9:00 - 10: 00
35-225

MIT ID# (last four digits) SOLUTIONS

Unified Quiz 2M

October 13, 2006

- Put the last four digits of your MIT ID # on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the actual answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units. Intermediate answers and final answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators and hardwritten "crib sheets" are allowed.***

EXAM SCORING

#1M (25%)	
#2M (25%)	
#3M (25%)	
#4M (25%)	
FINAL SCORE	

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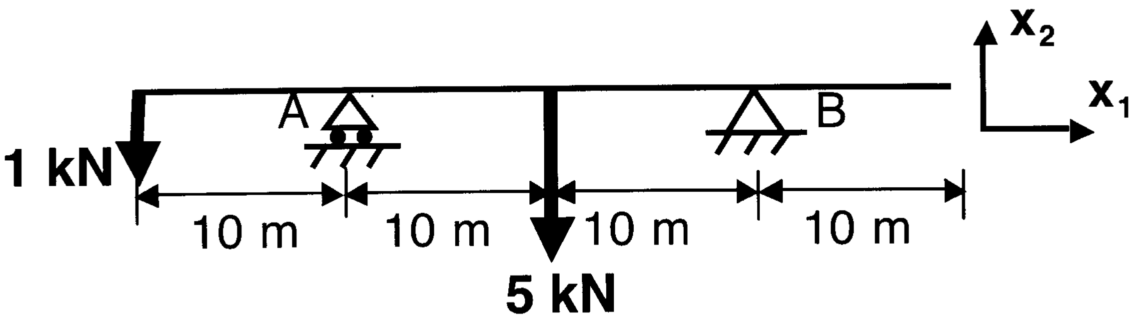
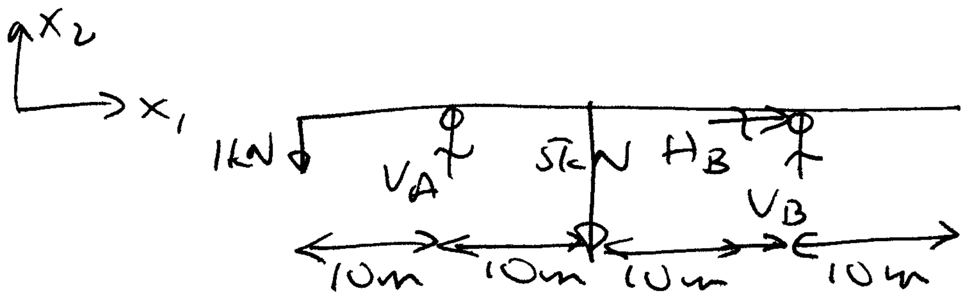
PROBLEM #1M (25%)

The 40-meter long structure, depicted in the figure below, is supported by a roller 10 meters in from its left end, and a pin, 10 meters in from its right end. It is subjected to a tip load of 1000 Newtons (downwards) at its left end and by a mid-span load of 5000 Newtons (downwards).



(a) Determine the reaction forces at support points A and B ***or*** indicate all information available to determine the reaction forces and the additional information that is needed to fully determine these forces.

First draw the Free Body Diagram



Have 3 reactions and 3 degrees of freedom, so this is statically determinate and can determine the reactions.

Use equations of equilibrium:

$$\sum F_1 = 0 \quad \xrightarrow{+} \quad \boxed{H_B = 0}$$

$$\sum F_2 = 0 \quad \uparrow + \quad -1kN + V_A - 5kN + V_B = 0$$

$$\Rightarrow V_A + V_B = 6\,kN$$

PROBLEM #1M (continued)

