Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2006

Problem Set 13

Name: ________________________

Due Date: 12/05/2006

	Time Spent (min)
S7	
S8	
T13	
T14	
T15	
Study Time	

Announcements:

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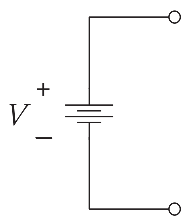
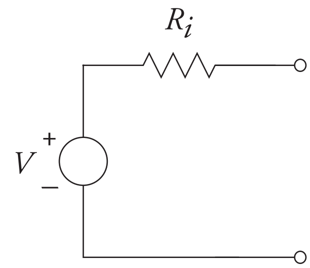
Problem S7 (Signals and Systems)

Note: Please do not use the bibles or other sources to solve this problem. As discussed in the notes and in class, one useful model for a battery is an ideal voltage source in series with an internal resistance, as shown below:

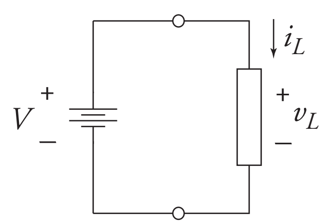




battery circuit element model for battery including internal resistance

For a typical car battery, $V = 12$ volts, and $R_i = 0.01\ \Omega$.

Now, suppose we attach the car battery to a load, such as the starter motor, as in the figure below:



The load will draw current from the battery, which will reduce the voltage of the battery somewhat. The goal is to design the load (the starter) to get as much power from the battery as possible.

1. Find the voltage across the load, v_L, if the current through the load is i_L. Hint: The voltage across the load is the same as the voltage across the battery.
2. Find the power dissipated by the load, $P_L = i_L v_L$, in terms of the current through the load.
3. Find the value of i_L that maximizes the power dissipated by the load. Also find the corresponding voltage across the load.
4. How much power can a typical 12 volt car battery supply to a starter? Does this answer seem reasonable to you? Note: 1 horsepower is equal to 746 Watts.
5. If the load were a resistor instead of a starter motor, what value of resistance would maximize the power dissipated across the load? This is often called the *impedance matching condition.*

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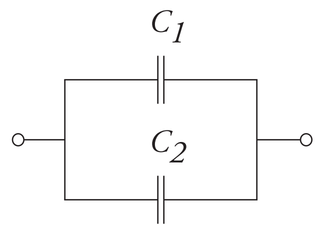
Problem S8 (Signals and Systems)

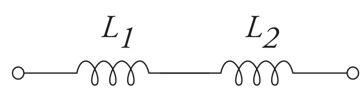
1. Using the constitutive law for capacitors and inductors, derive the equivalent capacitance and inductance for the following series and parallel configurations:

   (a)

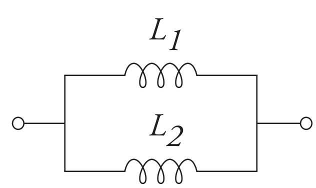


   (b)

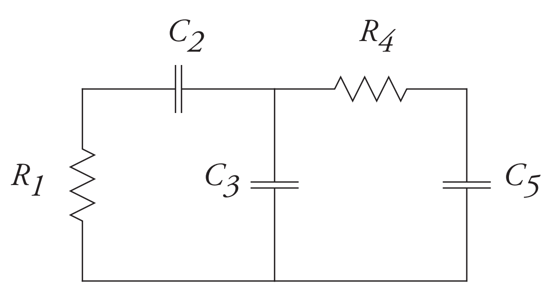
   (c)



   (d)



2. Find a set of differential equations that describe the dynamics of the circuit below, using the node method



where

$$R_1 = 1\ \Omega, \quad C_2 = 0.2\ \text{F}, \quad C_3 = 0.2\ \text{F}, \quad R_4 = 4\ \Omega, \quad C_5 = 0.5\ \text{F}$$

Note: You do not have to solve the differential equations.

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Ian A. Waitz

Problem T13 (Unified Thermodynamics)

An aircraft is traveling against a headwind. Assume R = 287 J/kg-K, c_p = 1003.5 J/kg-K, c_v = 716.5 J/kg-K, γ = 1.4. (LO# 4)

a) If the temperature of the atmosphere is 260K and the wind is moving at 10 m/s relative to the ground, what temperature would a thermometer read if it is fixed to the ground?

b) What temperature would a weather balloon read if it was carried along with the wind?

c) For a thermometer mounted on an airplane that is flying into the wind at 300 m/s relative to the moving air mass, what temperature is read?

d) If the thermometer were mounted on the leading edge of a fan blade in the front of the engine, and at the location of the thermometer the blade speed (orthogonal to the aircraft flight direction) was 250 m/s relative to the airplane, what temperature would be read by the thermometer?

Problem T14 (Unified Thermodynamics) (LO#4)

Consider a B777 flying at M=0.85 at 11km (T_{atm} = 217K, p_{atm} = 22.6kPa, γ = 1.4).

a) In the reference frame of the airplane, what are the static and stagnation (or total) temperatures, and static and stagnation (or total) pressures?

b) In the inlet of the engine, the flow is decelerated (adiabatically and quasi-statically) to about M=0.5 relative to the airplane before passing into the compressor. Again in the reference frame of the airplane, what are the stagnation and static pressures and temperatures at the entrance to the compressor?

c) The compressor operates quasi-statically and adiabatically and increases the total pressure of the flow by a factor of 35, before delivering it to the combustor at a Mach number of M= 0.05. What are the stagnation and static pressures and temperatures at the combustor inlet?

d) If the airliner were a supersonic transport flying at M=3 at 18km (T_{atm} = 217K, p_{atm} = 7.5kPa, γ = 1.4), what are the static and stagnation temperatures, and static and stagnation pressures in the reference frame of the aircraft?

e) What pressure ratio would be required for the compressor on the supersonic aircraft if the stagnation temperatures at the exit of the compressor were constrained by material limitations to be the same as those for the subsonic aircraft?

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Problem T15 (Unified Thermodynamics) (LO#4)

1) Consider a constant pressure heat exchanger. It takes in 40 kg/s of water at p = 5 MPa and T = 20 °C. Water exits the device at T = 1000 °C. Assume that changes in kinetic and potential energy across the device are negligible.
   a. Draw the process on a p-v diagram and show its relation to the vapor dome.
   b. Calculate the power required to heat the flow.
2) A cylinder fitted with a piston has a volume of 0.1 m^3 and contains 5 kg of water at 0.4MPa. Heat is transferred to the steam until the temperature is 300 °C, while the pressure remains constant.
   a. Draw the process on a p-v diagram and show its relation to the vapor dome.
   b. Calculate the heat and work for this process.
3) The mass flow rate into a steam turbine is 1.5 kg/s, and the heat transfer from the turbine is 8.5 kW. Determine the shaft power output of the turbine given the following conditions:

	Inlet	**Outlet**
Pressure	2.0 MPa	0.1 MPa
Temperature	350 °C	
Quality		100%
Velocity	50 m/s	100 m/s
Height above reference plane	6 m	3 m
g = 9.8 m/s^2		