T7 SOLUTIONS

a) GIVEN: $T_1 = 600K$, $T_2 = 300K$, $\eta_1 = \eta_2 + 0.2$

$$\eta_1 = 1 - \frac{T_i}{T_1} \; ; \quad \eta_2 = 1 - \frac{T_2}{T_i} \; ; \quad \eta_1 = \eta_2 + 0.2$$

$$\Longrightarrow 1 - \frac{T_i}{T_1} = 1 - \frac{T_2}{T_i} + 0.2 \Longrightarrow T_i^2 + 0.2 T_1 T_i - T_1 T_2 = 0$$

$$\Longrightarrow T_i = -0.1 T_1 + \sqrt{(0.1 T_1)^2 + T_1 T_2} \Longrightarrow \boxed{T_i = 368.5\,K}$$

b) GIVEN: $T_{COLD} = 243K$, $T_{HOT} = 473K$, $T_{AMB} = 303K$

FOR A CARNOT CYCLE, THE HEAT ADDED AND REJECTED IS RELATED TO THE RESERVOIR TEMPERATURE:

$\frac{Q_H}{T_H} + \frac{Q_L}{T_L} = 0$ AND THE EFFICIENCY IS $\eta = 1 - \frac{T_L}{T_H}$

1ST LAW FOR ENGINE 1

$$\Delta U = Q - W = 0 \; , \quad W = Q \quad \text{OR} \quad |W_{eng}| = |Q_H| - |Q_{AMB1}|$$

$$\left|\frac{Q_H}{T_H}\right| = \left|\frac{Q_{AMB1}}{T_{AMB}}\right| \quad \therefore |Q_{AMB1}| = \frac{T_{AMB}}{T_H}|Q_H|$$

$$\therefore |W_{ENG1}| = |Q_H|\left(1 - \frac{T_{amb}}{T_H}\right)$$

1ST LAW FOR ENGINE 2 $\quad |W_{eng2}| + |Q_L| = |Q_{AMB2}|$

$$\left|\frac{Q_{AMB2}}{T_{AMB}}\right| = \left|\frac{Q_L}{T_L}\right| \quad \therefore |W_{eng2}| = (Q_L)\left[\frac{T_{AMB}}{T_L} - 1\right]$$

$$|W_{eng1}| = |W_{eng2}|$$

$\therefore$

$$|Q_H|\left(1 - \frac{T_{amb}}{T_H}\right) = |Q_L|\left(\frac{T_{amb}}{T_L} - 1\right)$$

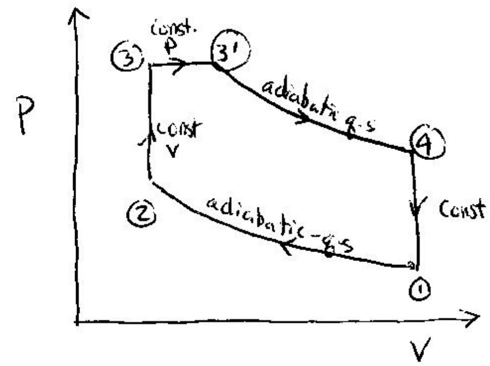
$$\boxed{\frac{Q_H}{Q_L} = \frac{\left(\frac{T_{amb}}{T_L} - 1\right)}{\left(1 - \frac{T_{amb}}{T_H}\right)} = 0.687}$$

T8 SOLUTIONS

GIVEN: $q_H = 2400$ kJ/kg   2/3 AT V=const, 1/3 AT p=const.

$T_1 = 293K$, $p_1 = 90\times10^3$ Pa, $r = 7 = \frac{V_1}{V_2} = \frac{V_4}{V_3}$

a)



LEG	Δu	q	w
1-2	+	0	+
2-3	+	+	0
3-3'	+	+	+
3'-4	−	0	−
4-1	−	−	0

AIR, so $C_v = 716.5$ J/kg-K, $C_p = 1003.5$ J/kg-K, $R = 287$ J/kg-K

1, b)

STATE 1: $T_1 = 293K$, $P_1 = 90\times10^3$ Pa $\therefore$ $v_1 = \frac{RT_1}{P_1} = 0.934 \frac{m^3}{kg}$

STATE 2: $v_2 = \frac{1}{7} v_1 = 0.133 \frac{m^3}{kg}$

q-s. adiab., so $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \Rightarrow P_2 = 1372\times10^3$ Pa

AND $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Longrightarrow T_2 = 638$ K

STATE 3:

$V_3 = V_2 = 0.133$ m³/kg p_3? T_3?

