Unified Quiz 6S

December 1, 2006

ID number (last four digits) SOLUTION

Quiz Instructions:

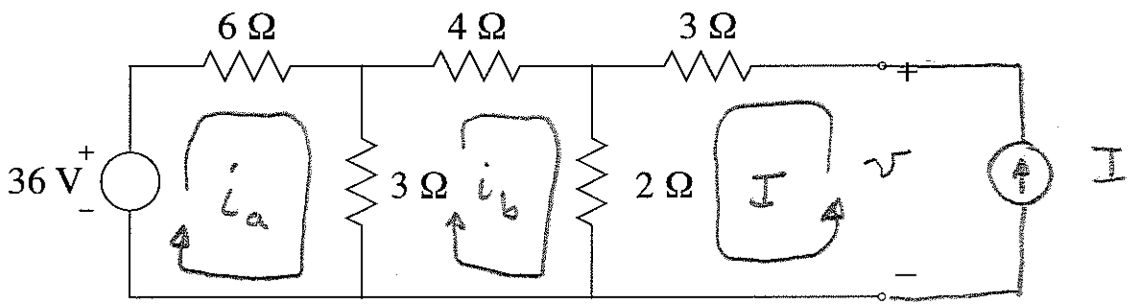
- One 8.5 x 11 crib sheet (both sides) allowed. No books or notes allowed.
- Calculators are not needed, and may not be used.
- Put the last 4 digits of your ID on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work — don't just write equations. Any problem (except multiple choice) without an explanation can receive no better than a "B" grade.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

Exam Scoring:

Problem 1 (40%)	
Problem 2 (40%)	
Problem 3 (20%)	
Total	

Problem 1 (40%) ID number (last four digits) SOLUTION

Find the Thevinin equivalent of the circuit



To find Thevinin equivalent, add test current I and find $v = V_T + R_T I$. To find v, use loop method, with 2 unknown loops, rather than node method, with 3 unknown potentials. Label loop currents as above. Application of KVL around i_a and i_b yield

$$(6+3)i_a - 3i_b = 36$$

$$-3i_a + (3+4+2)i_b = -2I$$

or

$$9i_a - 3i_b = 36$$

$$-3i_a + 9i_b = -2I$$

To find v, need i_b. Solve by Cramer's rule or row reduction:

Problem 1 ID number (last four digits) SOLUTION

9	−3	36	①
1	−1/3	4	①/9 = ③
−3	9	−2I	②
0	8	−2I + 12	② + 3·③ = ④
0	1	$-\frac{I}{4} + 1.5$	④/8 = ⑤

Therefore,

$$i_b = -I/4 + 1.5$$

But $v = 2(I + i_b) + 3I$

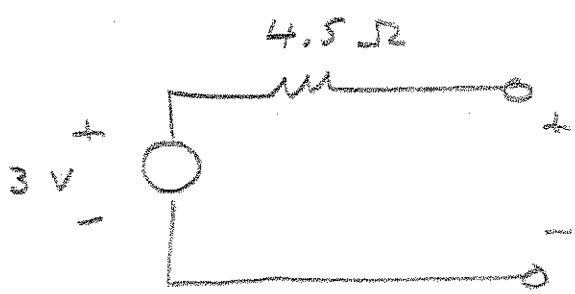
$$= 2\left(\frac{3}{4}I + 1.5\right) + 3I$$

$$= 4.5I + 3$$

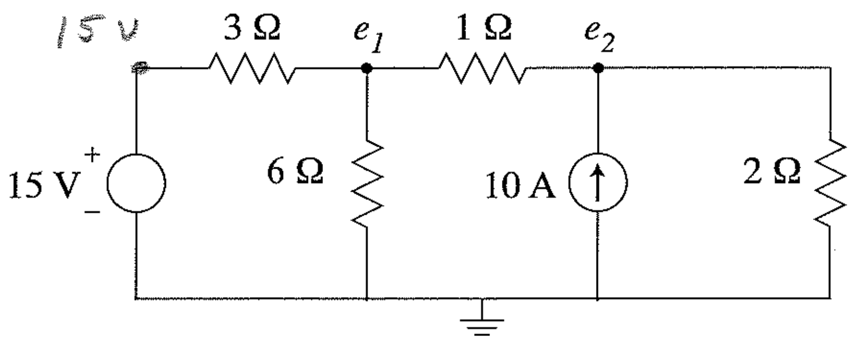
Therefore,

$$\boxed{\begin{array}{l} R_T = 4.5\ \Omega \\ V_T = 3\ V \end{array}}$$

and the Thevinin circuit is



Problem 2 (40%) **ID number (last four digits)** SOLUTION



For the circuit above, calculate the node potentials e_1 and e_2.

Use the node method. The node equations are

$$\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{1}\right) e_1 \quad - \frac{1}{1} e_2 = \frac{1}{3} \cdot 15$$

$$-\frac{1}{1} e_1 \quad + \left(\frac{1}{1} + \frac{1}{2}\right) e_2 = 10$$

Simplifying,

$$1.5 e_1 - e_2 = 5$$

$$-e_2 - 1.5 e_2 = 10$$

Solve by Cramer's rule or <u>row reduction</u>:

Problem 2 ID number (last four digits) SOLUTION

$\frac{3}{2}$	-1	5	①
1	$-2/3$	$10/3$	③ = ① · $\frac{2}{3}$
-1	$+3/2$	10	②
0	$\frac{5}{6}$	$\frac{40}{3}$	④ = ② + ③
0	1	16	⑤ = ④ · $\frac{6}{5}$

⑤ $\Rightarrow e_2 = 16$

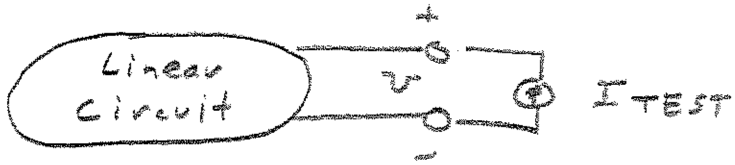
③ $\Rightarrow e_1 = \frac{2}{3} e_2 + \frac{10}{3} = \frac{32}{3} + \frac{10}{3} = \frac{42}{3} = 14$

Therefore,

$$e_1 = 14 \text{ V}$$
$$e_2 = 16 \text{ V}$$

Problem 3 (20%) **ID number (last four digits)** SOLUTION

The Thevinin equivalent resistance, R_T, of a circuit with a single pair of terminals can be found by finding the equivalent resistance of the circuit at the terminals with all the voltage sources replaced with short circuits and all the current sources replaced with open circuits. Explain why.



If the circuit is linear, then the solution for v will be of the form

$$v = a_1 V_1 + a_2 V_2 + \cdots + b_1 I_1 + b_2 I_2 + \cdots + R_T I_{TEST} \qquad (1)$$

where $a_1, a_2, \ldots b_1, b_2, \ldots R_T$ are constants, and $V_1, V_2, \ldots, I_1, I_2, \ldots$ are the strengths of the sources inside the circuit. If all we want to do is find R_T, we can turn all the source strengths to zero, so that (1) becomes

$$v = R_T I_{TEST} \qquad (2)$$

That is, R_T is the apparent resistance at the terminals. But a strength zero current source is an open circuit (since exactly zero current flows across an open circuit) and a strength zero

Problem 3 **ID number (last four digits)** SOLUTION

voltage source is a short circuit (since a short circuit has exactly zero potential across it). So setting the sources to zero is equivalent to shorting the voltage sources and opening the current sources.