$\sum M = 0$ (about A) $\circlearrowleft +$ $1kN(10m) - 5kN(10m) + V_B(20m) = 0$

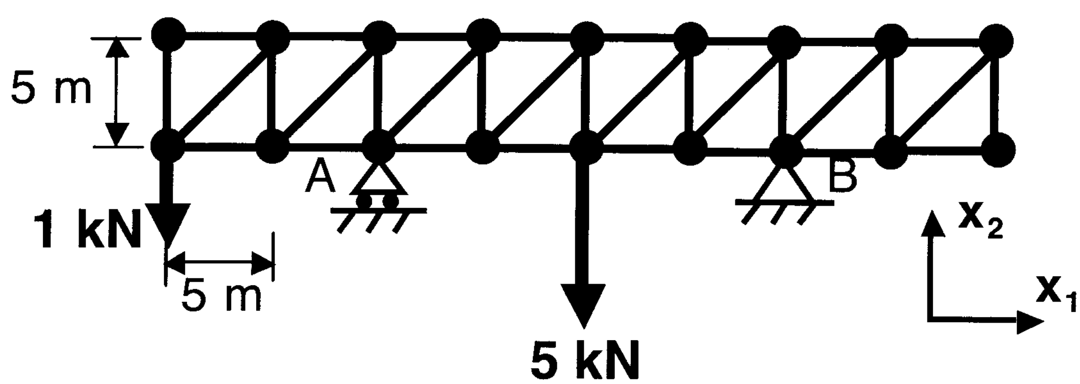
$\Rightarrow \boxed{V_B = 2kN}$

using $\sum F_2 = 0 \Rightarrow V_A + 2kN = 6kN$

$\Rightarrow \boxed{V_A = 4kN}$

Summarizing: $\boxed{\begin{array}{l} V_A = 4kN \\ V_B = 2kN \\ H_B = 0 \end{array}}$

(b) How does this change if the structure is replaced by an 8-bay truss, as depicted, of 5 meter by 5 meter units? Explain clearly.



Although the structure itself changed, the Free Body Diagram does not and thus the reactions $\boxed{\text{do not change}}$.

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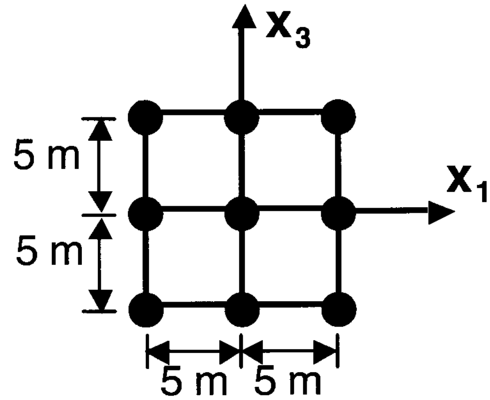
PROBLEM #2M (25%)

A set of three forces acts in the x_1-x_3 plane at node points in the depicted 5 meter by 5 meter grid. The force vectors and the (x_1, x_3) points at which they act are as follows:

$\underline{F}_A = (-5\text{ N})\,\underline{i}_1$ acts at (5m, 0m)
$\underline{F}_B = (3\text{ N})\,\underline{i}_1 - (2\text{ N})\,\underline{i}_3$ acts at (–5m, 0m)
$\underline{F}_C = (-10\text{ N})\,\underline{i}_3$ acts at (5m, 5m)



(a) Determine the force system acting at the origin that is equipollent to this force system.

The total force and moment with regard to the origin must be determined in finding the equipollent force system acting at the origin.

<u>Force</u>: Must have same magnitude and direction as effect of these three forces:

Find $\sum \underline{F}_i = \underline{F}_A + \underline{F}_B + \underline{F}_C$

$+\uparrow \overset{+}{\rightarrow}$ $= (-5N)\underline{i}_1 + (3N)\underline{i}_1 - (2N)\underline{i}_3 + (-10N)\underline{i}_3$

$$\Rightarrow \boxed{\underline{F}_{(0,0)} = (-2N)\underline{i}_1 + (-12N)\underline{i}_3}$$

<u>Moment</u>: Must include pure moment equal to sum of moments caused by three forces acting about origin.

