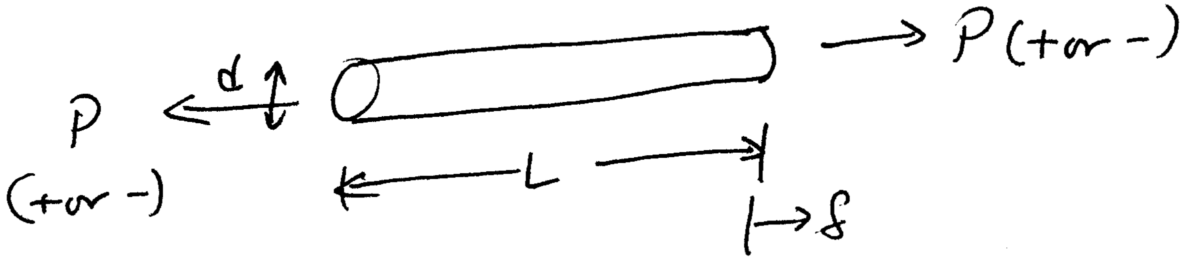
PAL
10/26/06

Unified Engineering Problem Set
Week 9 fall, 2006

SOLUTIONS

M 9.1 Bar is of a given length with a solid circular cross-section and it must carry a constant load, in tension or compression, of no greater magnitude than P.



δ = bar deformation

(a) List the constants: P, L

Requirement: Carry load of a magnitude no more than P

Needs: Deform as little as possible and weigh as little as possible

Design variables: bar diameter and material used

→ List items to be considered for minimization etc.:

mass/weight (m)
deformation (δ)
cost (c)

→ List key equations:

stress-strain: $\sigma = E\epsilon$ (1)
strain-displacement: $\epsilon = \delta / L$ (2)
stress-load: $\sigma = P/A$ (3)
area-diameter: $A = \pi (d/2)^2$ (4)
mass (weight) - density: $M = \rho A L$ (5)
cost: (cost/weight)(weight) = Cost (6)

→ List other variables, parameters:

E = modulus
d = diameter
A = area
ρ = density

The figures of merit are based on the overall items to be considered and expressing these in terms of geometrical and material parameters/properties

→ First consider deformation:

- From (2): $\delta = \epsilon L$
- use (1) to give: $\epsilon = \sigma / E$

  and thus: $\delta = \dfrac{\sigma L}{E}$

- Now use (3) in this:

$$\delta = \frac{PL}{EA}$$

- Finally use (4) to get this in terms of load, length, diameter, and modulus:

$$\delta = \frac{PL}{E\pi (d/2)^2}$$

$$\Rightarrow \boxed{\delta = \frac{4PL}{\pi E d^2}}$$

First <u>Figure of</u> <u>Merit</u>
(* 1)

→ Now consider mass/weight:

- from (5): $\text{Weight} = \rho A L$

  ρ: weight density

- use (4) to get in terms of (load), length and diameter:

$$\text{weight} = \rho \pi \left(\frac{d}{2}\right)^2 L$$

$\Rightarrow$ $\boxed{\text{weight} = \frac{\pi}{4}\rho d^2 L}$ Second Figure of Merit (* 2)

→ Finally consider cost:

- from (6):

$$\text{Cost} = C\,(\text{weight})$$

↑ cost/weight

- using the Second figure of Merit gets this in terms of key parameters:

$$\boxed{\text{Cost} = \frac{\pi}{4} C \rho d^2 L}$$

Third Figure of Merit (* 3)

(b) We have three equations that allow us to explore the possibilities in terms of the key items (deformation, weight, cost).

However, separately considering any one consideration and its minimization (in this case) is generally insufficient. For example, one can decrease deformation by continuing to increase bar diameter. The key is to consider the ability of any choice for other fixed conditions. This leads to considering

Tradeoffs!

If one considers the ability to provide a specified minimum deformation (call it δ_0) and first consider mass (weight). The first and second figures of merit can be combined for this consideration

from (*1): $\delta_0 = \dfrac{4PL}{\pi E d^2}$

$$\Rightarrow d^2 = \frac{4PL}{\pi E \delta_0}$$

Place this in (*2):

$$\text{weight} = \frac{\pi}{4}\rho\left(\frac{4PL}{\pi E\delta_0}\right)L$$

$$\Rightarrow \text{weight} = \frac{\rho P L^2}{E\delta_0}$$

Here, $\dfrac{PL^2}{\delta_0}$ is a constant so we assess the possibilities via the factor ρ/E

Material	ρ/E $\left[\text{lb/in}^3\right]/\left[\frac{10^6\,\text{lb}}{\text{in}^2}\right] = \left[\frac{1}{10^6\,\text{in}}\right]$
Wood	0.0122
Aluminum	0.0096
Titanium	0.010
Carbon fiber Composite	0.0022
Steel	0.0098
$\boxed{\text{Silicon Carbide}}$	0.0018

best choice to minimize mass/weight (for give deformation)

