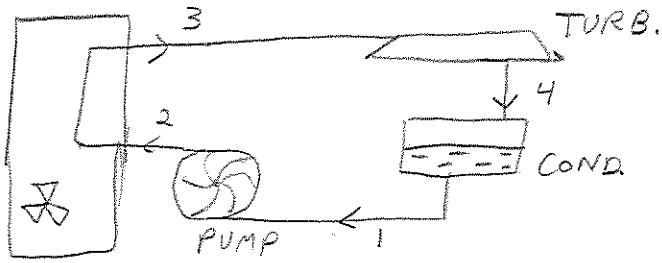
Unified Fall 2006 T17 solutions by Christy Edwards

Given:

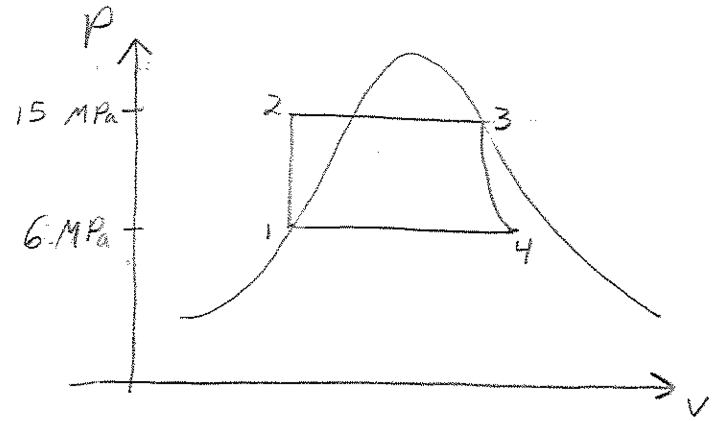


$p_1 = 6$ MPa
$p_3 = 15$ MPa
$x_3 = 1$ (Saturated Steam)
$x_1 = 0$ (Saturated Liquid)

Find: * Pv diagram
* Plant operating conditions

Solution:

a)



b.) $\left.\frac{m_f}{m_g+m_f}\right|_4 < 20\%$, $x \equiv \frac{m_g}{m_f+m_g}$, $\therefore$ $\boxed{x_4 \geq 80\%}$

Quality at 4 must be greater than 0.8.

c.) Find MASS FLOW

SFEE Across turbine:

$$\dot{W}_T = 500\text{ MW} = \dot{m}_{H_2O}(h_3 - h_4) \Rightarrow \dot{m}_{H_2O} = \frac{\dot{W}_T}{h_3 - h_4}$$

At 3: $h_3 = h_g|_{15\text{ MPa}} = 2610.5$ kJ/kg

At 4: $h_f|_{6\text{MPa}} = 1213.32$ kJ/kg

$h_g|_{6\text{MPa}} = 2784.3$ kJ/kg

} From Saturated Water Table

$$h_4 = h_f|_{6\text{MPa}}(1 - x_4) + h_g|_{6\text{MPa}}\, x_4$$
$$= 2470.104 \text{ kJ/kg}$$

$$\dot{m}_{H_2O} = \frac{500 \times 10^3 \text{ kW}}{\left(2610.5 \frac{kJ}{kg} - 2470.104 \frac{kJ}{kg}\right)} = \boxed{3,561 \frac{kg}{s} = \dot{m}_{H_2O}}$$

≈ 10x Aircraft Mass Flow

Note: Makes no distinction between vapor or liquid: It's <u>total</u> mass flow.

d.) <u>Find Main Pump Power</u>

<u>SFEE</u> Across Pump:

$$\dot{W}_p = \dot{m}_{H_2O}(h_2 - h_1)$$

$$h_1 = h_f|_{6MPa} = 1213.32 \text{ kJ/kg}$$ (From saturated water table)

