T13 SOLUTIONS

a) THIS IS A STEADY REFERENCE FRAME (AIR MOVING BY AT A CONSTANT SPEED OF 10 m/s WITH A TEMPERATURE OF 260K) THE TEMPERATURE MEASURED BY THE THERMOMETER IS HIGHER THAN 260 K SINCE THE FLOW STAGNATES ON THE SURFACE OF THE THERMOMETER (VIA A Q-S, ADIABATIC PROCESS WITH NO EXTERNAL WORK) AND THE KINETIC ENERGY IS CONVERTED TO ENTHALPY.

$C_p T_T = C_p T + \frac{c^2}{2}$ $\qquad 1003.5(T_T) = 1003.5(260) + \frac{10^2}{2}$

$\boxed{T_T = 260.05\,K}$

b) SINCE THE BALLOON IS MOVING WITH THE WIND, THERE IS NO RELATIVE VELOCITY DIFFERENCE BETWEEN THE AIR MASS AND THE FRAME OF REFERENCE OF THE BALLOON. THEREFORE, THE TEMPERATURE READ BY THE THERMOMETER IS THE SAME AS THE (STATIC) TEMPERATURE OF THE ATMOSPHERE

$\boxed{= 260\,K}$

c) FIRST YOU MUST PUT YOURSELF IN A STEADY REFERENCE FRAME (SEATED ON THE AIRPLANE NEXT TO THE THERMOMETER). YOU SEE THE FLOW MOVING TOWARDS YOU AT 300 m/s WITH A TEMPERATURE OF 260K. THE TEMPERATURE READ BY THE THERMOMETER IS HIGHER THAN 260K SINCE THE FLOW STAGNATES (VIA A Q-S, ADIAB. PROCESS W/ NO EXTERNAL WORK) AND THE KINETIC ENERGY IS COVERTED TO ENTHALPY.

$C_p T_T = C_p T + \frac{c^2}{2}$ $\qquad 1003.5(T_T) = 1003.5(260) + \frac{300^2}{2}$

$\boxed{T_T = 304.8\,K}$

d) AGAIN, PUT YOURSELF IN A STEADY REFERENCE FRAME (SEATED ON THE FAN BLADE NEXT TO THE THERMOMETER)

YOU SEE A FLOW MOVING TOWARDS YOU AT $C = \sqrt{300^2 + 250^2} =$

$= 390.5 \text{ m/s}$ AND $T = 260K$

$$C_p T_T = C_p T + \frac{C^2}{2} \qquad 1003.5(T_T - T) = \frac{390.5^2}{2}$$

$$\boxed{T_T = 336K}$$

T14 SOLUTIONS

3 EQNS ARE NEEDED: $\underbrace{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}_{\text{q-s adiab}}$ $\underbrace{\frac{T_T}{T} = 1 + \frac{\gamma-1}{2}M^2}_{\text{adiab + no ext. work}}$ $\underbrace{\frac{p_T}{p} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}}_{\text{adiab + no ext. work + q-s}}$

a) $\frac{T_T}{T} = 1 + 0.2(0.85)^2 = 1.1445$ $\boxed{T_T = 248\text{ K},\ T = T_{atm} = 217}$

$\frac{p_T}{p} = 1.604$ $\boxed{p_T = 36.2\text{ kPa} \quad p = p_{atm} = 22.6\text{ kPa}}$

b) T_T & p_T ARE SAME AS ABOVE (INLET IS ADIABATIC & q-S SO STAGNATION QUANTITIES ARE CONSTANT & WE HAVEN'T CHANGED REFERENCE FRAMES.)

$T = \frac{T_T}{1+\frac{\gamma-1}{2}M^2}$, $M = 0.5$ $\boxed{T_T = 248\text{ K} \qquad T = 236.2\text{ K}}$

$p = \frac{p_T}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}}$, $M = 0.5$ $\boxed{p_T = 36.2\text{ kPa} \qquad p = 30.5\text{ kPa}}$

c) $p_{T_{ext}} = 35(36.2\text{ kPa}) = 1267\text{ kPa}$

$p = \frac{p_T}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}}$, $M = 0.05$ $\boxed{p_T = 1267\text{ kPa},\ p = 1265\text{ kPa}}$