→ Now consider cost by using (∗3)

$$\Rightarrow \text{Cost} = \frac{\pi}{4} C\rho \left(\frac{4PL}{\pi E \delta_0}\right) L$$

giving... $\text{Cost} = \dfrac{CPL^2\rho}{E\delta_0}$

Here, $\dfrac{PL^2}{\delta_0}$ is, again, a constant, so we assess the possibilities via the factor $\dfrac{C\rho}{E}$

Material	$\frac{C\rho}{E}$ $\left[\frac{\$}{10^6\ \text{in}\cdot\text{lb}}\right]$
Wood	0.012
Aluminum	0.060
Titanium	0.232
Carbon Fiber Composite	0.176
Steel	0.013*
Silicon Carbide	0.254

best choice to minimize cost (for given displacement)

a close second

→ Finally, think about minimizing deformation for a fixed cross-section. Using (∗1):

$$\delta = \frac{4PL}{\pi E d^2}$$

Here, $\dfrac{4PL}{\pi d^2}$ is given, so we assess the possibilities via the factor $1/E$

Material	$1/E$ $\left[\frac{in^2}{10^6 lb}\right]$
Wood	0.552
Aluminum	0.095
Titanium	0.063
Carbon Fiber Composite	0.041
Steel	0.034
Silicon Carbide	0.017

best choice to minimize deformation (for given area)

__Final note__: There is no __final__ answer without further clarification of the objectives and the relative values of the various aspects. It depends upon decisions with regard to the __tradeoffs__.

M9.2 Compliance tensor

(a) Start with:

$$\epsilon_{mn} = S_{mnpq}\sigma_{pq}$$

This has the same form as the elasticity equation:

$$\sigma_{mn} = E_{mnpq}\epsilon_{pq}$$

and since the same symmetries exist for the compliance tensor as for the elasticity tensor, the full anisotropic equations have the same form.

Thus:

$$\boxed{\begin{aligned}
\epsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1133}\sigma_{33} + 2S_{1123}\sigma_{23} + 2S_{1113}\sigma_{13} + 2S_{1112}\sigma_{12}\\
\epsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33} + 2S_{2223}\sigma_{23} + 2S_{2213}\sigma_{13} + 2S_{2212}\sigma_{12}\\
\epsilon_{33} &= S_{1133}\sigma_{11} + S_{2233}\sigma_{22} + S_{3333}\sigma_{33} + 2S_{3323}\sigma_{23} + 2S_{3313}\sigma_{13} + 2S_{3312}\sigma_{12}\\
\epsilon_{23} &= S_{1123}\sigma_{11} + S_{2223}\sigma_{22} + S_{3323}\sigma_{33} + 2S_{2323}\sigma_{23} + 2S_{2313}\sigma_{13} + 2S_{2312}\sigma_{12}\\
\epsilon_{13} &= S_{1113}\sigma_{11} + S_{2213}\sigma_{22} + S_{3313}\sigma_{33} + 2S_{1323}\sigma_{23} + 2S_{1313}\sigma_{13} + 2S_{1312}\sigma_{12}\\
\epsilon_{12} &= S_{1112}\sigma_{11} + S_{2212}\sigma_{22} + S_{3312}\sigma_{33} + 2S_{1223}\sigma_{23} + 2S_{1213}\sigma_{13} + 2S_{1212}\sigma_{12}
\end{aligned}}$$

Likewise, these can be grouped into the 3 groups similar to that for E_{mnpq}:

extensional stresses to extensional strains $\left\{\begin{matrix} S_{1111} & S_{1122} \\ S_{2222} & S_{1133} \\ S_{3333} & S_{2233} \end{matrix}\right.$

shear stresses to shear strains
$$\begin{cases} S_{1212} & S_{1213} \\ S_{1313} & S_{1323} \\ S_{2323} & S_{1223} \end{cases}$$

COUPLING TERMS

extensional stresses to shear strains

or

shear stresses to extensional strains
$$\begin{cases} S_{1112} & S_{2212} & S_{3312} \\ S_{1113} & S_{2213} & S_{3313} \\ S_{1123} & S_{2223} & S_{3323} \end{cases}$$

(b) For the orthotropic case, all COUPLING TERMS are zero (in the principal axes of the material). Thus:

$$S_{1112} = 0 \qquad S_{2212} = 0 \qquad S_{3312} = 0$$
$$S_{1113} = 0 \qquad S_{2213} = 0 \qquad S_{3313} = 0$$
$$S_{1123} = 0 \qquad S_{2223} = 0 \qquad S_{3323} = 0$$

in addition, shear strains (stresses) in one plane do not cause shear strains (stresses) in another plane. Thus, ~~those~~ of the shear stress to shear strain terms become zero:

$$S_{1213} = 0$$
$$S_{1323} = 0$$
$$S_{1223} = 0$$

All other terms are nonzero and independent. This gives 9 independent compliance components as for the elasticity case. The resulting equations are:

$$\boxed{\begin{aligned}
\epsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1133}\sigma_{33} \\
\epsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33} \\
\epsilon_{33} &= S_{1133}\sigma_{11} + S_{2233}\sigma_{22} + S_{3333}\sigma_{33} \\
\epsilon_{23} &= 2 S_{2323}\sigma_{23} \\
\epsilon_{13} &= 2 S_{1313}\sigma_{13} \\
\epsilon_{12} &= 2 S_{1212}\sigma_{12}
\end{aligned}}$$