<u>Need h_2</u>

Given $T_2 = 280°C$

$$h_2 = 1232.09 \frac{kJ}{kg}$$ (From compressed liquid water table at 15 MPa)

$$\Rightarrow \dot{W}_p = \left(3561 \frac{kg}{s}\right)\left(1232.09 - 1213.32 \frac{kJ}{kg}\right)$$

$$\boxed{\dot{W}_p = 66.840 \text{ MW}}$$

e.) <u>Reactor Heat Input</u>

$$\dot{Q}_{IN} = \dot{m}_{H_2O}(h_3 - h_2) \quad \text{by SFEE}$$

$$= \left(3,561 \frac{kg}{s}\right)\left(2610.5 - 1232.09 \frac{kJ}{kg}\right)$$

$$\boxed{\dot{Q}_{IN} = 4,908.5 \text{ MW}}$$

A bit large, typically $\dot{Q}_{IN} \approx 3000$ MW

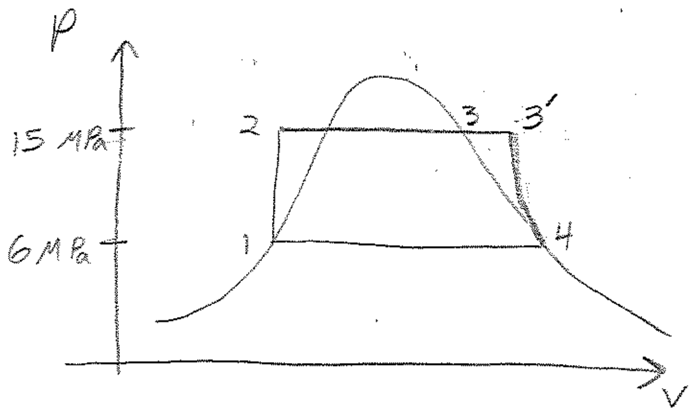
f.) <u>Thermal Efficiency</u>

$$\eta_{th} = \frac{\text{What we get}}{\text{What we pay for}} = \frac{\dot{W}_T - \dot{W}_p}{\dot{Q}_{IN}} = \frac{500 \text{ MW} - 66.84 \text{ MW}}{4,908.5 \text{ MW}}$$

$$\boxed{\eta_{th} = 8.82\%}$$

Quite low. Other plants might have $\eta_{th} \gtrsim 35\%$

Improvements: Superheat



$\eta_{th} \uparrow$ } With
$W_{NET} \uparrow$ } Superheating

g.) Superheated Vapor Table for 15 MPa:
Given $T_{3'} = 400°C$
$h_{3'} = 2975.44$

Net Cycle Work: $\dot{W}_{NET} = \dot{W}_T - \dot{W}_P$ (Need to recalculate)

$\dot{W}_T = \dot{m}_{H_2O}(h_{3'} - h_4)$

Now $x_4 = 1$, so state 4 at vapor saturation

In Saturated Water Tables, when $P = 6\ MPa$

$$h_4 = h_g|_{6MPa} = 2784.33\ \frac{kJ}{kg}$$

$$\dot{W}_T = \left(3{,}561\ \frac{kg}{s}\right)\left(2975.44 - 2784.33\ \frac{kJ}{kg}\right)$$
$$= 68.054\ MW$$

$\dot{W}_{NET} = \dot{W}_T - \dot{W}_P$ ← From previous parts of problem
$= 68.054 - 66.840\ MW$

$$\boxed{\dot{W}_{NET} = 1{,}214\ MW}$$

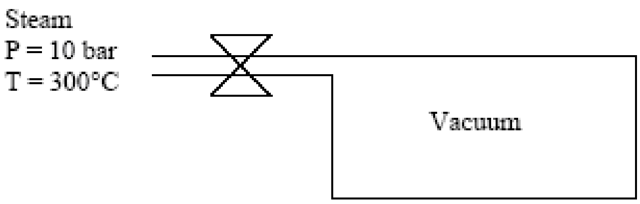
h.) $\eta_{th} = \dfrac{\dot{W}_{NET}}{\dot{Q}_{IN}}$

$$\dot{Q}_{IN} = \dot{m}(h_{3'} - h_2) = \left(3{,}561\ \frac{kg}{s}\right)\left(2975.44 - 1232.09\ \frac{kJ}{kg}\right)$$
$$= 6{,}208\ MW$$

$\Rightarrow \boxed{\eta_{th} = 19.6\ \%}$ Much improved.

