UE Fluids Quiz 1 Solution Fall '06

1. a)

Parameters		Units
frequency	F	$1/t$
freestream vel.	V	l/t
diameter	d	l
density	ρ	m/l^3
viscosity	μ	m/lt

10 pts, correct list

Note: This is a low-speed flow, so speed of sound a, and hence the Mach Number V/a will not be important.

b) $N = 5$ $K = 3$ (m, l, t) $\Rightarrow$ $N-K = 5-3 = 2$ Pi groups.

$$\Pi_1 = \frac{Fd}{V} \quad , \quad \Pi_2 = \frac{\rho V d}{\mu} = Re$$

many alternatives possible, eg $\frac{\rho F d^2}{\mu}$, etc.

$$\Pi_1 = \bar{f}(\Pi_2) \quad \text{or} \quad \frac{Fd}{V} = \bar{f}(Re)$$

5 pts

c) $V' = 2V$, $d' = \frac{1}{2} d$, so $Re' = \frac{\rho V' d'}{\mu} = \frac{\rho (2V)(\frac{1}{2}d)}{\mu} = \frac{\rho V d}{\mu} = Re$

$$\frac{F' d'}{V'} = \bar{f}(Re') = \bar{f}(Re) = \frac{Fd}{V}$$

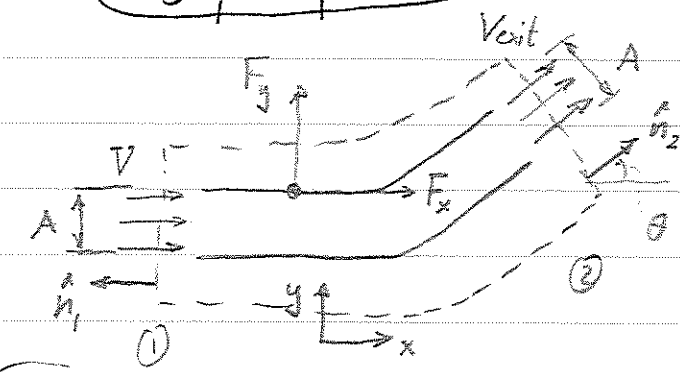
since $\bar{f}$ applies to any such flow, and $Re' = Re$, so we have dynamic similarity

$$\Rightarrow \boxed{F' = \frac{Fd}{V} \cdot \frac{V'}{d'} = F \frac{d}{V} \frac{2V'}{\frac{1}{2}d'} = 4F}$$

10 pts, including reason

5 pts for C.V.

2a) It's easiest to make C.V. enclose entire device as shown



Mass Eqn: $\oint \rho \vec{V} \cdot \hat{n}\, dA = 0$

$-\rho V A + \rho V_{exit} A = 0 \Rightarrow \boxed{V_{exit} = V}$ 10 pts

Note: $\hat{n}_2 = \underbrace{\cos\theta}_{n_x}\, \hat{i} + \underbrace{\sin\theta}_{n_y}\, \hat{j}$

b) x-Momentum Eqn $\left\{ \oint \rho \vec{V} \cdot \hat{n}\, \vec{V}\, dA + \oint p \hat{n}\, dA + \overbrace{F_x \hat{i} + F_y \hat{j}}^{\vec{R}} = 0 \right\} \cdot \hat{i}$

$-\rho V A (V) + \rho V_{exit} A (V_{exit} \cos\theta) + 0 + F_x = 0$ since $\oint p_{atm} \hat{n}\, dA = 0$ (constant p)

$\rho V^2 A (-1 + \cos\theta) + F_x = 0$

$\boxed{F_x = \rho V^2 A (1 - \cos\theta)}$ 10 pts

c) y-Momentum Eqn $\left\{ \oint \rho \vec{V} \cdot \hat{n}\, \vec{V}\, dA + \oint p \hat{n}\, dA + F_x \hat{i} + F_y \hat{j} = 0 \right\} \cdot \hat{j}$