From the lecture notes, the stress-strain relations for the orthotropic case using engineering constants are:

$$\epsilon_1 = \frac{1}{E_1}\left[\sigma_1 - \nu_{12}\sigma_2 - \nu_{13}\sigma_3\right]$$

$$\epsilon_2 = \frac{1}{E_2}\left[-\nu_{21}\sigma_1 + \sigma_2 - \nu_{23}\sigma_3\right]$$

$$\epsilon_3 = \frac{1}{E_3}\left[-\nu_{31}\sigma_1 - \nu_{32}\sigma_2 + \sigma_3\right]$$

$$\gamma_{23} = 1/G_{23}\ \sigma_{23}$$

$$\gamma_{13} = 1/G_{13}\ \sigma_{13}$$

$$\gamma_{12} = 1/G_{12}\ \sigma_{12}$$

To go from tensorial to engineering notation for stress and strain, note that:

$$\epsilon_{11} = \epsilon_1 \qquad \sigma_{11} = \sigma_1$$
$$\epsilon_{22} = \epsilon_2 \qquad \sigma_{22} = \sigma_2$$
$$\epsilon_{33} = \epsilon_3 \qquad \sigma_{33} = \sigma_3$$
$$2\epsilon_{23} = \gamma_{23} \qquad \sigma_{23} = \sigma_{23}$$
$$2\epsilon_{13} = \gamma_{13} \qquad \sigma_{13} = \sigma_{13}$$
$$2\epsilon_{12} = \gamma_{12} \qquad \sigma_{12} = \sigma_{12}$$

using these relations with the previous two sets of equations results in:

$$\boxed{\begin{aligned}
S_{1111} &= 1/E_1 \\
S_{1122} &= -\nu_{12}/E_1 = -\nu_{21}/E_2 && \leftarrow \text{reciprocity} \\
S_{1133} &= -\nu_{13}/E_1 = -\nu_{31}/E_3 && \leftarrow \text{reciprocity} \\
S_{2222} &= 1/E_2 \\
S_{2233} &= -\nu_{23}/E_2 = -\nu_{32}/E_3 && \leftarrow \text{reciprocity} \\
S_{3333} &= 1/E_3 \\
S_{2323} &= 1/(4G_{23}) \\
S_{1313} &= 1/(4G_{13}) \\
S_{1212} &= 1/(4G_{12})
\end{aligned}}\ \Bigg\}\ *$$

* Note factor of 4 in these cases. Look at one particular case:

$$\epsilon_{23} = 2S_{2323}\sigma_{23} \quad \underline{\text{and}} \quad \gamma_{23} = \frac{1}{G_{23}}\sigma_{23}$$

and $2\epsilon_{23} = \gamma_{23}$

$$\Rightarrow 2\varepsilon_{23} = 4S_{2323}\sigma_{23} = \gamma_{23} = \frac{1}{G_{23}}\sigma_{23}$$

$$\Rightarrow 4S_{2323} = \frac{1}{G_{23}}$$

$$\Rightarrow \boxed{S_{2323} = \frac{1}{4G_{23}}}$$

... same for other two cases

(c) We know that the compliance matrix is the inverse of the elasticity matrix or vice versa:

$$\underset{\sim}{E} = \underset{\sim}{S}^{-1}$$

For this orthotropic case, the compliance matrix is:

$$\underset{\sim}{S} = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0 \\ S_{1122} & S_{2222} & S_{2233} & 0 & 0 & 0 \\ S_{1133} & S_{2233} & S_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2S_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2S_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2S_{1212} \end{bmatrix}$$

We also found the relations between the components of the compliance tensor and the engineering constants. We can use these in the compliance matrix:

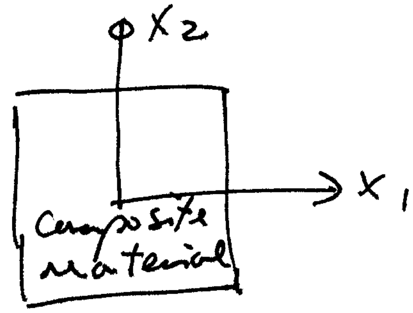
$$\underset{\sim}{S} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & -\nu_{13}/E_1 & 0 & 0 & 0 \\ -\nu_{21}/E_2 & 1/E_2 & -\nu_{23}/E_2 & 0 & 0 & 0 \\ -\nu_{31}/E_3 & -\nu_{32}/E_3 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2G_{12} \end{bmatrix} \quad (1)$$

