Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 9

Name: ______________________

Due Date: 11/07/2006

	Time Spent (min)
M9.1	
M9.2	
M9.3	
T1	
T2	
T3	
Study Time	

Announcements:

Unified Engineering Problem Set **Lectures: M20, M21, M22**
Week 9 Fall, 2006 **Units: M3.1 (part), M3.2**

M9.1 ***(10 points)*** A truss is to be made of a number of bars, all of the same solid circular cross-section and the same given length. Each bar must carry a constant load, in either tension or compression, of no greater a magnitude than P. The key design criteria at this point are that the bars are to deform as little as possible and weigh as little as possible. Cost is also a consideration. The design variables are the bar diameter and the material used to make the bar.

(a) Determine the figure(s) of merit that is/are pertinent in this case.

(b) For the materials listed in the accompanying table, indicate which you would choose for the bar depending upon which of the three design considerations are most important: minimization of deformation, minimization of weight, minimization of cost. Be sure to clearly explain your reasoning. Use figures as appropriate.

Material	**Density** [lb/in^3]	**Modulus** [Msi]	**Strain Limit**, %	**Acquisition Cost**, [$/lb]
Wood	0.022	1.81	0.35	0.97
Aluminum alloy (2000 series)	0.101	10.5	0.58	6.20
Titanium alloy (TI-6Al-4V)	0.162	16.0	0.73	23.25
Carbon fiber Composite	0.054	24.2	0.50	80.00
Steel (low carbon)	0.285	29.0	0.60	1.30
Silicon Carbide	0.108	60.5	2.20	141.00

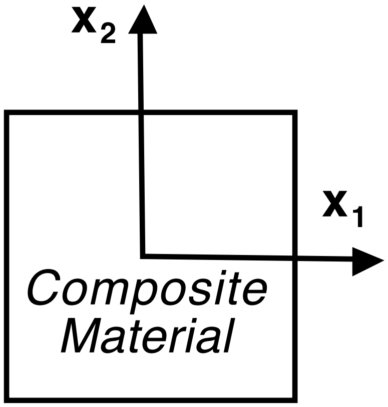
M9.2 ***(10 points)*** The compliance tensor is a key part of the overall set of stress-strain relationships. Let's explore these ties to the three-dimensional compliance tensor in the following way:

(a) Write out, in full, the tensorial version of the three-dimensional compliance relations for the complete anisotropic case. Group the components of the compliance tensor into the three groups (as done for the elasticity tensor in the lecture notes).

(b) Reduce these compliance relations to the orthotropic case and relate the engineering constants to the components of the compliance tensor for this case.

(c) Using the results of (a) and (b), as appropriate, show how to relate the engineering constants back to the components of the elasticity tensor.

M9.3 ***(10 points)*** A composite material is made with woven fibers in multiple directions and with a polymer matrix. A set of three experiments are performed on this composite material. The stresses applied in each case are noted and various strains measured. Note that the strain gage in the 1-direction broke during Experiment B and no readings were obtained. The stresses and strains for the three experiments are:



Experiment A

σ_{12} = 300 MPa
ε_{12} = 7200 μstrain

Experiment B

σ_{11} = 250 MPa
σ_{22} = 250 MPa
ε_{22} = 3850 μstrain

Experiment C

σ_{11} = 400 MPa
ε_{11} = 5300 μstrain
ε_{22} = - 1550 μstrain

Note that any stresses or strains not specified are equal to zero. Also, all strains are tensorial