$\frac{T_{T_{exit}}}{T_{T_{INLET}}} = (35)^{0.4/1.4} = 2.76$ $\quad T_{T_{exit}} = 2.76(248\text{ K}) = 685\text{ K}$

$T_{exit} = \frac{T_{T_{exit}}}{1+\frac{\gamma-1}{2}M^2}$, $M = 0.05$ $\boxed{T_T = 685\text{ K},\ T = 685\text{ K}}$

d) $\frac{T_T}{T} = 1 + \frac{\gamma-1}{2}M^2$, $M = 3$, $T_{atm} = 217\text{ K}$ $\boxed{T_T = 607.6\text{ K},\ T = T_{atm} = 217\text{ K}}$

$\frac{p_T}{p} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}$, $M = 3$, $p_{atm} = 7.5\text{ kPa}$ $\boxed{p_T = 275.5\text{ kPa},\ p = 7.5\text{ kPa}}$

e) IF $T_{T_{INLET}} = 607.6\text{ K}$, $T_{T_{exit}} = 685\text{ K}$ THEN $\frac{T_{T_{exit}}}{T_{T_{INLET}}} = 1.13$

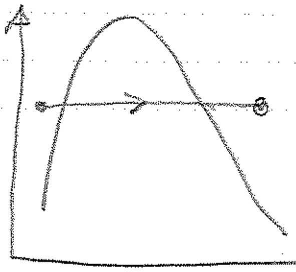
$\frac{p_{T_{exit}}}{p_{T_{INLET}}} = (1.13)^{\gamma/\gamma-1} = 1.52$

THIS SHOWS THAT AS $M \uparrow$ THE RAM PRESSURIZATION (DUE TO RAMMING AIR IN THE INLET) CAN BE QUITE SIGNIFICANT. INDEED, FOR FLIGHT AT VERY HIGH SPEEDS ($> M=3$) COMPRESSORS AREN'T REQUIRED. THE ENGINES ARE CALLED RAMJETS. NOTE THAT THEY STILL NEED SOMETHING TO GET THEM FROM $M=0$ TO $M=3$. IN SOME CASES A ROCKET IS USED (CRUISE MISSILES FOR EXAMPLE), IN OTHER CASES THE ENGINE IS CAPABLE OF BEING RE-CONFIGURED (BYPASSING FLOW AROUND THE COMPRESSOR) IN FLIGHT.

T15 SOLUTIONS (WAITZ)

1. CONSTANT PRESSURE HEAT EXCHANGER, 40 kg/s
$P = 5\,MPa$, $T_{IN} = 20°C$, $T_{OUT} = 1000°C$

a)



INITIAL CONDITIONS: $\left.\begin{matrix} P_1 = 5\,MPa \\ T_1 = 20°C \end{matrix}\right\} h_1 = 88.64 \frac{kJ}{kg}$

(from compressed liq. tables)

FINAL CONDITIONS: $\left.\begin{matrix} P_2 = 5\,MPa \\ T_2 = 1000°C \end{matrix}\right\} h_2 = 4625.7 \frac{kJ}{kg}$

(from superheated vapor tables)

b)

SFEE: $\dot{m}[h_2 - h_1] = \dot{Q} - \cancel{\dot{W}_s}^{\,0}$

$\dot{Q} = 40\left[4625.7 \times 10^3 - 88.64 \times 10^3\right]$

$\boxed{= 181\ MW}$

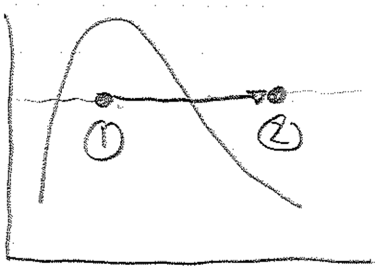
2. CYLINDER - CONSTANT PRESSURE HEATING
$\bar{V}_1 = 0.1\,m^3$, $m = 5\,kg$, $P = 0.4\,MPa$

$\frac{\bar{V}_1}{m} = v_1 = 0.02 \frac{m^3}{kg}$ THEN FROM STEAM TABLES

$v_f = 0.001084$ $v_{fg} = 0.4614$

$\bar{V}_1 = v_f + X_1 (v_{fg}) \quad \therefore \quad X_1 = 0.041$

a)



