location y is given by

$$d\phi \;=\; \frac{\lambda\, dy}{2\pi}\, \ln \sqrt{x^2 + y^2}$$

a) Determine the potential function $\phi(x)$ of the entire sheet along the x axis. (This is a subset of determining $\phi(x, y)$ in the entire plane).

b) Determine the horizontal velocity $u(x)$ of the sheet on the x-axis. Also determine the horizontal velocity $u_{\mathrm{point}}(x)$ of a point source placed on the origin, with the same total source strength Λ as the sheet. Explicitly state the relation between λ and Λ in this case.

c) Plot $u(x)$ and $u_{\mathrm{point}}(x)$ overlaid. Set the plot scale to ignore the singularity in u_{point} at $x = 0$. Describe how the two curves relate at large x distances.



From CRC Handbook:

$$\int \ln(x^2 + a^2)\, dx \;=\; x\, \ln(x^2 + a^2) \;-\; 2x \;+\; 2a\, \arctan(x/a) \;+\; C$$

Unified Engineering Problem Set **Lectures: M14, M15, M16**
Week 7 Fall, 2006 **Units: M2.2, M2.3**

M7.1 ***(10 points)*** Consider the stress equations of equilibrium.

(a) Write these equations in engineering notation.

(b) Reduce the full three-dimensional equations to their plane stress form.

M7.2 ***(10 points)*** A three-dimensional body is subjected to a state of stress such that the three extensional stresses are all tensile and constant, but of different magnitudes: A in the 1-direction, B in the 2-direction, and C in the 3-direction. The shear stress in the 2-3 plane varies linearly in both the x_2 and x_3 directions. The magnitude of this variation is equal to +D in x_2, and −E in x_3. There are no body forces.

Determine as much as you can about the overall stress field of this three-dimensional body using functional forms with further description as needed. Be sure to explain all reasoning clearly.

M7.3 ***(10 points)*** For each of the following two-dimensional displacement fields (no displacement in the x_3- direction), draw a neat sketch of a unit square (with the bottom left corner at the origin) before and after deformation (exaggerate deformation by a factor of 10). Calculate the associated two-dimensional strain field. Identify the "type" of deformation/strain that characterizes the displacement.

(a) $\underline{u} = (0.020\, x_2)\, \underline{i}_1 + (0.040\, x_1)\, \underline{i}_2$

(b) $\underline{u} = -(0.030\, x_2)\, \underline{i}_1 + (0.030\, x_1)\, \underline{i}_2$

(c) $\underline{u} = (-0.050\, x_1 + 0.030\, x_2)\, \underline{i}_1 + (0.030\, x_1 + 0.010\, x_2)\, \underline{i}_2$

(d) $\underline{u} = -(0.020)\, \underline{i}_1 + (0.030)\, \underline{i}_2$

(e) $\underline{u} = -(0.030\, x_1)\, \underline{i}_1 + (0.020\, x_2)\, \underline{i}_2$