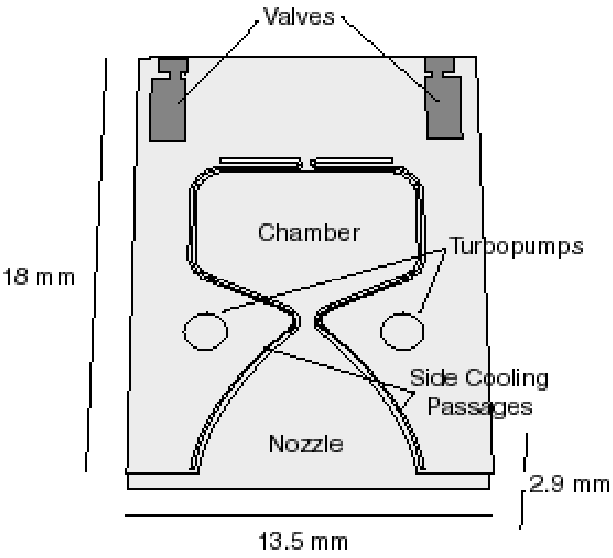
(a) Determine the in-plane engineering constants (all possible) and characterize the stress-strain behavior of the material.

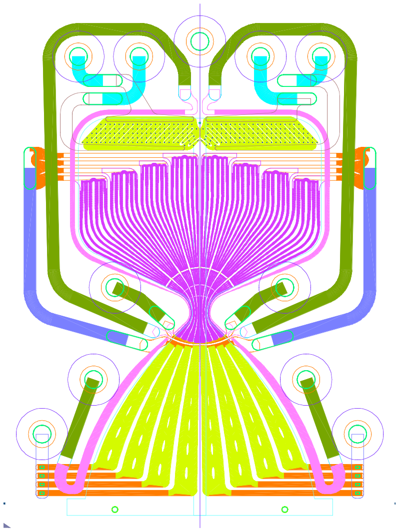
(b) Determine what the broken strain gage along the 1-direction in Experiment B should have read?

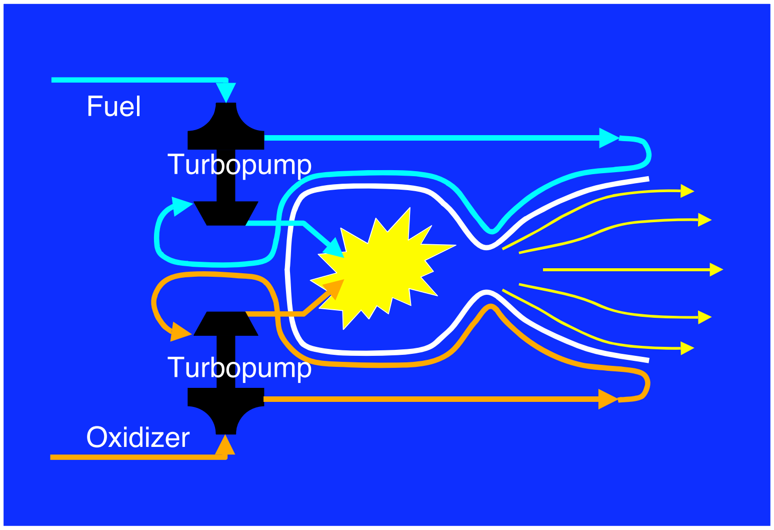
(c) Determine as many components of the compliance tensor as possible and put this in matrix form.

Problem T1. (Unified Thermodynamics)
Below is a schematic of liquid propellant (methane and oxygen), regeneratively-cooled, turbopump-pressurized micro-rocket designed by Adam London (MIT, PhD Thesis, 2000). The schematic on the left shows the overall architecture of the rocket engine. The schematic on the right shows the details of the coolant passages for the fuel and oxidizer. The schematic on the bottom shows the principal of operation (you can learn more about this kind of rocket engine by researching turbo-expander cycles).