$p_3' = p_3$

$q_{2-3+3-3'} = 2400 \frac{kJ}{kg} = q_{2-3} + q_{3-3'}$

$q_{2-3} = 1600 \frac{kJ}{kg}$ $q_{3-3'} = 800 \frac{kJ}{kg}$

PROCESS 2-3' IS CONST VOL ∴ $W = 0$

$\Rightarrow \Delta u = C_v \Delta T = q_{2-3} = 1600 \frac{kJ}{kg}$

$716.5 (T_3 - 638\,K) = 1600 \frac{kJ}{kg} \Rightarrow \underline{T_3 = 2871\,K}$

FROM IDEAL GAS $\boxed{P_3 = \frac{RT_3}{V_3} = \frac{287 \cdot 2871}{0.133} = 6195 \times 10^3\,Pa}$

STATE (3'): $P_3' = 6195 \times 10^3\,Pa$, T_3'?, V_3'?

KNOW $q_{3-3'} = 800 \frac{kJ}{kg}$ THEN FROM 1ST LAW,

$\Delta u = C_v \Delta T = q - w$ $\quad C_v (T_3' - T_3) = 800 \frac{kJ}{kg} - P_3 (V_3' - V_3)$

ALSO KNOW RELATIONSHIP BETWEEN V_3' & T_3' (IDEAL GAS)

$V_3' = \frac{RT_3'}{P_3}$ SO

$716.5 (T_3' - 2871) = 800 \times 10^3 - 6195 \times 10^3 \left(\frac{287\,T_3'}{6195 \times 10^3} - 0.133 \right)$

$\boxed{T_3' = 3668\,K}$ $\quad V_3' = 0.17 \frac{m^3}{kg}$, $P_3' = 6195 \times 10^3\,Pa$

STATE (4) $\quad V_4 = V_1 = 0.934 \frac{m^3}{kg}$ AND PROCESS IS q-s, adiab.

SO $\frac{T_4}{T_3'} = \left(\frac{V_3'}{V_4} \right)^{\gamma - 1} \Rightarrow T_4 = 1855\,K$

& $\frac{P_4}{P_3'} = \left(\frac{V_3'}{V_4} \right)^{\gamma} \Rightarrow P_4 = 570 \times 10^3\,Pa$

③

c)

$$\eta = \frac{W}{Q_{IN}} = \frac{q_{2-3} + q_{3-3'} + q_{4-1}}{q_{2-3} + q_{3-3'}}$$

$$q_{2-3} + q_{3-3'} = 2400 \text{ kJ/kg}$$

1^{st} LAW $\Delta U = q - W$ $\quad q_{4-1} = C_v (T_1 - T_4) + 0 \quad$ (const. vol, $\underset{=0}{\text{work}}$)

$$= 716.5(293 - 1855)$$

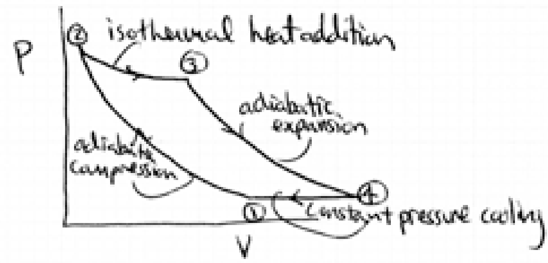
$$q_{4-1} = -1119 \text{ kJ/kg}$$

$$\boxed{\eta = \frac{2400 - 1119}{2400} = 0.53}$$

d) FOR OTTO $\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{7^{0.4}}$

$$\boxed{\eta_{OTTO} = 54\%}$$

SOLUTIONS BY WAITZ



a) LEG 1-2 Q-S ADIABATIC COMPRESSION

$$q = 0, \quad \Delta u = C_v(T_2 - T_1) = \cancel{q}^{\,0} - w \quad \therefore \quad w = -C_v(T_2 - T_1)$$
$$w_s = -C_p(T_2 - T_1)$$
$$w_f = R(T_2 - T_1)$$

