T11 Solutions (WAITZ)

a) $p_1 = 1\times10^6\,Pa$, $T_1 = 200K$, $C_1 = 50\frac{m}{s}$ $\dot{m} = 100\,kg/s$

$p_2 = 5\times10^6\,Pa$ via a q-s adiab. process $\therefore pv^{\gamma} = const.$ $\therefore q = 0$

$C_2 = 50\frac{m}{s}$

$p_1 v_1 = RT_1 \Rightarrow v_1 = 0.052\,\frac{m^3}{kg}$ $(R = 260\,J/kg\text{-}K)$

$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$, $\gamma = \frac{c_p}{c_v} = 1.1 \Rightarrow v_2 = 0.012\,\frac{m^3}{kg}$

SFEE $q^{\to 0} - w_s = h_2 - h_1 + \frac{C_2^2}{2} - \frac{C_1^2}{2}$

or $q^{\to 0} - w = u_2 - u_1 + \frac{C_2^2}{2} - \frac{C_1^2}{2}$

$w_f = R(T_2 - T_1) = w - w_s$

so

$w = -\left[c_v(T_2 - T_1) + \frac{C_2^2}{2} - \frac{C_1^2}{2}\right]$, $\frac{p_2 v_2}{R} = T_2 = 230.8K$

$w = -78.2\,\frac{kJ}{kg}$
or
$\dot{W} = \dot{m}w = -7.8\,MW$

$w_f = 260\,(230.8 - 200) = 8\,\frac{kJ}{kg}$
or
$\dot{W}_f = \dot{m}w_f = 0.8\,MW$

$w_s = -\left[c_p(T_2 - T_1) + \frac{C_2^2}{2} - \frac{C_1^2}{2}\right] = w - w_f$

$w_s = -86.2\,\frac{kJ}{kg}$ or $\dot{W}_s = -8.62\,MW$

b) $C_2 = 50\,m/s$, $T_2 = 230.8\,K$, $p_2 = 5\times10^6\,Pa$

$C_3 = 100\,m/s$, $T_3 = ?$, $p_3 = 5\times10^6\,Pa$ $\quad q_{IN} = 1300\,kJ/kg$

$$q - \cancel{W_s}^{\,0} = c_p(T_3 - T_2) + \frac{C_3^2}{2} - \frac{C_2^2}{2}$$

(NO SHAFT WORK BUT CAN STILL BE FLOW WORK)

$$1300\times10^3 = 2800(T_3 - 230.8) + \frac{100^2}{2} - \frac{50^2}{2}$$

$$T_3 = 693.7, \quad v_3 = \frac{RT_3}{p_3} = \frac{260(693.7)}{5\times10^6} = 0.036\,\frac{m^3}{kg}$$

$$\boxed{W_s = 0} \qquad W_f = R(T_3 - T_2)$$

$$\boxed{W_f = 260(693.7 - 230.8) = 120\,\frac{kJ}{kg} \quad \text{or} \quad \dot{W}_f = 1.2\,MW}$$

c) $$\boxed{W_{s_{TURBINE}} = -W_{s_{PUMP}} = 86.2\,\frac{kJ}{kg}} \quad \boxed{\text{or } \dot{W}_s = 8.6\,MW}$$

$$\cancel{q}^{\,0} - W_s = c_p(T_4 - T_3) + \frac{C_4^2}{2} - \frac{C_3^2}{2}$$

$$-86.2\times10^3 = 2800(T_4 - 693.7) + \frac{120^2}{2} - \frac{100^2}{2}$$

$$T_4 = 662\,K$$

$$W_f = R(T_4 - T_3) = 260(662 - 693.7) = -8.2\,kJ/kg$$

$$\boxed{W_f = -8.2\,\frac{kJ}{kg} \quad \text{or} \quad \dot{W}_f = -0.82\,MW}$$

T12 SOLUTIONS (WAITZ)

GIVEN
- $C_{p_C} = 1000\,J/kg\text{-}K$
- $C_{p_T} = 1200\,J/kg\text{-}K$
- $\dot{m} = 30\,kg/s$
- STAGNATION TEMPERATURE & PRESSURE PROFILES

a) POWER TO DRIVE HIGH PRESSURE COMPRESSOR (2C-3)

STEADY FLOW ENERGY EQN: $\dot{Q} - \dot{W}_s = \dot{m}\left(h_{T_3} - h_{T_{2C}}\right)$

$T_{T_{2C}} = 367K$, $T_{T_3} = 810K$ $\quad \dot{W}_s = \dot{m}\,1000\left[367 - 810\right] = \underline{-13.3\,MW}$

b) HEAT TRANSFER IN COMBUSTOR

$\dot{Q} - \cancel{\dot{W}_s}^{\,0} = \dot{m}\Delta h_T = 30\left[C_{p_T} T_{T_4} - C_{p_C} T_{T_3}\right] = 30\left[1200 \cdot 1532 - 1000 \cdot 810\right]$