For the orthotropic case, the elasticity matrix is:

$$\underset{\sim}{E} = \begin{bmatrix} E_{1111} & E_{1122} & E_{1133} & 0 & 0 & 0 \\ E_{1122} & E_{2222} & E_{2233} & 0 & 0 & 0 \\ E_{1133} & E_{2233} & E_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2E_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2E_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2E_{1212} \end{bmatrix} \quad (2)$$

Now take the inverse of the matrix expression of (1) and equate it to the matrix expression of (2). This will give relations between the components of the elasticity tensor and the engineering constants.

M 9.3



<u>Experiment A</u>: $\sigma_{12} = 300$ MPa
$\epsilon_{12} = 7200\ \mu$strain

<u>Experiment B</u>: $\sigma_{11} = 250$ MPa
$\sigma_{22} = 250$ MPa
$\epsilon_{22} = 3850\ \mu$strain

<u>Experiment C</u>: $\sigma_{11} = 400$ MPa
$\epsilon_{11} = 5300\ \mu$strain
$\epsilon_{22} = -1550\ \mu$strain

stresses and strains not specified are zero (except ϵ_{11} for Experiment B as the gage broke)

(a) Experiments B and C show that extensional stresses cause only extensional strains and Experiment A shows that shear stress causes only shear strain. Thus, this material behaves <u>at most</u> as an orthotropic material.

Since the out-of-plane stresses (σ_{3j}) are also zero in all three experiments, we only need to use the in-plane form of the equations. We end up with:

$$\epsilon_{11} = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{21}}{E_2}\sigma_{22} \qquad (1)$$

$$\epsilon_{22} = \frac{-\nu_{12}}{E_1}\sigma_{11} + \frac{1}{E_2}\sigma_{22} \qquad (2)$$

$$2\epsilon_{12} = \frac{1}{G_{12}}\sigma_{12} \qquad (3)$$

→ using the results of Experiment A in (3):

$$14{,}400 \times 10^{-6} = \frac{1}{G_{12}}\,(300 \times 10^{6}\ Pa)$$

$$\Rightarrow \boxed{G_{12} = 20.8 \times 10^{9}\ Pa = 20.8\ GPa}$$

→ use the results of Experiment (C) (since there is only one stress applied) in (1):

$$5300 \times 10^{-6} = \frac{1}{E_1}\,(400 \times 10^{6}\ Pa)$$

$$\Rightarrow \boxed{E_1 = 75.5 \times 10^{9}\ Pa = 75.5\ GPa}$$

and in (2) along with the value for E_1:

$$-1550\times10^{-6} = -\frac{\nu_{12}}{(75.5\times10^9\,Pa)}(400\times10^6\,Pa)$$

$$\Rightarrow \boxed{\nu_{12} = 0.293}$$

→ use the results of Experiment B in (2) along with the values for ν_{12} and E_1:

$$3850\times10^{-6} = \frac{-0.293}{(75.5\times10^9\,Pa)}(250\times10^6\,Pa) + \frac{1}{E_2}(250\times10^6\,Pa)$$

$$\Rightarrow 3850\times10^{-6} = -970\times10^{-6} + \frac{1}{E_2}(250\times10^6\,Pa)$$

$$\Rightarrow 4820\times10^{-6} = \frac{1}{E_2}(250\times10^6\,Pa)$$

$$\Rightarrow \boxed{E_2 = 51.9\times10^9\,Pa = 51.9\,GPa}$$

One can also use the reciprocity relation:

$$\nu_{21}E_1 = \nu_{12}E_2$$

and the values determined:

$$\nu_{21}(75.5\,GPa) = (0.293)(51.9\,GPa)$$

to get: $\boxed{\nu_{21} = 0.201}$

This gives the in-plane engineering constants for this orthotropic material.

With these 5 constants (4 independent) and the equations with which we started, the in-plane stress-strain behavior of the material is characterized.

(b) Use the results from Experiment B and the results from (a) in equation (1) to determine ϵ_{11}:

$$\epsilon_{11} = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{21}}{E_2}\sigma_{22}$$

$$\Rightarrow \epsilon_{11} = \frac{1}{(75.5\,GPa)}(250\,MPa) - \frac{(0.201)}{(51.9\times10^{9}\,GPa)}(250\,MPa)$$

$$= 3311\times10^{-6} - 968\times10^{-6}$$

$$\Rightarrow \boxed{\epsilon_{11} = 2343\,\mu strain}$$

(c) Begin by writing the stress-strain equations in matrix form:

$$\begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{12}\end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/2G_{12}\end{bmatrix}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}$$

