Unified Quiz 4M
November 17, 2006

- Put the last four digits of your MIT ID # on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. In some cases, the reasoning is worth as much (or more) than the actual answers.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units. Intermediate answers and final answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- Calculators and handwritten “crib sheets” are allowed.
- Unified Handout entitled “Review of Stress, Strain, and Elasticity” allowed.

EXAM SCORING

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1M (25%)</td>
<td></td>
</tr>
<tr>
<td>#2M (25%)</td>
<td></td>
</tr>
<tr>
<td>#3M (25%)</td>
<td></td>
</tr>
<tr>
<td>#4M (25%)</td>
<td></td>
</tr>
<tr>
<td>FINAL SCORE</td>
<td></td>
</tr>
</tbody>
</table>
PROBLEM #1M (25%)

An orthotropic material has a general loading applied resulting in stresses of $s_{11}$, $s_{22}$, and $s_{12}$, as indicated in the accompanying figure. The stresses may not be aligned with the main axes of the material. A three-gage strain gage rosette is placed on the material and aligned in such a way that the strain in the $x_1$-direction, $e_{11}$, and in the $x_2$-direction, $e_{22}$, are measured along with the strain at a $45^\circ$ angle to the $x_1$-direction, $e_{45}$.

\[
\begin{align*}
\sigma_{11} & & \sigma_{12} \\
\sigma_{12} & & \sigma_{22}
\end{align*}
\]

\[\text{I} = \text{strain gage}\]

Determine the principal in-plane strains or clearly indicate why they cannot be determined.
PROBLEM #1M (continued)
**PROBLEM #2M (25%)**

A square block of a very compliant material is in the $x_2 - x_3$ plane as pictured below to the left. This block initially has *unit dimensions* on each side. The slab is then placed into a form so that it undergoes deformation consistent with the boundaries of the form, as pictured below to the right. The dimensions are shown in values *normalized to unit dimensions*.

An engineer performs an analysis and indicates that the following strain field should result:

\[
\varepsilon_{22} = 0.02 \quad \varepsilon_{33} = -0.08 x_2 \quad \varepsilon_{23} = -0.04 x_3
\]

It is known that the strain does not vary with $x_1$.

(a) Determine whether this strain field is consistent with the deformation described. Clearly give your reasoning. Use equations as appropriate.
PROBLEM #2M (continued)
PROBLEM #2M (continued)

(b) The form is now changed so that the deformations are increased by a factor of ten. How will this affect the values of the in-plane strains? Describe clearly using equations as necessary.
PROBLEM #3M (25%) 

As an engineer, you have been presented with the statement:

“The modulus of this unidirectional carbon/epoxy material is 300 GPa”

(a) List any assumptions and limitations in the use of this information. Be sure to clearly describe the source(s) of these assumptions and limitations and their ramifications.
(b) **List** the factors that contribute at lower levels of lengthscale to this statement. Clearly **indicate** the relative importance of each, the lengthscale(s) at which they operate, and any associate limitations.
PROBLEM #4M (25%) 

A unidirectional composite ply has the following elastic constants:

\[
\begin{align*}
E_1 &= 120 \text{ GPa} \\
E_2 &= 20 \text{ GPa} \\
\nu_{12} &= 0.30 \\
\nu_{21} &= 0.05 \\
G_{12} &= 35 \text{ GPa}
\end{align*}
\]

These are referenced with the \( x_1 \) direction along the fibers and the \( x_2 \) direction perpendicular to the fibers. The unidirectional ply is loaded by a stress of 100 MPa at an angle 30° off the fiber direction as pictured. Strain gages are placed on the surface of the ply parallel and perpendicular to the fiber direction (also shown).

Can the strain readings be determined? If so, do so. If not, clearly explain why not, using equations as appropriate.
PROBLEM #4M (continued)