F1a. Before touchdown:

After touchdown:

Airfoil sheds a vortex which contains all the airfoil's initial circulation. Airfoil is left with zero circulation.

F1b. Since initial circulation is zero, must have \( \Gamma_v = -\Gamma \)

Velocity seen by airfoil:

\[
\frac{w}{V_\infty} = \frac{\Gamma}{2\pi d}, \quad |\vec{V}|^2 = V_\infty^2 + \left(\frac{\Gamma}{2\pi d}\right)^2 \approx V_\infty^2 \quad \text{if} \quad w \ll V_\infty
\]

Net lift force/span is perpendicular to apparent velocity:

\[
F' = \rho |\vec{V}| \Gamma \times \vec{V}_\infty \Gamma
\]

Take components \( \perp \) and \( \parallel \) to \( \vec{V}_\infty \)

\[
L' = F' \frac{V_\infty}{|\vec{V}|} \perp \vec{V}_\infty \rightarrow C_L = \frac{L'}{\frac{1}{2} \rho V_\infty^2 c} = \frac{2\Gamma}{c V_\infty}
\]

\[
D' = F' \frac{w}{|\vec{V}|} \parallel \vec{V}_\infty \rightarrow C_d = \frac{D'}{\frac{1}{2} \rho V_\infty^3} = \frac{2\Gamma w}{c V_\infty^2 V_\infty} = C_L \frac{w}{V_\infty}
\]

\[
\text{or} \quad C_d = \frac{2\Gamma \Gamma}{c V_\infty 2\pi d V_\infty} = \frac{1}{4\pi d} C_L^2
\]

\( C_d \) decreases as \( \frac{1}{\text{time}} \)
As airfoil thickness goes to zero, the panel results approach the result from thin airfoil theory (assumptions become more valid).

Viscous results approach inviscid results as \( Re \) increases (as viscosity gets smaller).
Unified problems MT-103
Solutions

M1: Minimum mass objective with a strength requirement, need to maximize $\sigma_f/E$

Compute $\sigma_f/E$ for available materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_f/MPa$</th>
<th>$E/Kg/m^3$</th>
<th>$KPa/Kg/m^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>220</td>
<td>79600</td>
<td>27.8</td>
</tr>
<tr>
<td>Al</td>
<td>380</td>
<td>2800</td>
<td>125</td>
</tr>
<tr>
<td>Ti</td>
<td>850</td>
<td>4500</td>
<td>18.8</td>
</tr>
<tr>
<td>CFRP</td>
<td>700</td>
<td>1500</td>
<td>46.7</td>
</tr>
<tr>
<td>Wood</td>
<td>30</td>
<td>6000</td>
<td>50</td>
</tr>
<tr>
<td>SiC</td>
<td>300</td>
<td>3000</td>
<td>100</td>
</tr>
</tbody>
</table>

CFRP works best for bars in truss.

Now decide on truss configuration

Design considerations:
- Minimize number of bars (simplicity is good)
- Aim to have all bars carrying similar loads
Candidates

\[ \begin{align*}
\text{(1)} & \quad \text{\Diagram 1} \\
\text{(2)} & \quad \text{\Diagram 2} \\
\text{(3)} & \quad \text{\Diagram 3} \\
\end{align*} \]

(2) looks simplest - smallest number of bars - probably lowest mass

suspect (1) may have more bars at some other - so more efficient

In any case go with (2)
\[ \sum F_x = 0 : \quad H_A + 0 = 0 \implies H_A = 0 \]

\[ \sum F_y = 0 : \quad V_A + V_B - 10 = 0 \]

\[ \sum M_A = 0 : \quad V_B - 10 \cdot 2 = 0 \implies V_B = +20 \text{ kN} \]

\[ \therefore V_A = -10 \text{ kN} \]

Mo J @ A

\[ \sum F_y = 0 : \quad F_{AC} \sin 45^\circ - 10 = 0 \]

\[ F_{AC} = +10\sqrt{2} \text{ kN} \]

\[ \sum F_x = 0 : \quad F_A \cos 45^\circ + F_{AB} = 0 \]

\[ F_{AB} = -10 \text{ kN} \]
\[ \Sigma F_y = 0 \quad F_{Ec} \sin \theta + 20 = 0 \]

\[ F_{Ec} = -\frac{20 \cdot \sqrt{5}}{2} = -22.4 \text{ kN} \]

Critical bar is BC - carries highest load.