Now look at the compliance tensor relations also written in matrix form and with the zero stresses ($\sigma_{13}, \sigma_{23}, \sigma_{33}$) and strains ($\epsilon_{13}, \epsilon_{23}, \epsilon_{33}$) ignored/eliminated:

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} = \begin{bmatrix} S_{1111} & S_{1122} & 2S_{1112} \\ S_{2211} & S_{2222} & 2S_{2212} \\ S_{1211} & S_{1222} & 2S_{1212} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}$$

We know the COUPLING TERMS are zero so we are left with 5 terms. We compare these directly with those from the matrix expression using engineering constants and find:

$$S_{1111} = \frac{1}{E_1} = \frac{1}{75.5\times10^9 Pa} = 1.32\times10^{-11}\frac{1}{Pa}$$

$$S_{1122} = S_{2211} = \frac{-\nu_{21}}{E_2} = \frac{-\nu_{12}}{E_1} = \frac{-0.201}{51.9\times10^9 Pa} = -0.387\times10^{-11}\frac{1}{Pa}$$

$$S_{2222} = \frac{1}{E_2} = \frac{1}{51.9\times10^9 Pa} = 1.93\times10^{-11}\frac{1}{Pa}$$

$$2S_{1212} = \frac{1}{2G_{12}} = \frac{1}{2(20.8\times10^9)Pa} = 2.40\times10^{-11}\frac{1}{Pa}$$

$$\Rightarrow S_{1212} = 1.20\times10^{-11}\frac{1}{Pa}$$

Summarizing:

$$\boxed{\begin{array}{l} S_{1111} = 1.32\times10^{-11}\frac{1}{Pa} \quad S_{2222} = 1.93\times10^{-11}\frac{1}{Pa} \\ S_{1122} = S_{2211} = -0.387\times10^{-11}\frac{1}{Pa} \\ S_{1212} = 1.20\times10^{-11}\frac{1}{Pa} \\ S_{1112} = S_{2212} = S_{1211} = S_{1222} = 0 \end{array}}$$

$\leftarrow$ also shown in experiment

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In matrix form:

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} = \begin{bmatrix} 1.32 & -0.387 & 0 \\ -0.387 & 1.93 & 0 \\ 0 & 0 & 2(1.20) \end{bmatrix} \times \left[10^{-11}\frac{1}{Pa}\right] \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}$$

Solution to T1 by Waitz. (Unified Thermodynamics)

It is understood at this point in Unified that you have only had one or two lectures on thermodynamics, and no propulsion lectures. So what is expected is that you recognize conversions between various forms of energy when they occur and know the difference between transfers of energy called "work" and transfers of energy called "heat".

- This is a neat process. I will describe it generally first, and then describe the energy exchange processes in more detail in the following bullets. The liquid fuel and oxidizer are pumped along the walls of the combustion chamber and rocket nozzle. They are used to cool the walls. In doing so, they pick-up energy and undergo a phase change to become gases. These higher energy gases are used to drive the liquid fuel and oxidizer pumps (like pulling yourself up by your bootstraps). Then the gases are injected into the combustion chamber where they react. Then they are accelerated through the nozzle.

- The liquid fuel and oxidizer enter the device with internal, kinetic and chemical energy.

- The pumps do work on the liquid fuels, raising the pressure and increasing the internal energy.

- These fuels then pass by the walls of the combustion chamber and nozzle where heat is transferred from the hot walls, increasing the internal energy of the fuels. There is a phase change (liquid to vapor).

- The higher energy gases then do work on the turbines that drive each turbopump. In the process, the gases lose internal and kinetic energy. If this process were ideal, all the energy would go into increasing the energy of the liquid fuels, but it is not ideal so there is typically some waste heat generated.

- The gases then enter the combustion chamber where they react, converting chemical energy into internal/thermal energy (very high temperature and pressure).

- The nozzle converts the high internal energy into high kinetic energy. In the process, some of the energy of the reacted gases near the walls is transferred to the walls (and then on to the fuels being used to cool the walls).

- Although the liquid fuels coming in have energy in several forms, and the gases leaving the nozzle have energy in several forms, the primary energy conversion for the device as a whole is the conversion of the chemical energy of the liquid fuels to kinetic energy of the propulsive gases.

T2 Solution Fall 2006

a.) Given: $p_1 = 300\ psi = 2{,}068{,}427\ Pa$
$T_1 = 22°C = 295.15\ K$
$T_2 = 27°C = 300.15\ K$

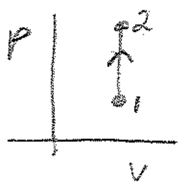
Concepts Used: $pv = RT$; State of a system; $W = \int p\,dv$

Given two properties for initial conditions, therefore state is defined.

$$v_1 = \frac{RT_1}{p_1} \qquad R_{CO_2} = \frac{\mathcal{R}}{MW} = \frac{8.314}{44.0} = 0.18895\ \frac{kJ}{kg\text{-}K}$$

$$v_1 = \frac{(0.18895\times10^3)(295.15)}{2{,}068{,}427} = 0.02696\ \frac{m^3}{kg}$$

Given path to final state (Rigid Tank $\Rightarrow \Delta v = 0$)



And given final temperature.