$\sum M_{(0,0)} = \sum M_i = \sum (\underline{r}_i \times \underline{F}_i)$

$\circlearrowleft +$ $= (5m)\underline{i}_1 \times (-5N)\underline{i}_1 + (-5m)\underline{i}_1 \times [(3N)\underline{i}_1 - (2N)\underline{i}_3]$
$+ [(5m)\underline{i}_1 + (5m)\underline{i}_3] \times (-10N)\underline{i}_3$

$= 10\text{ N}\cdot\text{m} - 50\text{ N}\cdot\text{m}$

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PROBLEM #2M (continued)

$$\Rightarrow \boxed{M_{(0,0)} = -40\ N\cdot m} \quad (\circlearrowleft \text{ in } x_1 - x_3$$

(Note change in sense of $\underline{i}_2$ in defining this, otherwise get $+40N\cdot m$ acting about $+\underline{i}_2$)

Summarizing:

$$\boxed{\begin{aligned} F_{(0,0)} &= (-2N)\underline{i}_1 + (-12N)\underline{i}_3 \\ M_{(0,0)} &= -40\ N\cdot m \end{aligned}}$$

(b) Can this system be put in equilibrium by applying one force at any one node of the grid? If so, what is that force and what is the location? If not, explain why not and how many forces are needed and how these forces and their location can be determined. Clearly explain your reasoning.

One force must have the same magnitude and opposite direction for equilibrium. This is:

$$\underline{F} = 2N\underline{i}_1 + 12N\underline{i}_3$$

This would need to be put at a point where it would cause exactly the opposite moment:

$$M = 40\ N\cdot m \quad \circlearrowleft^{+}$$

Inspection shows $\boxed{\text{it cannot be done with one force}}$

It can clearly $\boxed{\text{be done by two forces}}$. Put one at any node equal to $\underline{F}$. Now construct a couple using this node and any other and adjust the magnitude of the couple forces to achieve $M = 40N\cdot m$. Sum the two forces at the first node into one but note that adding the two couple forces caused no change in the overall force

$\Rightarrow$ equilibrium achieved

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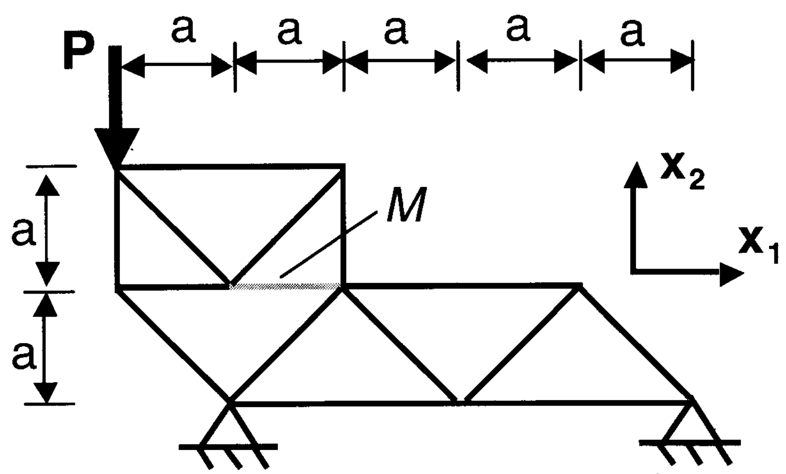
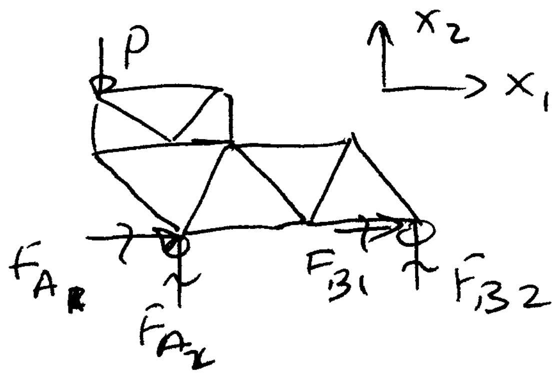
PROBLEM #3M (25%)

The following truss with various triangular bays is loaded at one tip by a load P as shown in the figure below.



(a) What is the "class/category" of this structural configuration (Dynamic, Statically Determinate, Statically Indeterminate)? Clearly explain your reasoning.