This determines cross-section:

\[ \frac{22.4 \times 10^3}{\sigma_f} = A_{crit} \]

\[ \frac{22.4 \times 10^3}{700 \times 10^6} = 31.9 \times 10^{-6} \text{ m}^2 = 31.9 \text{ mm}^2 \]

Total length of bars:

\[ = L_{AC} + L_{AB} + L_{BC} \]

\[ 2\sqrt{2} + 1 + \sqrt{5} = 6.06 \text{ m} \]

\[ \text{Total mass} = 1500 \times 6.06 \times 31.9 \times 10^{-6} = 0.29 \text{ kg} \]

\[ \frac{1}{2} \text{ a 16! seems right!} \]
Assume weight of cable is constant per horizontal length.

\[ F_{ED} = 10 \text{ N/m} \]

\[ \sum F_x = 0 \quad \Rightarrow \quad -H_A + H_B = 0 \quad \Rightarrow \quad H_A = H_B \]

\[ \sum F_y = 0 \quad \Rightarrow \quad V_A + V_B = 100 \times 10 - 500 = 0 \quad \Rightarrow \quad V_B = 650 \text{ N} \]

\[ \sum M_A = 0 \quad \Rightarrow \quad V_B \times 100 - 500 \times 30 - 1000 \times 50 = 0 \quad \Rightarrow \quad V_A = 1500 - 650 = 850 \text{ N} \]

The structure is apparently statically indeterminate.

Apply method of sections, just to left of C.

\[ H_A = \text{constant} \]

\[ V_C = 10 \text{ N} \]

\[ V_C = 850 \text{ N} \]

\[ \sum M_C = 0 \quad \Rightarrow \quad 850 \times 100 - 30 \times 1000 = 0 \quad \Rightarrow \quad V_C = 10 \text{ N} \to H_C \]
\[ \sum F_x = 0: \quad -H_A + H_c = 0 \quad \text{(tension in cable)} \]

\[ \sum F_y = 0 \quad 850 - 10 \times 30 + V_c = 0 \]

\[ V_c = -550 \text{ N} \]

\[ \sum M_A = 0 \quad -38 \times 10 \times 15 + V_c \cdot 30 + H_c \cdot 10 \cdot 9 = 0 \]

\[ -4500 - 550 \times 30 = H_c \]

\[ H_c = 1100 \text{ KN} \]

At center of cable, consider RHS

\[ \delta_D \]

\[ \delta_D \]

V_D - 10 \times 50 + 650 = 0

\[ V_D = -150 \text{ N} \]

\[ \sum M_D = 0 \quad 650 \times 50 - 4.2 \times 10^3 \delta_D = 0 \]

\[ \delta_D = \frac{650 \times 50}{4.2 \times 10^3} \]
\[ \sigma = \frac{650 \times 50 - 500 \times 25}{11 \times 10^{-3}} = 18 \text{ m}\epsilon \]

6) Consider only horizontal component of tension in cable (much larger than vertical)

\[ \sigma = \frac{11 \times 10^{-3}}{1000 \times 10^{-6}} = 11.0 \text{ MPa} \]

Young's modulus = 26 GPa

\[ \text{Strain} = \frac{11 \times 10^{-6}}{2 \times 10^9} = 5.5 \times 10^{-9} = 5500 \mu \text{E} \]

Change in length = \[ 5500 \times 10^{-6} \times 100 = 0.55 \text{ m} \]

= 0.55 m \rightarrow This is on the same order as the dip of the cable so it is likely to result in an appreciable change in geometry which would need to be accounted for.

Note: 26 GPa is a low modulus – equivalent to nylon or polyester rope.