So $v_2 = v_1 = 0.0269625\ \frac{m^3}{kg}$
$T_2 = 300.15\ K$

$$\therefore\ p_2 = \frac{RT_2}{v_2} = \frac{(188.95)\,300.15}{0.02696} = \boxed{2.103\ MPa}$$
$$= 305.1\ psi$$

b.) $\int p\,dv = 0$ since $dv = 0$

$\therefore$ $\boxed{\text{No Work Was Done}}$

c.) Given: $p_3 = 230\ psi = 1{,}585{,}794\ Pa$
$$\frac{dp}{dv} = 1\times10^5\ \frac{MPa}{m^3/kg}$$

Concepts Used:

Here, a new path is specified. Instead of $v = \text{constant}$, we have $\frac{dp}{dv} = 1\times10^5\ \frac{MPa\cdot kg}{m^3}$

Integrate $\int dp = \int 1\times10^5\,dv$

$$p = 10^5 v + C$$

Using initial conditions to solve for C

$$P_2 = 10^5 v_2 + C$$

$$2.103 \text{ MPa} = 10^5 (0.02696) + C$$

$$\Rightarrow C = -2{,}694 \text{ MPa}$$

$$P = 1\times10^5 \frac{\text{MPa}}{\text{m}^3/\text{kg}} \cdot v - 2{,}694 \text{ MPa} \Bigg\} \text{ New Path Fully Defined}$$

Given $P_3 = 230 \text{ psi} = 1.5858 \text{ MPa}$, this equation can be used to solve for v_3.

$$P_3 = 1.5858 = 1\times10^5 v_3 - 2{,}694 \text{ MPa}$$

$$\Rightarrow v_3 = 0.0269573 \frac{\text{m}^3}{\text{kg}}$$

Using ideal gas equation

$$T_3 = \frac{P_3 v_3}{R} \quad (\text{units } Pa)$$

$$= \frac{(1.5858\times10^6)(0.0269573)}{188.95} = 226.238 \text{ K}$$

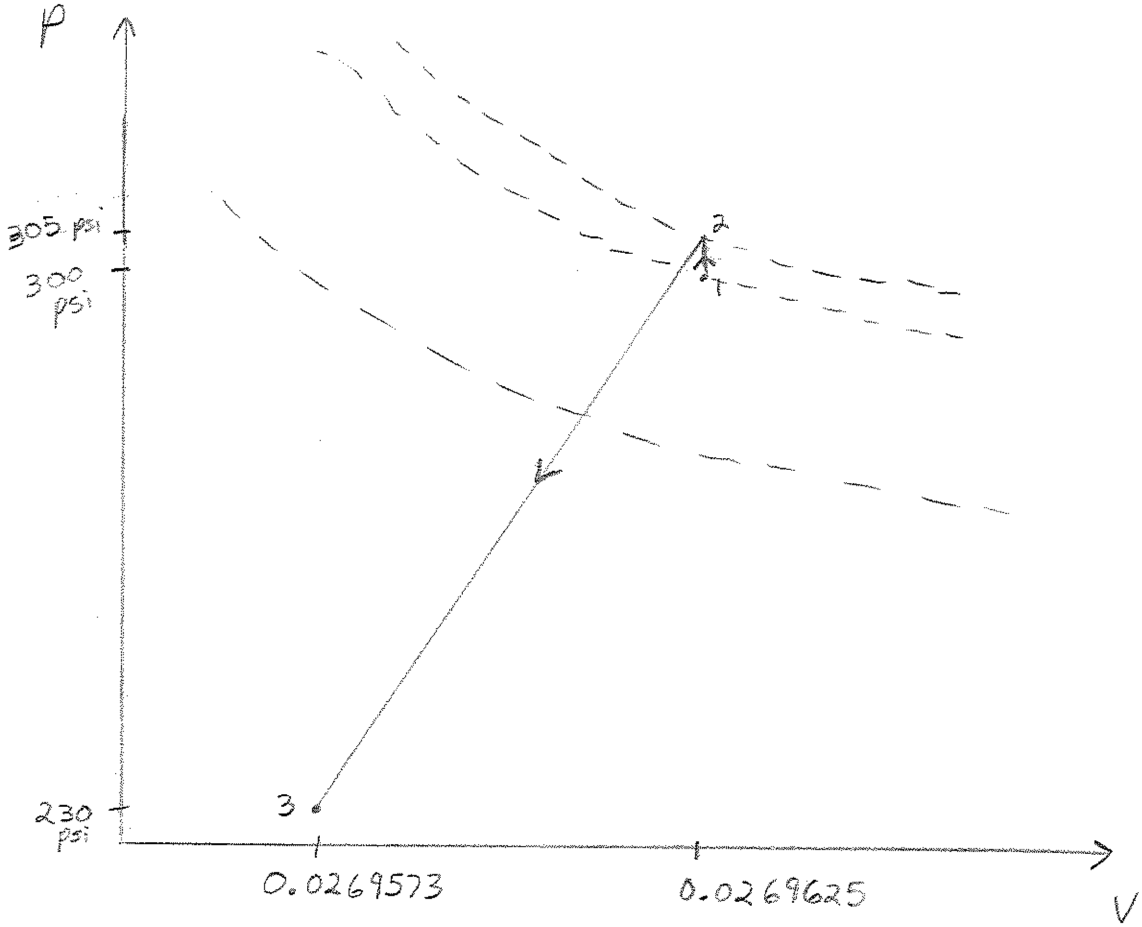
$$\therefore \boxed{T_3 \doteq 226 \text{ K} = -47.15\,°C}$$