Problem 1

GIVEN:



ASSUMPTIONS: • Neglect potential and kinetic energy effects.

CONCEPTS:
- Conservation of mass.
- First Law in Control Volume form.
- Two-phase media.

SOLUTION:

a) Applying mass conservation:

$$\frac{dm_{CV}}{dt} = \dot{m}_{\text{in}}$$

Integrating in time:

$$m_{CV,2} - m_{CV,1} = \Delta m_{\text{in}}$$

The tank is empty at the beginning, $m_{CV,1} = 0$:

$$m_{CV,2} = \Delta m_{\text{in}}$$

Applying First Law in Control Volume:

$$\frac{dU_{CV}}{dt} = \dot{m}_{\text{in}} h_{\text{in}}$$

Integrating in time:

$$U_{CV,2} - U_{CV,1} = \Delta m_{\text{in}} h_{\text{in}}$$

Since the tank is empty at the beginning, $U_{CV,1} = 0$:

$$U_{CV,2} = \Delta m_{\text{in}} h_{\text{in}}$$

From mass conservation, $m_{CV,2} = \Delta m_{\text{in}}$:

$$U_{CV,2} = m_{CV,2} h_{\text{in}} \Rightarrow u_2 = h_{\text{in}}$$

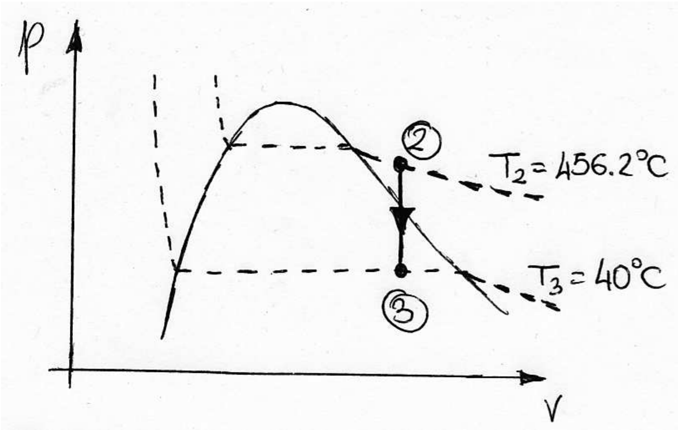
Then, the final specific internal energy is known:

$$u_2 = h_{\text{in}} = h(p = 10 \text{ bar}, T = 300°\text{C}) = 3051.2 \text{ kJ/kg}$$

The final pressure is also known, $p_2 = 10$ bar. Then, T_2 can be obtained by interpolation:

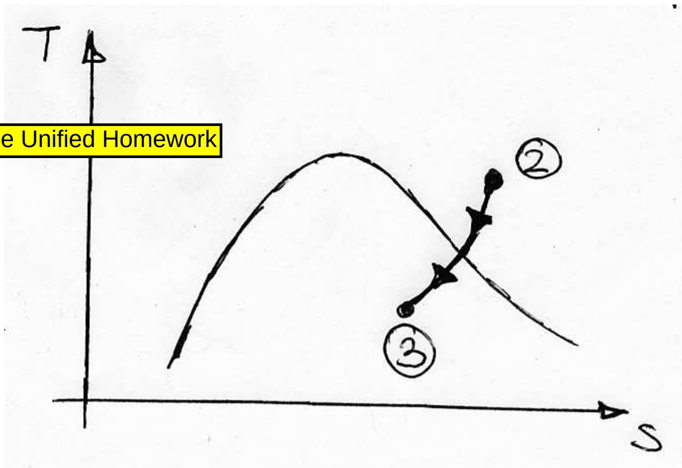
$$\left.\begin{array}{l} p = 1 \text{ MPa},\ T = 400^{\circ}\text{C},\ u = 2957.3 \text{ kJ/kg} \\ p = 1 \text{ MPa},\ T = 500^{\circ}\text{C},\ u = 3124.4 \text{ kJ/kg} \end{array}\right\} T_2 = 456.2^{\circ}\text{C}$$

b) The p-v diagram is:



The T-s diagram is:

Not part of the Unified Homework



c) The process 2-3 is a constant volume heat rejection. The specific volume at point 3 is:

$$v_3 = v_2 = 0.3333 \text{ m}^3/\text{kg}$$

This specific volume is between the specific volumes of the saturated vapor and the saturated liquid at 40°C:

$$v_f(T = 40^{\circ}\text{C}) = 0.001008 \text{ m}^3/\text{kg}$$

$$v_g(T = 40^{\circ}\text{C}) = 19.52 \text{ m}^3/\text{kg}$$

Then, the final state is under the vapor dome. The final pressure is the saturation pressure:

$$p_3 = p_{\text{sat}}(T = 40^{\circ}\text{C}) = 7.384 \text{ kPa}$$

d) The rejected heat can be obtained using First Law:

$$U_3 - U_2 = Q$$

The final specific internal energy is:

$$u_3 = (1 - x_3)u_f(T = 40^{\text{o}}\text{C}) + x_3 u_g(T = 40^{\text{o}}\text{C})$$

x_3 is calculated from:

$$x_3 = \frac{v_3 - v_f(T = 40^{\text{o}}\text{C})}{v_g(T = 40^{\text{o}}\text{C}) - v_f(T = 40^{\text{o}}\text{C})} = 0.01702$$

Then:

$$u_3 = (1 - x_3)u_f(T = 40^{\text{o}}\text{C}) + x_3 u_g(T = 40^{\text{o}}\text{C}) = 206.1 \text{ kJ/kg}$$

where $u_f(T = 40^{\text{o}}\text{C}) = 167.56$ kJ/kg and $u_g(T = 40^{\text{o}}\text{C}) = 2430.1$ kJ/kg.

The internal energy at point 2 is:

$$u_2 = h_{\text{in}} = 3051.2 \text{ kJ/kg}$$

Finally, the mass in the tank is:

$$m_{\text{vessel}} = \frac{V}{v_3} = 9 \times 10^{-3} \text{ kg}$$

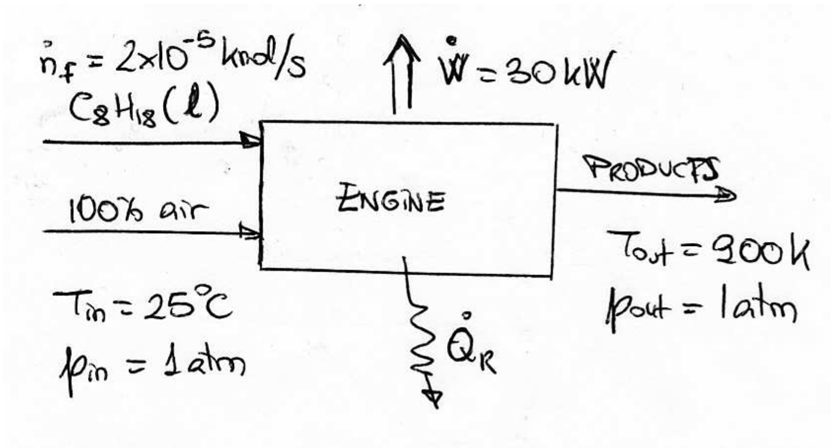
Then:

$$Q = m_{\text{vessel}}(u_3 - u_2) = -25.61 \text{ kJ}$$

This heat has negative sign because is rejected heat.

Problem 5

GIVEN:



ASSUMPTIONS:
- Steady flow.
- Neglect kinetic and potential energy effects.

CONCEPTS:
- Stoichiometry.
- First Law in CV form.
- Enthalpy of formation.