Draw the Free Body Diagram:



There are 4 reactions and this is a 2-D system and thus it has 3 degrees of freedom

# of reactions > # degrees of freedom

$\Rightarrow$ $\boxed{\text{Statically Indeterminate}}$

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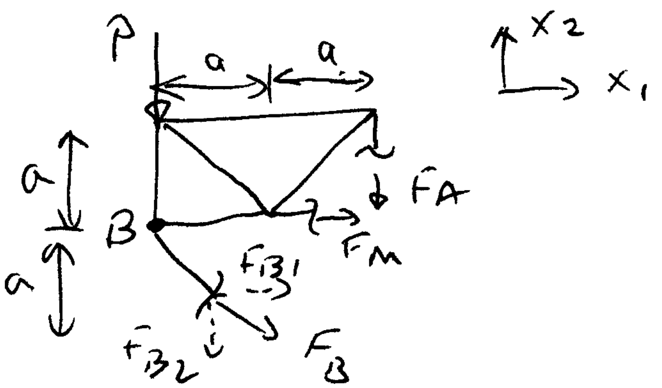
PROBLEM #3M (continued)

(b) Determine the load in the horizontal bar of the truss labeled "M" (highlighted in the figure) ***or*** indicate the additional information needed in order to determine this load.

Do a Method of Section:



The B-bar makes a 45° angle with x_1, so:

$$F_{B1} = \sqrt{2}/2\ F_B$$

$$F_{B2} = \sqrt{2}/2\ F_B$$

Use equilibrium:

$$\sum F_1 = 0 \quad \xrightarrow{+} \quad \Rightarrow \frac{\sqrt{2}}{2} F_B + F_M = 0$$

$$\sum F_2 = 0 \quad \uparrow + \quad \Rightarrow -P - F_A - \frac{\sqrt{2}}{2} F_B = 0$$

$$\sum M = 0 \quad \circlearrowleft + \quad \Rightarrow -F_A(2a) = 0 \Rightarrow F_A = 0$$

(take about point B)

use in $\sum F_2 \Rightarrow F_B = -\sqrt{2}\, P$

use in $\sum F_1 \Rightarrow -P + F_M = 0$

$$\Rightarrow \boxed{F_M = P}$$

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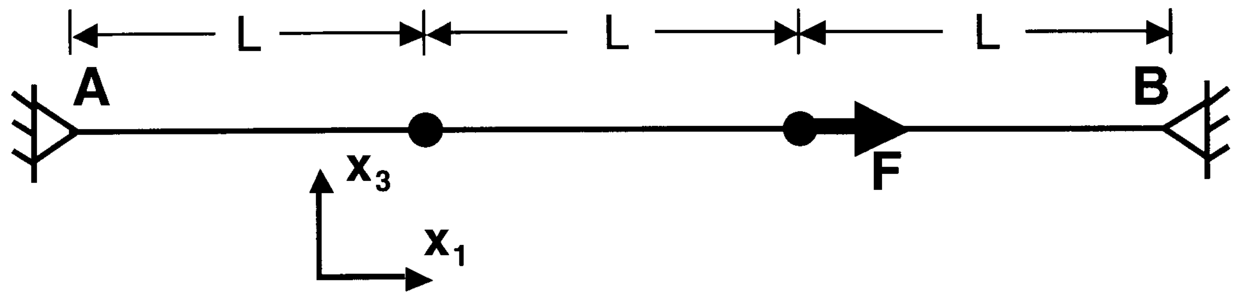
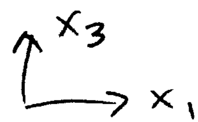
PROBLEM #4M (25%)

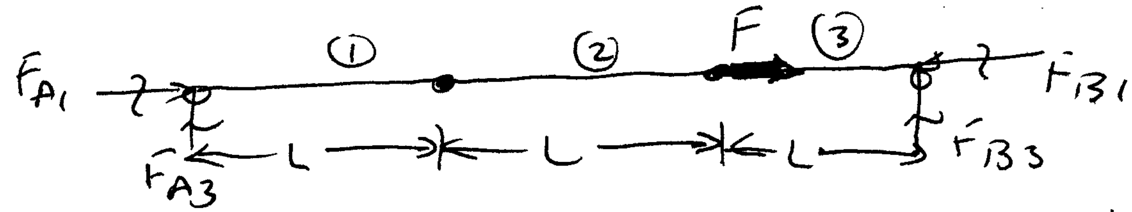
A collinear, three-bar system, with each bar of the same material, area, and length, L, is pinned at each end, and is loaded at the junction between the second and third bar by a load, F, acting along the line of the bars. This configuration is depicted below.