d.) $$W = \int_{v_2}^{v_3} P\,dv = \int_{v_2}^{v_3} \left(1\times10^5 \frac{\text{MPa}}{\text{m}^3/\text{kg}} \cdot v - 2{,}694 \text{ MPa}\right) dv$$

$$W = \frac{1\times10^{11} v^2}{2} - 2{,}694\times10^6 v \Bigg]_{v_2}^{v_3}$$

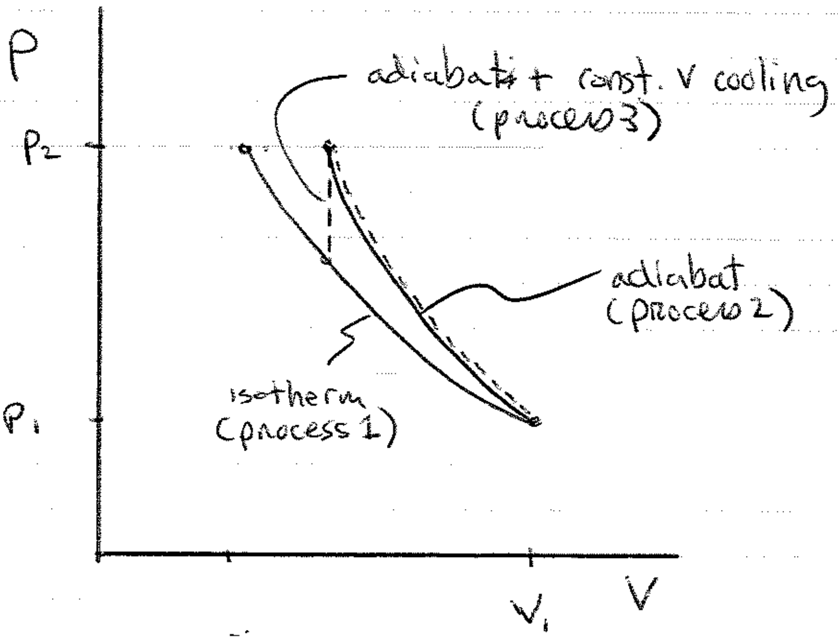
$$\boxed{W = -9.549 \text{ J/kg}}$$

This is only the the work done against the wall of the container and does not include the work against the atmospheric pressure done by the gases that leave the tank.


P
305 psi
300 psi
230 psi
2
1
3
0.0269573
0.0269625
V

SOLUTIONS TO T3 BY WAITZ

a)



b) $p_1 v_1 = RT_1$ $p_1 = 101.3 \times 10^3\,Pa$, $T_1 = 288\,K$, $v_1 = 0.81595\,\frac{m^3}{kg}$

$p_2 = 308.7 \times 10^3\,Pa + 101.3 \times 10^3\,Pa = 410 \times 10^3\,Pa$

CAN'T CALCULATE THE WORK UNTIL YOU KNOW HOW MUCH MASS IS COMPRESSED, SO WAIT UNTIL IT IS IN THE BOTTLE TO CALCULATE IT (BASED ON KNOWING $V_{bottle} * 0.6$ (% filled w/ ~~water~~ air))

① ISOTHERMAL COMPRESSION : $T_2 = 288\,K$

So $v_2 = \frac{RT_2}{p_2} = 0.2016\,\frac{m^3}{kg}$; $\rho_2 = 4.96\,\frac{kg}{m^3}$

$V_{air} = \underbrace{(2.3 \times 10^{-3}\,m^3)(0.6)}_{\text{fraction filled w/ air}} = 0.00138\,m^3$