SOLUTION:

The stoichiometric reaction is:

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 12.5 \times 3.76N_2$$

Using First Law in Control Volume form:

$$0 = -\dot{Q}_R - \dot{W} + \sum_R \dot{n}_R \overline{h}_R - \sum_P \dot{n}_P \overline{h}_P$$

The reactants enthalpy flow is given by:

$$\sum_R \dot{n}_R \overline{h}_R = \dot{n}_f(\overline{h}_{C_8H_{18}} + 12.5\overline{h}_{O_2} + 47\overline{h}_{N_2}) = \dot{n}_f \overline{h}^0_{f,C_8H_{18}(l)} = -5.002 \text{ kW}$$

The products enthalpy flow is given by:

$$\sum_P \dot{n}_P \overline{h}_P = \dot{n}_f(8\overline{h}_{CO_2} + 9\overline{h}_{H_2O} + 47\overline{h}_{N_2}) = -80.929 \text{ kW},$$

where

$$\overline{h}_{CO_2} = \overline{h}^0_{f,CO_2} + \Delta\overline{h}_{CO_2}|_{T=900\text{ K}} = -393522 \text{ kJ/kmol} + 28030 \text{ kJ/kmol} = -365492 \text{ kJ/kmol}$$

$$\overline{h}_{H_2O} = \overline{h}^0_{f,H_2O} + \Delta\overline{h}_{H_2O}|_{T=900\text{ K}} = -241826\text{ kJ/kmol} + 21937\text{ kJ/kmol} = -219889\text{ kJ/kmol}$$

$$\overline{h}_{N_2} = \overline{h}^0_{f,N_2} + \Delta\overline{h}_{N_2}|_{T=900\text{ K}} = 0\text{ kJ/kmol} + 18223\text{ kJ/kmol} = 18223\text{ kJ/kmol}$$

Finally:

$$\dot{Q}_R = -\dot{W} + \sum_R \dot{n}_R\overline{h}_R - \sum_P \dot{n}_P\overline{h}_P = 45.93\text{ kW}$$

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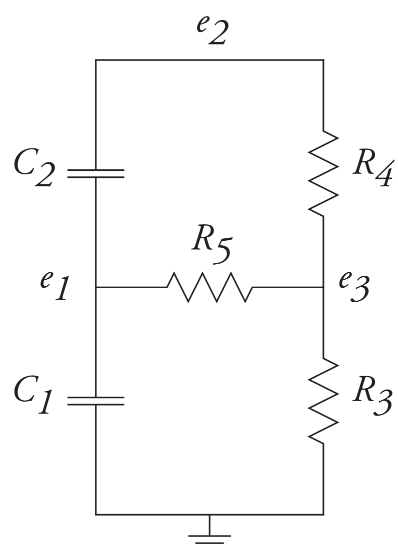
Problem S9 (Signals and Systems) SOLUTION



Write down the node equations, using impedances:

$$\begin{aligned}(C_1 s + C_2 s + G_5)E_1 \quad - C_2 s E_2 \quad - G_5 E_3 &= 0\\ -C_2 s E_1 + (C_2 s + G_4)E_2 \quad - G_4 E_3 &= 0\\ -G_5 E_1 \quad -G_4 E_2 + (G_3 + G_4 + G_5)E_3 &= 0\end{aligned}$$

Using the component values

$$C_1 = 1 \text{ F}, \quad C_2 = 2 \text{ F}, \quad R_3 = 2\ \Omega, \quad R_4 = 1\ \Omega, \quad R_5 = 1\ \Omega$$

we have that

$$\begin{bmatrix} 3s+1 & -2s & -1 \\ -2s & 2s+1 & -1 \\ -1 & -1 & 2.5 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = M(s)\underline{E}$$

In order to have a non-trivial solution, must have that

$$\det[M(s)] = 5s^2 + 3.5s + 0.5$$

The roots are

$$s_1 = -\frac{1}{2}, \quad s_2 = -\frac{1}{5}$$

Solve for the characteristics vectors. Define

$$M_1 = M(s_1) = \begin{bmatrix} -0.5 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 2.5 \end{bmatrix}$$