$\dot{Q} = \underline{30.9\,MW}$

c) $\beta = \dfrac{\dot{m}_B}{\dot{m}_C}$, THE FAN IS DRIVEN BY THE LOW PRESSURE TURBINE ASSUMING BOTH ARE ADIABATIC GIVES:

all mass flow through fan?

$$\left(\dot{m}_C + \dot{m}_B\right)\left(C_{p_C} T_{T_{2.4}} - C_{p_C} T_{T_2}\right) = \dot{m}_C\left(C_{p_T} T_{T_{5.4}} - C_{p_T} T_{T_{5.5}}\right)$$

$$\dot{m}_C(\beta + 1)\left(C_{p_C} T_{T_{2.4}} - C_{p_C} T_{T_2}\right) = \dot{m}_C\left(C_{p_T} T_{T_{5.4}} - C_{p_T} T_{T_{5.5}}\right)$$

$$\beta = \frac{C_{p_T}}{C_{p_C}} \frac{\left(T_{T_{5.4}} - T_{T_{5.5}}\right)}{\left(T_{T_{2.4}} - T_{T_2}\right)} - 1 = \frac{1200}{1000} \frac{(1032 - 865)}{(367 - 300)} - 1 = \underline{\underline{2}}$$

d) POWER TO DRIVE FAN $= \dot{m}_C\left[C_{p_T} T_{T_{5.5}} - C_{p_T} T_{T_{5.4}}\right]$

$= \underline{-6\,MW}$

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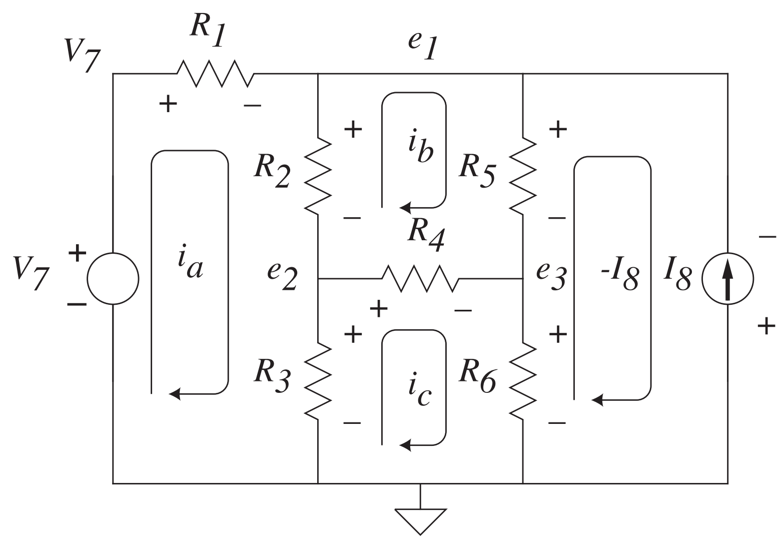
Problem S5 (Signals and Systems) SOLUTION



The component values are:

$$
\begin{aligned}
R_1 &= 1\ \Omega \\
R_2 &= 1\ \Omega \\
R_3 &= 4\ \Omega \\
R_4 &= 6\ \Omega \\
R_5 &= 1\ \Omega \\
R_6 &= 6\ \Omega \\
I_8 &= 3\ \text{A} \\
V_7 &= 5\ \text{V}
\end{aligned}
$$

Draw the loop currents as shown above. Note that there are three unknown currents, i_a, i_b, and i_c. Applying KVL around each loop results in the equations

$$
\begin{aligned}
i_a: &\quad (R_1+R_2+R_3)i_a - R_2 i_b - R_3 i_c = V_7 \\
i_b: &\quad -R_2 i_a + (R_2+R_4+R_5)i_b - R_4 i_c = -R_5 I_8 \\
i_c: &\quad -R_3 i_a - R_4 i_b + (R_3+R_4+R_6)i_c = -R_6 I_8
\end{aligned}
$$

Plugging in component values (and ignoring units for now), we have

$$
\begin{aligned}
6\,i_a - i_b - 4\,i_c &= 5 \\
-i_a + 8 i_b - 6\,i_c &= -3 \\
-4\,i_a - 6 i_b + 16\,i_c &= -18
\end{aligned}
$$

Solving by row reduction, or Matlab, gives

$$\begin{aligned}
i_a &= -\frac{43}{45}\text{ A} \\
i_b &= -\frac{19}{9}\text{ A} \\
i_c &= -\frac{97}{45}\text{ A}
\end{aligned}$$