$m_0 = 0.00685\,kg$ $(= V_{air}\,\rho_2)$

$W = mRT \ln \frac{v_2}{v_1} = (0.00685)(287)(288) \ln \left(\frac{0.2016}{0.81595}\right)$

$$\boxed{W_{ISOTHERMAL} = -791\,J}$$

(2) ADIABATIC COMPRESSION $pV^{\gamma} = \text{const.}$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$V_2 = \left[\frac{(101300)(0.816)^{1.4}}{410000}\right]^{1/1.4} = 0.3\,\frac{m^3}{kg} \quad \therefore \quad \rho_2 = 3.33\,\frac{kg}{m^3}$$

$$V_{air} = 0.00138\,m^3 \quad \therefore \quad m_{(2)} = 0.0046\,kg$$

$$T_2 = \frac{P_2 V_2}{R} = \frac{410000 \cdot 0.3}{287} = 428.6\,K$$

$$\Delta U = \cancel{Q}^{\text{adiabatic}} - W \quad \therefore \quad mC_v(T_2 - T_1) = -W$$

$$= (0.0046)(716.5)(428.6 - 288) = 463.4\,J$$

$$\boxed{W_{adiabatic} = -463.4\,J}$$

(3) NO ADDITIONAL VOLUME CHANGE FOR SECOND LEG OF THE THIRD PROCESS SO

$$\boxed{W_{adiab.+cooling} = -463.4\,J}$$

c) HEAT ADDED?

PROCESS (1) IS ISOTHERMAL SO $\Delta U = Q - W = 0$

SO $Q = W$

$$\boxed{\text{SO } Q_{ADDED} = -791\,J \text{ FOR ISOTHERMAL}}$$

PROCESS (2) IS ADIABATIC SO $\boxed{Q_{ADDED} = 0 \text{ FOR ADIAB.}}$

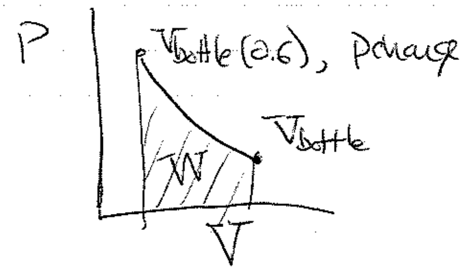
PROCESS (3) NO WORK DURING COOLING PROCESS SO

$$\Delta U = Q = mC_v \Delta T$$

$$= 0.0046(716.5)(288K - 428.6K)$$

$$\boxed{Q_{ADDED} = -463.4\,J \text{ FOR PROCESS (3)}}$$

d) WOULD LIKE TO EXTRACT AS MUCH WORK AS POSSIBLE FROM EXPANDING THE GAS, FROM THE CHARGED PRESSURE TO THE PRESSURE WHEN VOLUME = V_{BOT}. THE FORCE ASSOCIATED WITH THIS WORK IS WHAT PUSHES ON THE WATER, SO WE CAN EXPAND QUASI-STATICALLY & ADIABATICALLY FROM P_{CHARGE} & $V_{BOTTLE}(0.6)$ TO V_{BOTTLE}.



BY INSPECTION, WE CAN RULE OUT THE THIRD PROCESS - IT STARTS AT A LOWER PRESSURE - SO IT IS <u>NOT</u> A GOOD IDEA TO LET THE BOTTLE SIT ON THE LAUNCHER FOR A LONG TIME

WHAT ABOUT PROCESSES 1 & 2? COMPARING THEM IS INTERESTING. THEY BOTH START AT THE SAME PRESSURE AND VOLUME (BUT NOT SPECIFIC VOLUME! THEY IS A DIFFERENT AMOUNT OF MASS IN THE BOTTLE IN EACH CASE). AND THE BOTH EXPAND VIA A Q.S, ADIAB PROCESS TO THE SAME FINAL VOLUME. **SO THE WORK IS THE SAME.**

$$W = \int_{V_{INIT}}^{V_{FINAL}} (P - P_{atm})\, dV \quad \& \quad \frac{P}{P_{INIT}} = \left(\frac{V_{INIT}}{V}\right)^{\gamma}$$

$$\Rightarrow W = \frac{P_{INIT}V_{INIT} - P_{FINAL}V_{FINAL}}{\gamma - 1} - P_{atm}(V_{final} - V_{INIT})$$

BUT PROCESS 1 (ISOTHERMAL) HAS MORE MASS IN THE BOTTLE THAN PROCESS 2 (ADIABATIC). ASSUMING YOU COULD ADJUST FOR THIS BY CHANGING THE EMPTY WEIGHT OF THE BOTTLE, IS THERE ANY PREFERENCE? YES - WHEN THE AIR EXPANDS TO V_{BOTTLE}, THE PRESSURE IS STILL GREATER THAN ~~ATM~~ ATMOSPHERIC PRESSURE (IT IS ABOUT 2 ATMOSPHERES). SO THE AIR THAT GETS EXPELLED (AFTER THE WATER) ALSO PROVIDES AN IMPULSE TO THE ROCKET. AND, THE GREATER THE DENSITY, THE GREATER THE IMPULSE - SO THERE SHOULD BE A SMALL BENEFIT TO PROCESS 1.