Row reduction yields

$$M_1' = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & -1.5 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, a characteristic vector is

$$\underline{E}_1 = \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix}$$

Similarly, define

$$M_2 = M(s_2) = \begin{bmatrix} 0.4 & 0.4 & -1 \\ 0.4 & 0.6 & -1 \\ -1 & -1 & 2.5 \end{bmatrix}$$

Row reduction yields

$$M_1' = \begin{bmatrix} 1 & 1 & -2.5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, a characteristic vector is

$$\underline{E}_2 = \begin{bmatrix} 2.5 \\ 0 \\ 1 \end{bmatrix}$$

The general solution is

$$\underline{e}(t) = a \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix} e^{-0.5t} + b \begin{bmatrix} 2.5 \\ 0 \\ 1 \end{bmatrix} e^{-0.2t}$$

The initial conditions are that

$$e_1(0) = 3 \text{ V} = a + 2.5b$$
$$e_2(0) = 6 \text{ V} = 1.5a$$

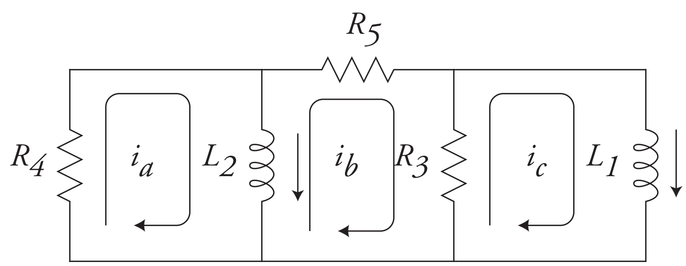
Solving for a and b,

$$a = 4$$
$$b = -0.4$$

The node voltages as a function of time are therefore

$$e_1(t) = \left(4e^{-0.5t} - 1e^{-0.2t}\right) \text{ V}$$
$$e_2(t) = \left(6e^{-0.5t}\right) \text{ V}$$
$$e_3(t) = \left(4e^{-0.5t} - 0.4e^{-0.2t}\right) \text{ V}$$

Problem S10 (Signals and Systems) SOLUTION



Label the loop currents as above. Using impedances, we can write the loop equations (KVL) as

$$\begin{aligned}(L_2 s + R_4) I_a \quad - L_2 s I_b \quad &= 0\\ -L_2 s I_a + (L_2 s + R_3 + R_5) I_b \quad - R_3 I_c &= 0\\ -R_3 I_b + (L_1 s + R_3) I_c &= 0\end{aligned}$$

Plug in the component values

$$L_1 = 1 \text{ H}, \quad L_2 = 1 \text{ H}, \quad R_3 = 1\ \Omega, \quad R_4 = 1\ \Omega, \quad R_5 = 0.5\ \Omega$$

Then

$$\begin{bmatrix} s+1 & -s & 0 \\ -s & s+1.5 & -1 \\ 0 & -1 & s+1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = M(s)\underline{I} = \underline{0}$$

Non-trivial solutions exist when

$$\det[M(s)] = 0 = 2.5s^2 + 3s + 0.5$$

Therefore, the characteristic values are

$$\begin{aligned} s_1 &= -1 \\ s_2 &= -0.2 \end{aligned}$$

Next, solve for the charactersitic values. Define

$$M_1 = M(s_1) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0.5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Row reduce to obtain

$$M_1' = \begin{bmatrix} 1 & 0.5 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, a characteristic vector is

$$\underline{I}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, define

$$M_2 = M(s_2) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 1.3 & -1 \\ 0 & -1 & 0.8 \end{bmatrix}$$

Row reduce to obtain

$$M_2' = \begin{bmatrix} 1 & -0.25 & 0 \\ 0 & 1 & -0.8 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, a characteristic vector is

$$\underline{I}_2 = \begin{bmatrix} -0.2 \\ 0.8 \\ 1 \end{bmatrix}$$

So, the general solution is

$$\underline{i}(t) = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{-t} + b \begin{bmatrix} -0.2 \\ 0.8 \\ 1 \end{bmatrix} e^{-0.2t}$$