$-\rho V A (0) + \rho V_{exit} A (V_{exit} \sin\theta) + 0 + F_y = 0$

10 pts $\boxed{F_y = -\rho V^2 A \sin\theta}$ F_y pushes <u>down</u> 5 pts

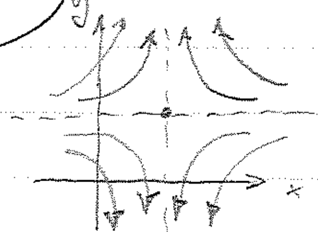
d) $F_x = 1000\ \text{kg/m}^3 \cdot (30\ \text{m/s})^2 \cdot 0.01\ \text{m}^2 \cdot \overbrace{(1 - \cos 60°)}^{0.5} = 4500\ \text{N}$ 5 pts

3a) Method 1): Integrate equation of stream-line : $\frac{dy}{dx} = \frac{v}{u}$

$$\frac{dy}{dx} = \frac{y-1}{1-x} \rightarrow \frac{dy}{y-1} = -\frac{dx}{x-1} \rightarrow \ln(y-1) = -\ln(x-1) + \not{c}$$

$$y - 1 = \frac{\not{c}}{x-1}$$

5 pts



10 pts $\boxed{(x-1)(y-1) = \not{c}}$ hyperbolas centered on $(1,1)$

Method 2) Find $\psi(x,y)$ if possible. Check : $\nabla \cdot \vec{V} \overset{?}{=} 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \overset{?}{=} 0$$

$$-1 + 1 \overset{\checkmark}{=} 0 \quad \text{OK}, \ \psi(x,y) \text{ exists.}$$

$$u = \frac{\partial \psi}{\partial y} = 1 - x \Rightarrow \psi = (1-x)y + f(x)$$

$$-v = \frac{\partial \psi}{\partial x} = 1 - y \Rightarrow \psi = (1-y)x + g(y)$$

must have $f(x) = x$, $g(y) = y$

$$\therefore \psi(x,y) = -xy + x + y$$

plot $\psi(x,y) = \not{c}$ for different $\not{c}$'s

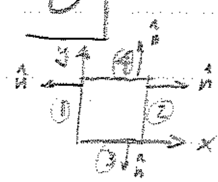
b) $\dot{\mathcal{V}}' \equiv \oint \vec{V} \cdot \hat{n}\, dA$

1) Easiest to apply Gauss's Theorem : $\oint \vec{V} \cdot \hat{n}\, dA = \iint \nabla \cdot \vec{V}\, dx dy$

10 pts

In this case $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so $\boxed{\oint \vec{V} \cdot \hat{n}\, dA = \iint 0\, dx dy = 0}$

2) Could also explicitly integrate on each side :



①: $\int \vec{V} \cdot \hat{n}\, dA = \int_0^1 -u\, dy = \int_0^1 (x-1)\, dy = x - 1 = -1$ since $x = 0$ on ①

②: $\int \vec{V} \cdot \hat{n}\, dA = \int_0^1 u\, dy = \int_0^1 (1-x)\, dy = 1 - x = 0$ since $x = 1$ on ②

③: $\int \vec{V} \cdot \hat{n}\, dA = \int_0^1 -v\, dx = \int_0^1 (1-y)\, dx = 1 - y = 1$ since $y = 0$ on ③

+ ④: $\int \vec{V} \cdot \hat{n}\, dA = \int_0^1 v\, dx = \int_0^1 (y-1)\, dx = y - 1 = 0$ since $y = 1$ on ④

sum : $\boxed{\oint \vec{V} \cdot \hat{n}\, dA = 0}$

c) This flow satisfies mass conservation, since $\nabla \cdot (\rho \vec{V}) = \rho \nabla \cdot \vec{V} = \rho \cdot 0 \overset{\checkmark}{=} 0$

5 pts