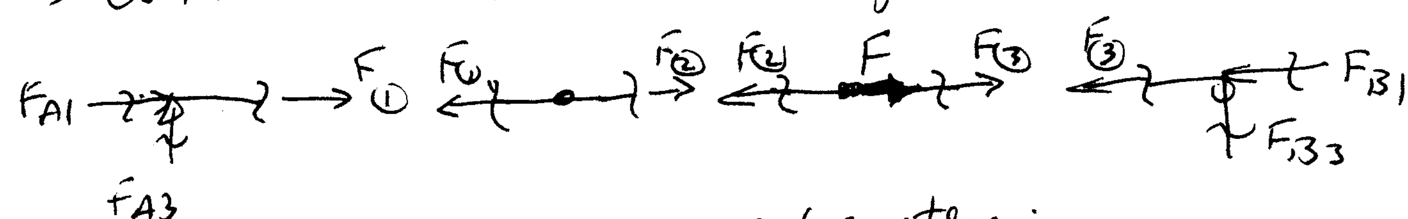
(a) Set up the equations available to determine the reaction loads.

Draw the Free Body Diagram:





→ Cut each bar and consider separate subsystems:



Do equilibrium for each subsystem:

first: $\sum F_1 = 0 \overset{+}{\rightarrow} \quad F_{A1} + F_{①} = 0 \Rightarrow F_{A1} = -F_{①}$ (1)

$\sum F_3 = 0 \uparrow + \quad F_{A3} = 0$

second: $\sum F_1 = 0 \overset{+}{\rightarrow} \quad -F_{①} + F_{②} = 0 \Rightarrow F_{①} = F_{②}$ (2)

$\sum F_3 = 0$ no info

third: $\sum F_1 = 0 \overset{+}{\rightarrow} \quad -F_{②} + F + F_{③} = 0$ (3)

$\sum F_3 = 0$ no info

PROBLEM #4M (continued)

fourth: $\sum F_1 = 0 \ \xrightarrow{+} \quad -F_{③} + F_{B1} = 0 \Rightarrow F_{③} = -F_{B1} \quad (4)$

$\sum F_3 = 0 \ \uparrow + \quad \Rightarrow \quad F_{B3} = 0$

Note: ~~no~~ moments for each subsystem since loads are through node points.

→ Have 4 equations and 5 unknowns ($F_{①}$, $F_{②}$, $F_{③}$, F_{A1}, F_{B1})

→ use constitutive relations. For a bar:

$F = \frac{EA}{L}\delta$ E = modulus of elasticity, A: Area, L = length } same for all 3 beams

so:

$F_{①} = \frac{EA}{L}\delta_{①} \quad (5)$

$F_{②} = \frac{EA}{L}\delta_{②} \quad (6)$

$F_{③} = \frac{EA}{L}\delta_{③} \quad (7)$

Now have 7 equations in 8 unknowns (added $\delta_{①}$, $\delta_{②}$, $\delta_{③}$)

→ use compatibility:

Three deformation must sum to zero since two supports are rigid in x_1:

$\delta_{①} + \delta_{②} + \delta_{③} = 0 \quad (8)$

$\boxed{\text{8 equations in 8 unknowns} \Rightarrow \text{set}}$

PROBLEM #4M (continued)

(extra credit)

(b) Determine the reactions loads ***or*** indicate what additional information is needed to do such.

→ Use (5) and (6) and (7) in (8):
(note 4/~~EA~~ cancel out)

$$\Rightarrow F_{①} + F_{②} + F_{③} = 0$$

Use (2):

$$2F_{②} + F_{③} = 0 \Rightarrow F_{③} = -2F_{②} \quad (9)$$

and then (3):

$$-F_{②} + F - 2F_{②} = 0$$

$$\Rightarrow F_{②} = F/3$$

fixing from (2): $F_{①} = F/3$

and from (9): $F_{③} = -2F/3$

→ Finally use (1) and (4):

$$\boxed{\begin{aligned} F_{A1} &= -F/3 \\ F_{A3} &= 0 \\ F_{B1} &= 2F/3 \\ F_{B3} &= 0 \end{aligned}}$$