We then obtain

$$\begin{aligned}
i_1 &= i_a = -\frac{43}{45}\text{ A} \\
v_1 &= i_1 R_i = -\frac{43}{45}\text{ V} \\
i_2 &= i_a - i_b = \frac{52}{45}\text{ A} \\
v_2 &= i_2 R_2 = \frac{52}{45}\text{ V} \\
i_3 &= i_a - i_c = \frac{6}{5}\text{ A} \\
v_3 &= i_3 R_3 = \frac{24}{5}\text{ V} \\
i_4 &= i_c - i_b = -\frac{2}{45}\text{ A} \\
v_4 &= i_4 R_4 = -\frac{4}{15}\text{ V} \\
i_5 &= i_b + I_8 = \frac{8}{9}\text{ A} \\
v_5 &= i_5 R_5 = \frac{8}{9}\text{ V} \\
i_6 &= i_c + I_8 = \frac{38}{45}\text{ A} \\
v_6 &= i_6 R_6 = \frac{76}{15}\text{ V} \\
i_7 &= -i_a = \frac{43}{45}\text{ A} \\
v_7 &= V_7 = 5\text{ V} \\
i_8 &= I_8 = 3\text{ A} \\
v_8 &= -v_5 - v_6 = -\frac{268}{45}\text{ V}
\end{aligned}$$

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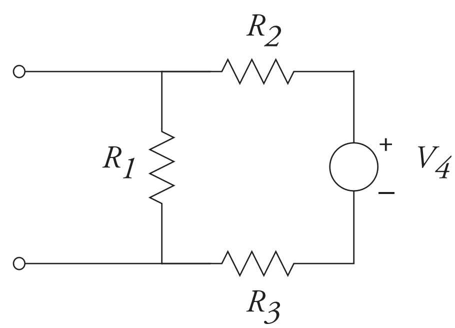
Problem S6 (Signals and Systems) SOLUTION

Find the Thevinin and Norton equivalent circuits for the circuits below. Hint: Add a test current to the terminals, and then determine the voltage at the terminals as a function of the test current. You should find that the terminal voltage can be expressed as

$$v = V_T + R_T I_{\text{test}}$$

1.



where

$$R_1 = 2\ \Omega, \quad R_2 = 4\ \Omega, \quad R_3 = 3\ \Omega, \quad V_4 = 12\ V$$

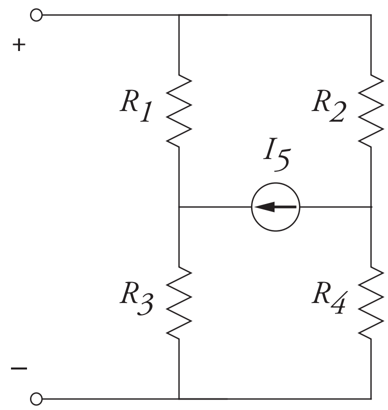
SOLUTION: The circuit is a voltage divider, so the open-circuit voltage is

$$V_{OC} = V_T = \frac{R_1}{R_1 + R_2 + R_3} V_4 = \frac{8}{3}\ \text{V}$$

The Thevinin resistance can be found by looking at the equivalent resistance at the terminals, setting all the sources to zero. Since a voltage source of strength zero is a short circuit, we have that

$$R_T = R_1 || (R_2 + R_3) = 2\ \Omega || 7\ \Omega = \frac{2 \cdot 7}{2 + 7} \Omega = \frac{14}{9}\ \Omega$$

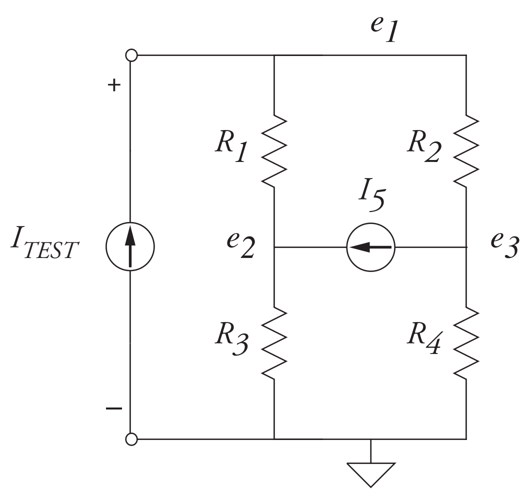
2.



where

$$R_1 = 1\ \Omega, \quad R_2 = 3\ \Omega, \quad R_3 = 3\ \Omega, \quad R_4 = 1\ \Omega, \quad I_5 = 8\ \text{A}$$

SOLUTION: There are several ways to proceed. The most direct is to add a test current, and use the node method to solve. Use the following analysis circuit:



Applying KCL at each node yields the node equations:-G_2

$$\begin{aligned} (G_1 + G_2)e_1 \quad - G_1 e_2 \quad - G_2 e_3 &= I_{TEST} \\ -G_1 e_1 + (G_1 + G_3)e_2 \quad &= I_5 \\ -G_2 e_1 \quad + (G_2 + G_4)e_3 &= -I_5 \end{aligned}$$

Plugging in values,

$$\begin{aligned} \frac{4}{3}e_1 \quad - e_2 \quad - \frac{1}{3}e_3 &= I_{TEST} \\ -e_1 + \frac{4}{3}e_2 \quad &= 8 \\ -\frac{1}{3}e_1 \quad + \frac{4}{3}e_3 &= -8 \end{aligned}$$

Solving by row reduction or Matlab yields

$$e_1 = 2I_{TEST} + 8$$

Therefore,

$$\begin{aligned} V_T &= 8 \text{ V} \\ R_T &= 2\,\Omega \end{aligned}$$