<u>Describe the conversions of energy (internal, potential, kinetic and chemical) and exchanges of heat and work for this system.</u> (LO#1, LO#2, LO#3)





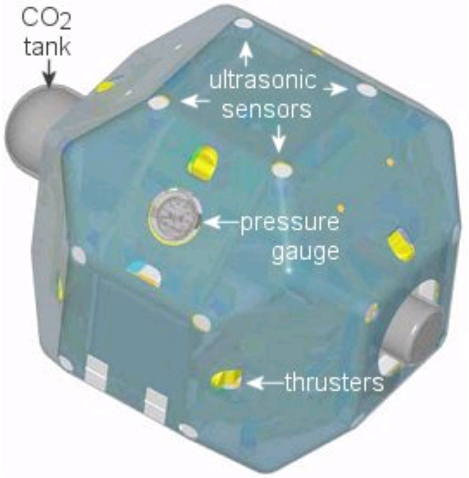


UNIFIED ENGINEERING Fall 2006
Christine Edwards

Problem T2. (Unified Thermodynamics)

The MIT Space Systems Lab (SSL) has developed a small satellite system called SPHERES that is currently being tested by the astronauts and cosmonauts on the International Space Station (ISS). The propulsion system is pneumatic, consisting of a CO_2 tank that ejects gas out of the thrusters.

MIT undergrads flight-test a SPHERE onboard NASA's KC-135 reduced gravity aircraft.



A CAD-model of a SPHERES satellite.

Assume that at the beginning of an ISS test, the tank is filled with gaseous CO_2 with the initial conditions p_i = 300 psi and T_i = 22°C.

a) As the astronaut places the CO_2 tank inside of the SPHERE, the temperature of the tank increases to 27°C. Assume the tank is rigid. What is the pressure in the tank at this condition?

b) How much work was done during the heating process in part a)?

c) After the tank is in the SPHERE, the SPHERE maneuvers itself so that it flies in a circle inside the ISS node. CO_2 is let out of the tank such that the pressure decreases to 230 psi. This time, assume the walls of the tank contract quasi-statically a small amount such that $dp/dv = 1e05$ Mpa*kg/m^3. What is the final temperature in the tank? (Assume that the initial conditions for part c are the final conditions of part a)

d) How much work was done during the depressurization process in part c)?

e) Sketch processes a) and c) on a p-v diagram. To make the sketch clearer you may want to accentuate the changes in volume with pressure for the second process. Also, include a series of isotherms in the background as a reference.

UNIFIED ENGINEERING

Fall 2006
Ian A. Waitz

Problem T3. (Unified Thermodynamics)

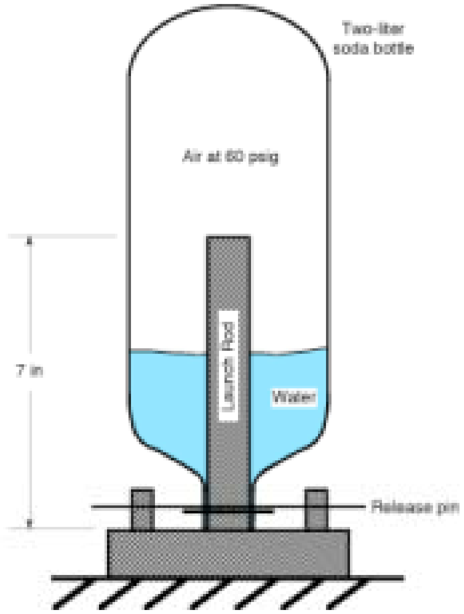
Consider the following two processes for charging a water rocket with high pressure air. For each, the initial condition is the same: 101.3 kPa and T=288 K. Assume that the bottle walls are rigid, the bottle volume is 2.3 x 10^{-3} m^3 and that it is 40% filled with water. Assume that c_p = 1003.5 J/kg-K, c_v = 716.5 J/kg-K, and R = 287 J/kg-K.

Process 1: isothermal compression to a pressure of 45 psig (308.7 kPa)

Process 2: adiabatic compression to 45 psig

Process 3: adiabatic compression to 45 psig followed by constant volume cooling to T = 288 K

a) Sketch each process on the same p-v diagram.
b) How much work is done by the gas for each process?
c) How much heat is transferred to the gas for each process?
d) Which process do you think is most advantageous if your objective is to maximize the height that the water rocket will go? Why? Note that during the launch, the air in the bottle can be assumed to expand via a quasi-static, adiabatic process. Support your answer with a calculation.



(LO#4, LO#5)