Apply the initial conditions

$$\begin{aligned} i_1(0) &= 10 \text{ A} = i_c(0) = a + b \\ i_2(0) &= 0 \text{ A} = i_a(0) - i_b(0) = a - b \end{aligned}$$

Solving for a and b yields

$$a = b = 5 \text{ A}$$

Therefore,

$$\begin{aligned} i_1(t) &= \left(5e^{-t} + 5e^{-0.2t}\right) \text{ A} \\ i_2(t) &= \left(5e^{-t} - 5e^{-0.2t}\right) \text{ A} \end{aligned}$$

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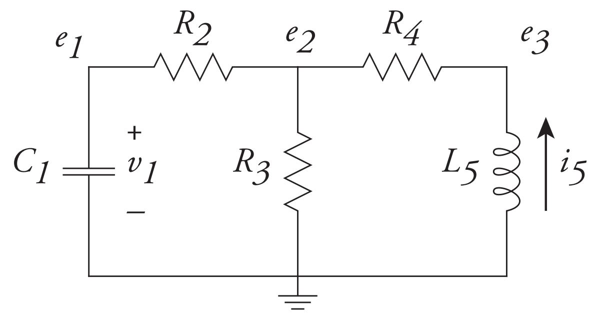
Problem S11 (Signals and Systems) SOLUTION

Label the node potentials as shown:



The node equations, using impedances, are

$$\begin{aligned}(C_1 s + G_2)E_1 \quad - G_2 E_2 \quad &= 0\\ -G_2 E_1 + (G_2 + G_3 + G_4)E_2 \quad - G_4 E_3 &= G_2 v_1\\ - G_4 E_2 + \left(\frac{1}{L_5 s} + G_4\right) E_3 &= 0\end{aligned}$$

Plugging in the component values,

$$C_1 = 0.25\ \text{F}, \quad R_2 = 4\ \Omega, \quad R_3 = 4\ \Omega, \quad R_4 = 1\ \Omega, \quad L_5 = 1\ \text{H}$$

we have that

$$\begin{aligned}(0.25s + 0.25)E_1 - 0.25E_2 \quad &= 0\\ -0.25E_1 \ + 1.5E_2 \quad - E_3 &= 0\\ -E_2 + \left(\frac{1}{s} + 1\right) E_3 &= 0\end{aligned}$$

or in matrix form,

$$\begin{bmatrix} 0.25s + 0.25 & -0.25 & 0 \\ -0.25 & 1.5 & -1 \\ 0 & -1 & \frac{1}{s} + 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = M(s)\underline{E} = \underline{0}$$

Non-trivial solutions exist for

$$\det[M(s)] = \frac{2s^2 + 7s + 5}{16s} = 0$$

The roots are

$$\begin{aligned}s_1 &= -1\\ s_2 &= -2.5\end{aligned}$$

Next, find the characteristic vectors. Define

$$M_1 = M(s_1) = \begin{bmatrix} 0 & -0.25 & 0 \\ -0.25 & 1.5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Row reduce to obtain

$$M_1' = \begin{bmatrix} 1 & -6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A characteristic vector is then

$$\underline{E}_1 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, define

$$M_2 = M(s_2) = \begin{bmatrix} -0.375 & -0.25 & 0 \\ -0.25 & 1.5 & -1 \\ 0 & -1 & 0.6 \end{bmatrix}$$

Row reduce to obtain

$$M_2' = \begin{bmatrix} 1 & 0.666\overline{6} & 0 \\ 0 & 1.666\overline{6} & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

A characteristic vector is then

$$\underline{E}_2 = \begin{bmatrix} -0.4 \\ 0.6 \\ 1 \end{bmatrix}$$

The general solution is

$$\underline{e}(t) = a \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} e^{-t} + b \begin{bmatrix} -0.4 \\ 0.6 \\ 1 \end{bmatrix} e^{-2.5t}$$