LEG 2-3 Q-S ISOTHERMAL EXPANSION

$$\Delta u = 0 = q - w \qquad q = w \qquad w = RT_2 \ln \frac{V_3}{V_2}$$
$$w_f = R(T_3 - T_2) = 0$$
$$w_s = w = RT_2 \ln \frac{V_3}{V_2}$$

LEG 3-4 Q-S ADIABATIC EXPANSION

$$q = 0 \qquad \Delta u = C_v(T_4 - T_3) = \cancel{q} - w \quad \therefore \quad w = -C_v(T_4 - T_3)$$
$$w_s = -C_p(T_4 - T_3)$$
$$w_f = R(T_4 - T_3)$$

LEG 4-1 Q-S CONST P COOLING

$$dh = \delta q + \cancel{v\,dp}^{\,0} \quad \therefore \quad q = C_p(T_1 - T_4), \quad W_f = R(T_1 - T_4)$$
$$\Delta u = q - w \Rightarrow C_v(T_1 - T_4) = C_p(T_1 - T_4) - w$$
$$\therefore \quad w = (C_p - C_v)(T_1 - T_4) = R(T_1 - T_4) = W_f$$
$$\therefore \quad W_s = 0$$

b) NET WORK $= -C_v(T_2-T_1) + RT_2 \ln\frac{V_3}{V_2} - C_v(T_4-T_3) + R(T_1-T_4)$

$-$ NET HEAT $= RT_2 \ln\frac{V_3}{V_2} + C_p(T_1-T_4)$

$0? = -C_v(T_2-T_1) - C_v(T_4-T_3) + R(T_1-T_4) - C_p(T_1-T_4)$

$(R = C_p - C_v)$

$0? = -C_v(T_2) + \cancel{C_vT_1} - \cancel{C_vT_4} + C_vT_3 + \cancel{C_pT_1} - \cancel{C_vT_1} - \cancel{C_pT_4} + \cancel{C_vT_4} - \cancel{C_pT_1} + \cancel{C_pT_4}$

$0? = C_v(T_3-T_2) \quad T_3 = T_2$ ✓

YES NET WORK $-$ NET HEAT $= 0$ $\boxed{\therefore \; Q = W}$ ✓

c) NET FLOW WORK $= R(T_2-\cancel{T_1}) + R(\cancel{T_4}-T_3) + R(\cancel{T_1}-\cancel{T_4})$

$= R(T_2-T_3) \; \boxed{=0}$ SINCE $T_2 = T_3$ ✓

NET SHAFT WORK $= -C_p(T_2-T_1) + RT_2 \ln\frac{V_3}{V_2} - C_p(T_4-T_3)$

$-$ NET WORK $= -C_v(T_2-T_1) + RT_2 \ln\frac{V_3}{V_2} - C_v(T_4-T_3) + R(T_1-T_4)$

$0? = -\left[(C_p-C_v)(T_2-T_1) - (C_p-C_v)(T_4-T_3) - R(T_1-T_4)\right]$

$= -R\left[T_2 - \cancel{T_1} + \cancel{T_4} - T_3 + \cancel{T_1} - \cancel{T_4}\right] = -R\left[\cancel{T_2-T_3}\right] = 0$

$\boxed{\text{NET WORK} = \text{NET SHAFT WORK}}$ ✓

d) THE FLOW WORK IS THE WORK ASSOCIATED WITH THE RELATIVE CHANGE IN PRESSURE * VOLUME AS FLUID PASSES THROUGH A DEVICE

* NOTE THAT THE ABOVE RESULTS ARE GENERAL FOR ANY CLOSED CYCLE:

$Q_{cycle} = W_{cycle}$

$W_{cycle} = W_{s\,cycle}$

$W_{f\,cycle} = 0$

(2) FROM THE SOLUTIONS FOR T7:

$T_1 = 297K$, $T_2 = 638K$, $T_3 = 1400K$, $T_4 = 652K$

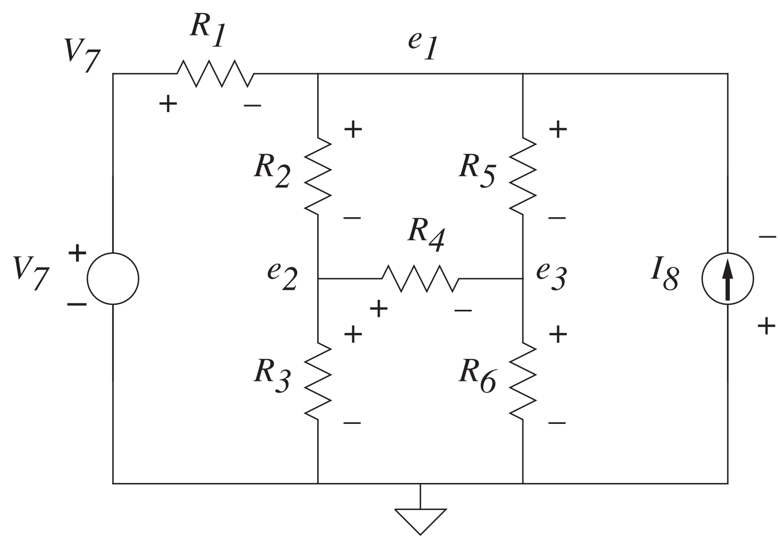
Turbine work $= C_v(T_3 - T_4) = 716.5(1400 - 652) = \boxed{W = 536\ kJ/kg}$

Turbine shaftwork: $C_p(T_3 - T_4) = 1003.5(1400 - 652) = \boxed{W_s = 751\ kJ/kg}$

Turbine flow work $= R(652 - 1400) = \boxed{W_f = -215\ kJ/kg}$

Unified Engineering I **Fall 2006**

Problem S3 (Signals and Systems) SOLUTION



1. To begin, we must label each of the elements of the circuit with a polarity (+/− signs) in order to be able to speak about the element voltages. Note that this labeling is arbitrary, so you may get an answer with different signs. I have labeled the elements as above. I have also assigned a ground node, labeled the one known node (V_7), and labeled the remaining three nodes e_1 , e_2, and e_3. Using the node method, the node equations are then, in order,

$$\begin{aligned}(G_1+G_2+G_5)e_1 \quad &- G_2e_2 &- G_5e_3 &= I_8\\ -G_2e_1 \quad &+ (G_2+G_3+G_4)e_2 &-G_4e_3 &= 0\\ -G_5e_1 \quad &- G_4e_2 &+ (G_4+G_5+G_6)e_3 &= 0\end{aligned}$$

Plugging in values, using $G = 1/R$, we have (leaving out the units)

$$\begin{aligned}3e_1 \quad &- e_2 &- e_3 &= 3\\ -e_1 \quad &+ 1.41\overline{6}\overline{6}e_2 &-0.1\overline{6}\overline{6}e_3 &= 0\\ -e_1 \quad &-0.1\overline{6}\overline{6}e_2 &+ 1.41\overline{6}\overline{6}e_3 &= 0\end{aligned}$$

Solve using a calculator or Matlab or row reduction, we have that

$$\begin{aligned}e_1 &= \frac{40}{7}\text{ V} \approx 5.7143\text{ V}\\ e_2 &= \frac{32}{7}\text{ V} \approx 4.5714\text{ V}\\ e_3 &= \frac{32}{7}\text{ V} \approx 4.5714\text{ V}\end{aligned}$$