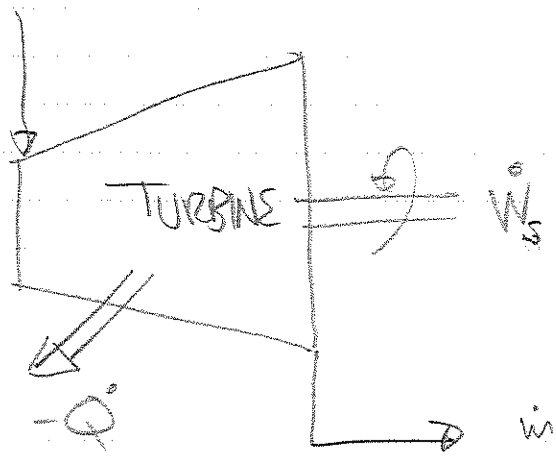
STATE 2 IS SUPERHEATED VAPOR, $h_2 = 3066.75\ kJ/kg$

b) $h_1 = h_f + X_1 h_{fg} = 604.74 + 0.041 \times 2133.8 = 692 \frac{kJ}{kg}$

$Q = m[h_2 - h_1] = 5\left[3066.75 \times 10^3 - 692 \times 10^3\right] = 11.9\,MJ$

3. STEAM TURBINE

$\dot{m}_i = 1.5\ \mathrm{kg/s}$
$P_i = 2\ \mathrm{MPa}$
$T_i = 350^\circ\mathrm{C}$
$C_i = 50\ \mathrm{m/s}$
$Z_i = 6\ \mathrm{m}$



$-\dot{Q} = 8.5\ \mathrm{kW}$

$\dot{m}_e = 1.5\ \mathrm{kg/s}$
$P_e = 0.1\ \mathrm{MPa}$
$x_e = 100\%$
$C_e = 100\ \mathrm{m/s}$
$Z_e = 3\ \mathrm{m}$

$$\dot{m}\left[h_e + \frac{C_e^2}{2} + gZ_e\right] - \dot{m}\left[h_i + \frac{C_i^2}{2} + gZ_i\right] = \dot{Q}_{cv} - \dot{W}_s$$

FROM STEAM TABLES $h_e = 2675.5\ \mathrm{kJ/kg}$, $h_i = 3137.0\ \mathrm{kJ/kg}$

$$\Longrightarrow \dot{W}_{s_{cv}} = 678.2\ \mathrm{kW}$$

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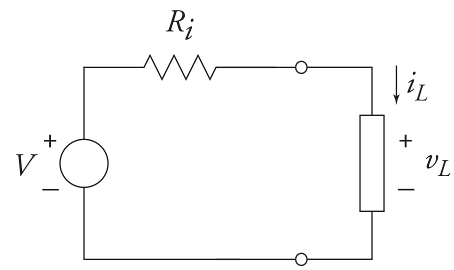
Problem S7 (Signals and Systems) SOLUTION

1. Redraw the circuit with the Thevinin model for the battery:



Since this is a single loop circuit, the current i_L flows through each element, inobvious direction. Applying KVL to the loop yields

$$R_i i_L + v_L - V = 0$$

Therefore,

$$v_L = V - R_i i_L$$

2. The power dissipated across the load is

$$\begin{aligned} P_L &= i_L v_L \\ &= i_L (V - R_i i_L) \\ &= i_L V - R_i i_L^2 \end{aligned}$$

3. To maximize the power across the load, treat above expression as function of i_L, take first derivative with respect to i_L, and set to zero:

$$\begin{aligned} 0 &= \frac{dP_L}{di_L} \\ &= V - 2R_i i_L \end{aligned}$$

Solving for i_L, we have that

$$i_L = \frac{V}{2R_i}$$

The corresponding voltage is

$$\begin{aligned} v_L &= V - R_i i_L \\ &= \frac{V}{2} \end{aligned}$$

4. For a battery voltage $V = 12$ volts, and internal resistance $R_i = 0.01\ \Omega$,

$$P_L = i_L v_L = \frac{V}{2R_i}\frac{V}{2} = \frac{V^2}{4R_i} = 3600\ \mathrm{W} = 4.83\ \mathrm{HP}$$

This seems reasonable to me at least. A 4 or 5 HP engine (like a lawn mower engine) would certainly be powerful enough to turn over a car engine.

