Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 11

Name: ________________________

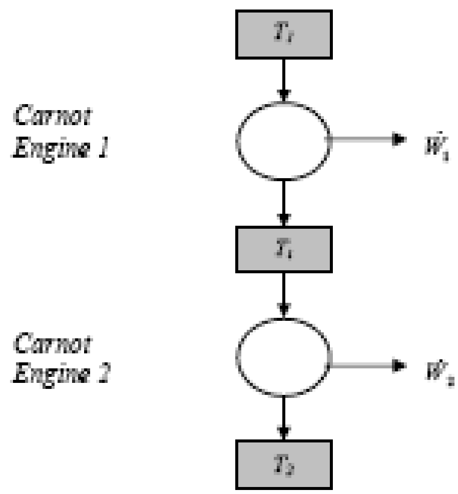
Due Date: 11/21/2006

	Time Spent (min)
T7	
T8	
T9	
T10	
S3	
S4	
Study Time	

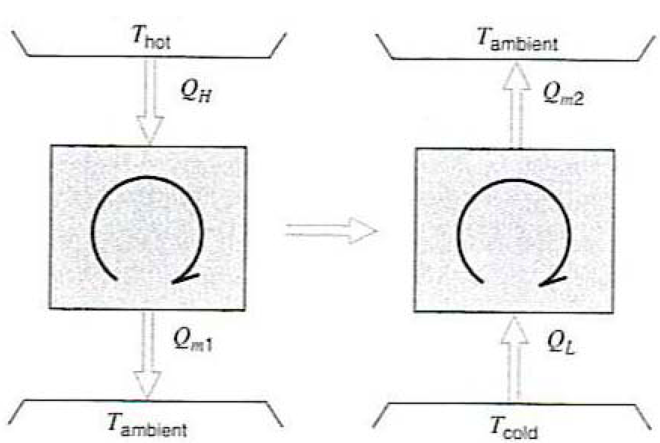
Announcements:

Problem T7 (Unified Thermodynamics)

a) Two Carnot engines operate in series between two reservoirs maintained at 600K and 300K, respectively, as shown below. The energy rejected by the first engine is input into the second engine. If the first engine's efficiency is 20 percent greater than the second engine's efficiency, what is the intermediate temperature T_i. (LO#4, LO#6)



b) We wish to produce refrigeration at -30°C. A reservoir, shown below, is available at 200°C and the ambient temperature is 30°C. Work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir Q_H to the heat transferred from the -30°C reservoir Q_L, assuming all processes are quasi-static. (LO#4, LO#6)



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Problem T8 (Unified Thermodynamics)

In the air-standard Otto cycle all the heat transfer q_H occurs at constant volume. It would be more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2400 kJ/kg, that the pressure and temperature at the beginning of the compression process are 90 kPa, 20ºC, and that the compression ratio is 7.

a) Sketch the process in a *p-v* diagram and label all states.

b) Calculate the maximum pressure and temperature.

c) Find the thermal efficiency of this cycle.

d) Compare the results with those of a conventional Otto cycle having the same given variables.

(LO#3, LO#4, LO#6)

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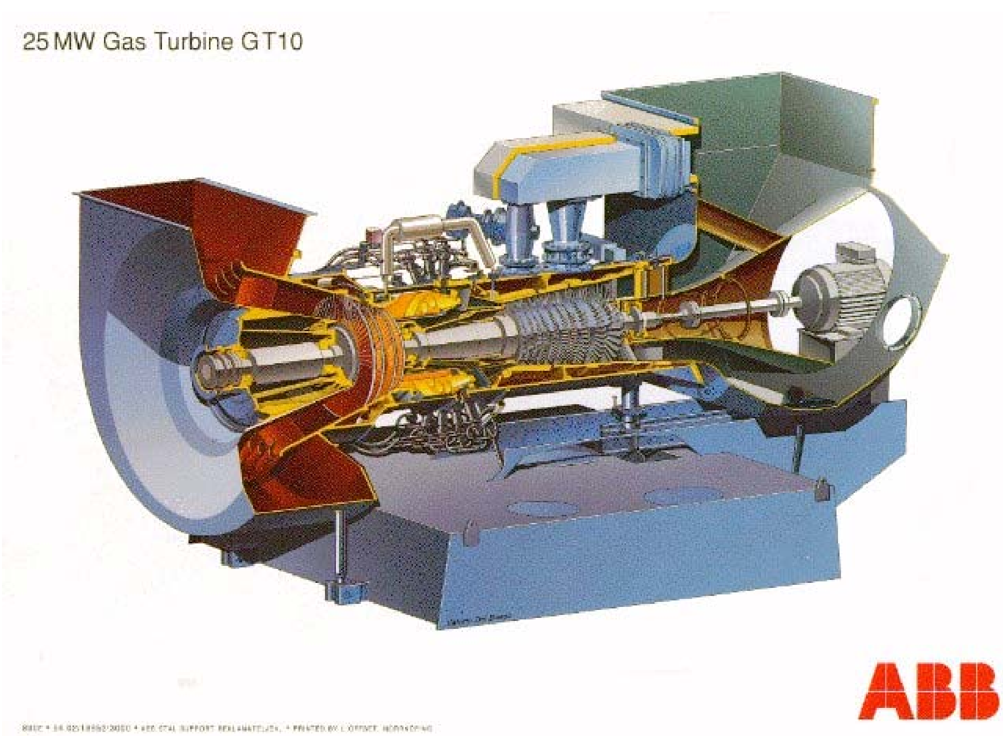
Problem T9 (Unified Thermodynamics)