The element voltages are then found by differencing node potentials:

$$
\begin{aligned}
v_1 &= V_7 - e_1 = -\frac{5}{7}\,V \\
v_2 &= e_1 - e_2 = \frac{8}{7}\,V \\
v_3 &= e_2 - 0 = \frac{32}{7}\,V \\
v_4 &= e_2 - e_3 = 0\,V \\
v_5 &= e_1 - e_3 = \frac{8}{7}\,V \\
v_6 &= e_3 - 0 = \frac{32}{7}\,V \\
v_7 &= 5\,V \\
v_8 &= 0 - e_1 = -\,\frac{40}{7}\,V
\end{aligned}
$$

The currents are found from the constitutive relations:

$$
\begin{aligned}
i_1 &= \frac{v_1}{R_1} = -\frac{5}{7}\text{ A} \\
i_2 &= \frac{v_2}{R_2} = \frac{8}{7}\text{ A} \\
i_3 &= \frac{v_3}{R_3} = \frac{8}{7}\text{ A} \\
i_4 &= \frac{v_4}{R_4} = \frac{0}{7}\text{ A} \\
i_5 &= \frac{v_5}{R_5} = \frac{8}{7}\text{ A} \\
i_6 &= \frac{v_6}{R_6} = \frac{10}{7}\text{ A} \\
i_8 &= I_8 = 3\text{ A}
\end{aligned}
$$

To find i_7, we must apply KCL at the V_7 node, which implies that $i_7 + i_1 = 0$, so that $i_7 = 5/7$ A.

2. The net power is

$$\sum_{n=1}^{8} i_n v_n = \left(\frac{-5}{7}\cdot\frac{-5}{7} + \frac{8}{7}\cdot\frac{8}{7} + \frac{8}{7}\cdot\frac{32}{7} + 0\cdot 0 + \frac{8}{7}\cdot\frac{8}{7} + \frac{8}{7}\cdot\frac{32}{7} + \frac{-5}{7}\cdot 5 + 3\cdot\frac{-40}{7}\right)\text{W} = 0\text{ W}$$

which is zero by direct calculation.

Problem S4 (Signals and Systems)

In this problem, you will consider a bit more *Tellegen's Theorem,* introduced in Problem S3.

1. Make up a set of voltages and currents that satisfy KVL and KCL. For example, leaving out units, take

$$v_1 = -2, \quad v_2 = -1, \quad v_3 = 5, \quad v_4 = -4, \quad v_5 = -5, \quad v_6 = 9, \quad v_7 = 2, \quad v_8 = -4$$

$$i_1 = 1, \quad i_2 = -1, \quad i_3 = -2, \quad i_4 = 1, \quad i_5 = -2, \quad i_6 = -1, \quad i_7 = -1, \quad i_8 = -4$$

Then

$$\sum_{n=1}^{8} i_n v_n = 0$$

as it should be.

2. For any circuit with N nodes, label the nodes $n = 1, 2, \ldots N$. Define the current flowing through a circuit element from node m to node n to be i_{mn}, and the voltage across the element to be v_{mn}. Then the total power of the circuit is

$$\mathbb{P} = \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} i_{mn} v_{mn}$$

The factor of $\frac{1}{2}$ appears in the sum because the power of each element appears twice, since $i_{mn} = -i_{nm}$, and $v_{mn} = -v_{nm}$. Since KVL holds, we can define node potentials, $e_1, e_2, \ldots e_N$. The element voltages are then given by

$$v_{mn} = e_m - e_n$$

Then the sum becomes

$$\begin{aligned} \mathbb{P} &= \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} i_{mn} (e_m - e_n) \\ &= \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} i_{mn} e_m - \frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} i_{mn} e_n \end{aligned}$$

The sums can be grouped as

$$\mathbb{P} = \frac{1}{2} \sum_{m=1}^{N} e_m \left(\sum_{n=1}^{N} i_{mn} \right) - \frac{1}{2} \sum_{n=1}^{N} e_n \left(\sum_{m=1}^{N} i_{mn} \right)$$

The first term in parentheses is the sum of the currents flowing out of node m, which is zero by KCL. The second term in parentheses is the sum of the currents flowing into node n, which is also zero by KCL. Hence,

$$\mathbb{P} = 0$$

for any circuit which satisfies KCL and KVL.