5. If $v_L = V/2$, and $i_L = V/2R_i$, then the load resistance is

$$R_L = \frac{v_L}{i_L} = R_i$$

That is, to maximize power, the load impedance should be the same as the output impedance of the driving circuit. This is the *impedance matching condition.*

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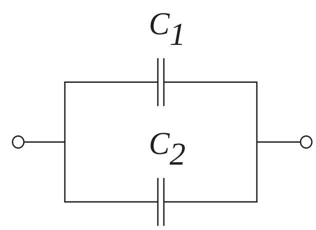
Problem S8 (Signals and Systems) SOLUTION

1. Using the constitutive law for capacitors and inductors, derive the equivalent capacitance and inductance for the following series and parallel configurations:

   (a)



For each capacitor, the current is $i = C\,dv/dt$. Therefore, the total current is

$$i = i_1 + i_2 = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} = (C_1 + C_2)\frac{dv}{dt}$$

Therefore, the equivalent capacitance is $C = C_1 + C_2$.

   (b)

For each capacitor, $i/C = dv/dt$. Therefore,

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt} = \frac{i}{C_1} + \frac{i}{C_2}$$

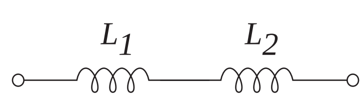
So

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

   (c)



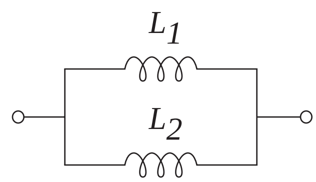
For each inductor, $v = L\, di/dt$. In the series connection, the current through the two inductors is the same. Therefore,

$$v = v_1 + v_2 = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = (L_1 + L_2)\frac{di}{dt}$$

Therefore,

$$L = L_1 + L_2$$

(d)



For each inductor, $di/dt = v/L$. Therefore,

$$\frac{di}{dt} = \frac{d}{dt}(i_1 + i_2) = \frac{v}{L_1} + \frac{v}{L_2}$$

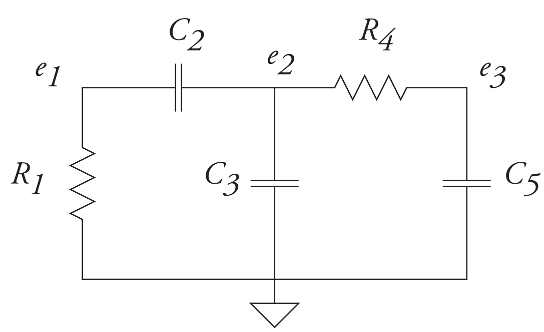
Therefore,

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

or

$$L = \frac{L_1 L_2}{L_1 + L_2}$$

2. Find a set of differential equations that describe the dynamics of the circuit below, using the node method. Label the nodes as shown:



We can wrtie the node equations "by inspection":

$$\begin{aligned}
C_2\frac{de_1}{dt} + G_1 e_1 \qquad\qquad - C_2\frac{de_2}{dt} \qquad\qquad\qquad &= 0 \\
-C_2\frac{de_1}{dt} + C_2\frac{de_2}{dt} + C_3\frac{de_2}{dt} + G_4 e_2 \qquad - G_4 e_3 &= 0 \\
-G_4 e_2 + C_5\frac{de_3}{dt} + G_4 e_3 &= 0
\end{aligned}$$

Using the component values

$$R_1 = 1\ \Omega, \quad C_2 = 0.2\ \mathrm{F}, \quad C_3 = 0.2\ \mathrm{F}, \quad R_4 = 4\ \Omega, \quad C_5 = 0.5\ \mathrm{F}$$

we have that

$$\begin{aligned}
0.2\frac{de_1}{dt} + e_1 \qquad & - 0.2\frac{de_2}{dt} & &= 0 \\
-0.2\frac{de_1}{dt} \qquad & + 0.4\frac{de_2}{dt} + 0.25e_2 & - 0.25e_3 &= 0 \\
& -0.25e_2 \quad + 0.5\frac{de_3}{dt} & + 0.25e_3 &= 0
\end{aligned}$$