MIT's gas turbine power plant operates on a Brayton cycle. The current operating conditions are shown at http://cogen.mit.edu/unified/. (The login name is "unified", the password is a popular MIT acronym.) Assume the cycle is ideal and the gas behaves as an ideal gas with constant specific heats $c_p = 1.0035$kJ/kg-K, and $c_v = 0.7165$kJ/kg-K. The maximum temperature at entrance to the turbine is 1400K. From the web page, determine current atmospheric conditions and also the compressor discharge pressure (labeled COMP DEL PRESS). Use this to determine the pressure ratio for the cycle. (LO #4, LO#6)

a) Starting from the measured atmospheric conditions, calculate the pressure and temperature at each point for the ideal cycle.
b) Calculate the thermal efficiency and work per kilogram for the ideal cycle.
c) Assume the compressor discharge temperature is fixed. What is the difference in efficiency between a hot summer day and a cold winter day?
d) How does the efficiency you calculated in part (b) compare with the actual efficiency of the plant? [You can determine this from the data on the web page. If you get a full listing of variables, the fuel flow is listed under "total gas energy flow" and the net power is listed under "active load". If you use the buttons next to the picture, you need to add up the main and primary fuel flows to get the total gas energy flow. Note that 1 BTU is 1055 Joules, and 1 KBTU/s is 10^3BTU/s.] Why is the actual efficiency different from that you calculated in part (b)?
e) You modeled the compressor as quasi-static and adiabatic. Test how good this assumption is by comparing the temperature rise you calculate across the compressor with the measured temperature rise across the compressor.



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Problem T10 (Unified Thermodynamics)

Consider the following air standard cycle:

Leg 1-2: adiabatic compression
Leg 2-3: constant temperature heat addition
Leg 3-4: adiabatic expansion
Leg 4-1: constant pressure cooling

Assume all processes are quasi-static. (LO# 4, LO#6)

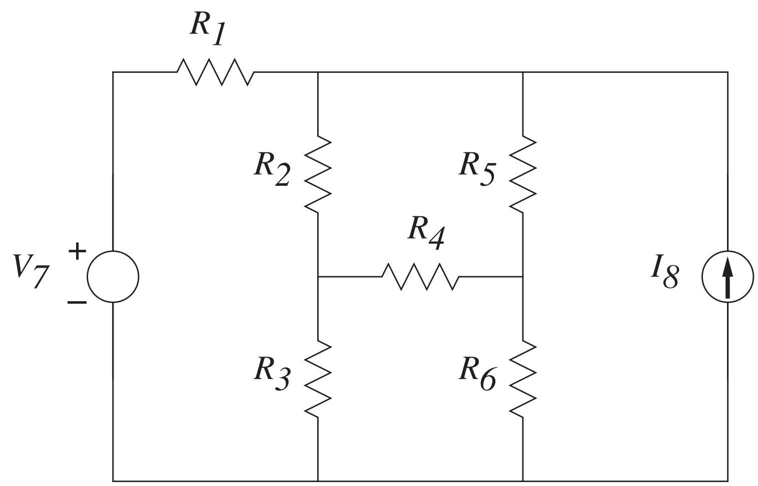
a) Derive expressions for the shaft work, flow work, work, and heat transfer for each leg of the cycle in terms of the temperatures $T_1, T_2, T_3, T_4, v_2, v_3, c_v, c_p$, and R.

b) Using these expressions, show that the net work for the cycle is equal to the net heat for the cycle.

c) Show that the net flow work for the cycle is zero and that the net shaft work is equal to the net work for the cycle.

d) For the ABB gas turbine problem T9, calculate the work, shaft work, and flow work for the turbine. What fraction of the shaft work is used to drive the compressor?

Problem S3 (Signals and Systems)



1. For the circuit above, find the branch voltages and branch currents using the node method. The component values are:

$$\begin{aligned} R_1 &= 1\ \Omega \\ R_2 &= 1\ \Omega \\ R_3 &= 4\ \Omega \\ R_4 &= 6\ \Omega \\ R_5 &= 1\ \Omega \\ I_8 &= 3\ \text{A} \\ V_7 &= 5\ \text{V} \end{aligned}$$

2. Verify that the net power dissipation of the circuit is zero, that is, that

$$\sum_{n=1}^{7} i_n v_n = 0 \tag{1}$$

The net power dissipation for *every* circuit is zero, no matter what the circuit elements are, so long as Kirchhoff's laws are satisfied. This result can be viewed as a consequence of the first law of thermodynamics, applied to circuits. This result is known as *Tellegen's Theorem.*

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Problem S4 (Signals and Systems)

In this problem, you will consider a bit more *Tellegen's Theorem,* introduced in Problem S3.

1. For the circuit in Problem S3, find (make up) 7 element voltages and 7 element currents (all nonzero) such that KCL and KVL are satisfied, *but which do not satisfy the constitutive law for the elements.* In other words, you are to make up random voltages and currents, subject only to the constraint that KVL and KCL are satisfied. For that choice, show that

$$\sum_{n=1}^{7} i_n v_n = 0 \tag{1}$$

   That is, the fact that the net power in a circuit is zero is a consequence of KVL and KCL, not of the choice of circuit elements.

2. Prove Tellegen's theorem is true for any circuit, using the following steps. First, for any circuit with N nodes, label the nodes $n = 1, 2, \ldots N$. Then define the current flowing through a circuit element from node m to node n to be i_{mn}, and the voltage across the element to be v_{mn}. Then the total power of the circuit is

$$\mathbb{P} = \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} i_{mn} v_{mn}$$

   The factor of $\frac{1}{2}$ appears in the sum because the power of each element appears twice, since $i_{mn} = -i_{nm}$, and $v_{mn} = -v_{nm}$. Note also that $i_{mm} = 0$ and $v_{mm} = 0$. Express each element voltage v_{mn} in terms of the node potentials, e_i. Then show that the sum above must always be equal to zero, if KVL and KCL hold.