The initial conditions on the capacitor and inductor are

$$v_1(0) = 2 \text{ V}, \quad i_5(0) = 1 \text{ A}$$

Since $v_1 = e_1 - 0 = e_1$, the I.C. on the capacitor yields

$$-4a - 0.4b = 2$$

Now, the voltage across the inductor is $v_5 = 0 - e_3 = -e_3 = -ae^{-t} - be^{-2.5t}$. Since the impedance of the inductor is $L_5 s$, we have that the current through the inductor is

$$i_5(t) = \frac{-ae^{-t}}{L_5(-1)} + \frac{-be^{-2.5t}}{L_5(-2.5)} = ae^{-t} + 0.4be^{-2.5t}$$

The I.C. on the inductor implies that

$$a + 0.4b = 1$$

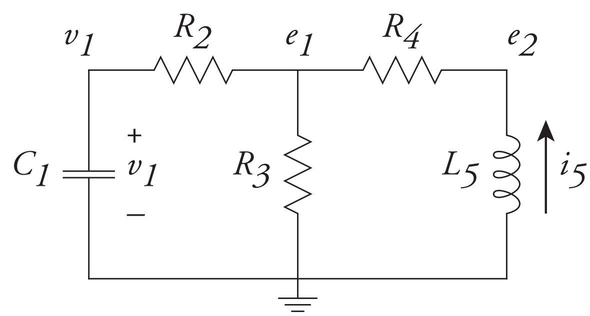
Solving for a and b, we have that

$$\begin{aligned} a &= -1 \text{ V} \\ b &= 5 \text{ V} \end{aligned}$$

Therefore, the final solution is

$$\begin{aligned} v_1(t) &= \left(4e^{-t} - 2e^{-2.5t}\right) \text{ V} \\ i_5(t) &= \left(-e^{-t} + 2e^{-2.5t}\right) \text{ V} \end{aligned}$$

Problem S12 (Signals and Systems) SOLUTION



Use the node voltages as labeled above. The node equations are

$$(G_2 + G_3 + G_4)e_1 - G_4 e_2 = G_2 v_1$$
$$-G_4 e_1 + G_4 e_2 = i_5$$

Using the component values

$$C_1 = 0.25 \text{ F}, \quad R_2 = 4\ \Omega, \quad R_3 = 4\ \Omega, \quad R_4 = 1\ \Omega, \quad L_5 = 1 \text{ H}$$

we have that

$$1.5e_1 - e_2 = 0.25v_1$$
$$-e_1 + e_2 = i_5$$

Solve for e_1, e_2 using Cramer's rule or row reduction to obtain

$$e_1 = 0.5v_1 + 2i_5$$
$$e_2 = 0.5v_1 + 3i_5$$

The constitutive law for the capacitor is

$$\frac{dv_1}{dt} = \frac{1}{C_1} i_1$$

Use KCL at the v_1 node to obtain i_1:

$$i_1 + G_2(v_1 - e_1) = 0$$

Solving for i_1, and using component values,

$$i_1 = -G_2(0.5v_1 - 2i_5) = -0.125v_1 + 0.5i_5$$

Then

$$\frac{dv_1}{dt} = \frac{1}{C_1} i_1 = -0.5v_1 + 2i_5$$

The constitutive law for the inductor is

$$\frac{di_5}{dt} = \frac{1}{L_5} v_5$$

Using $v_5 = -e_2$, and the value of L_5, we have that

$$\frac{di_5}{dt} = -0.5 v_1 - 3 i_5$$

Therefore, the dynamics matrix is

$$A = \begin{bmatrix} -0.5 & 2 \\ -0.5 & -3 \end{bmatrix}$$

The eigenvalues are the roots of

$$\det(sI - A) = 0 = s^2 + 3.5s + 2.5$$

The eigenvalues are then $s = -1, -2.5$, just as in Problem S11.