UE Fluids   Quiz 2 Solution   Fall '06

1)

a) $V_\theta = \frac{C}{r}$, $V_r = 0$ is an irrotational, incompressible flow. From table: $V_\theta = -\frac{\Gamma}{2\pi r}$

so for this case, $C = -\frac{\Gamma}{2\pi}$. Also from table for this flow: $\boxed{\psi = \frac{\Gamma}{2\pi}\ln r = -C\ln r}$

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Alternatively, can obtain $\psi(r,\theta)$ from $\frac{\partial \psi}{\partial r} = -V_\theta = -\frac{C}{r} \Rightarrow \boxed{\psi = -C\ln r} + f(\theta)$

$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = V_r = 0 \Rightarrow \psi = \text{const}$ ← equal, set to 0 arbitrarily

b) $\boxed{\dot{\mathcal{V}}' = \int \vec{V}\cdot\hat{n}\,dA = \int_{R_1}^{R_2} V_\theta\,dr = \int_{R_1}^{R_2}\frac{C}{r}\,dr = C(\ln R_2 - \ln R_1) = C\ln\left(\frac{R_2}{R_1}\right)}$

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Alternatively: use streamfunction: $\boxed{\dot{\mathcal{V}}' = \psi_1 - \psi_2 = C(\ln R_2 - \ln R_1)}$ same result

c) This is an irrotational flow (it's on the table), so $\boxed{\zeta = 0}$ 10

Alternatively, can compute directly: $\zeta = \nabla\times\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_\theta) - \frac{1}{r}\overset{0}{\frac{\partial V_r}{\partial \theta}} = \frac{1}{r}\frac{\partial}{\partial r}\left(\overset{\text{constant}}{r\cdot\frac{C}{r}}\right) = 0$

d) Helmholtz Equation: $\frac{D\zeta}{Dt} = 0$, or ζ = constant along streamlines.

So... Since $\zeta = 0$ in bend, it will stay at $\zeta = 0$ in straight section.

We have parallel flow in straight section, so $v = 0$, and $\zeta = \overset{0}{\frac{\partial v}{\partial x}} - \frac{\partial u}{\partial y} = 0$

So $\frac{\partial u}{\partial y} = 0$, or $\boxed{u(y) = \text{const.}}$ Correct answer is (iii)

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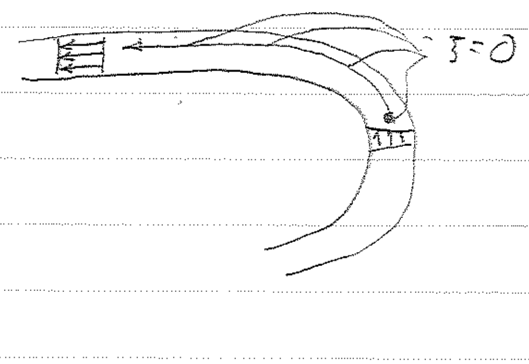

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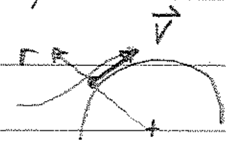
2).

a) $V_r = \frac{\partial\phi}{\partial r} = V_\infty\cos\theta - \frac{K}{2\pi}\frac{\cos\theta}{r^2} = \left(V_\infty - \frac{K}{2\pi}\frac{1}{r^2}\right)\cos\theta$



On cylinder surface: $V_r = 0$

at $r=R$: $V_\infty - \frac{K}{2\pi}\frac{1}{R^2} = 0 \Rightarrow \boxed{K = 2\pi R^2 V_\infty}$ (10)

b) $V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -V_\infty\sin\theta - \frac{K}{2\pi}\frac{\sin\theta}{r^2} - \frac{\Gamma}{2\pi r} = -V_\infty\left[1+\frac{R^2}{r^2}\right]\sin\theta - \frac{\Gamma}{2\pi R}$

On cylinder, $r=R$, $V_r=0$, so $V = \sqrt{V_r^2 + V_\theta^2} = |V_\theta| = \left|-2V_\infty\sin\theta - \frac{\Gamma}{2\pi R}\right|$

A: $\sin\theta = 1$, $V = \left|2V_\infty + \frac{\Gamma}{2\pi R}\right|$ (5)

B: $\sin\theta = -1$, $V = \left|2V_\infty - \frac{\Gamma}{2\pi R}\right|$ (5)

C: $\sin\theta = 0$, $V = \left|\frac{\Gamma}{2\pi R}\right|$ (5)

c) i) From a) $R^2 = \frac{K}{2\pi V_\infty}$, so to hold R constant if $V_2 = 2V$, $\boxed{K_2 = 2K}$ (5)

ii) $L' = \rho V_\infty \Gamma$, so to hold L' constant if $V_2 = 2V$, $\boxed{\Gamma_2 = \frac{1}{2}\Gamma}$ (5)

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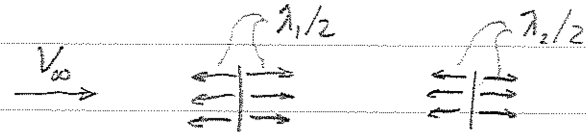
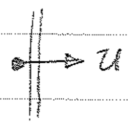
3)

a) Velocity components:



At point A: $u = V_\infty - \frac{\lambda_1}{2} - \frac{\lambda_2}{2} = 0$ (required)

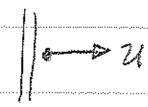


$$1 - \frac{\lambda_1}{2} - \left(\frac{-1}{2}\right) = 0$$

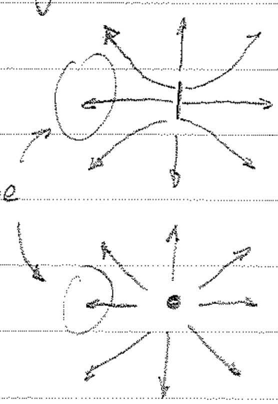
$$\boxed{\lambda_1 = 3}$$ (10)

b) At point B: $u = V_\infty + \frac{\lambda_1}{2} + \frac{\lambda_2}{2}$

$$\boxed{u = 1 + \frac{3}{2} - \frac{1}{2} = 2}$$ (5)



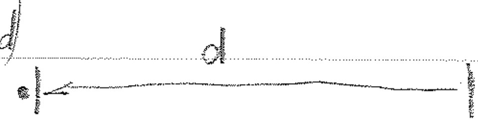
c) At large distances $(d >> l)$, the velocity field of a source panel becomes the same as that of a point source with the same total strength $\Lambda = \lambda \cdot l$



So far away, velocity of source panel is $V_r = \frac{\lambda l}{2\pi r}$

For this case, at point A,

$$u = V_\infty - \frac{\lambda_1}{2} - \frac{\lambda_2 l}{2\pi d} = 0 \text{ (required)}$$



$$1 - \frac{\lambda_1}{2} + \frac{1}{20\pi} = 0$$

$$\boxed{\lambda_1 = 2 + \frac{1}{10\pi}}$$ (10)