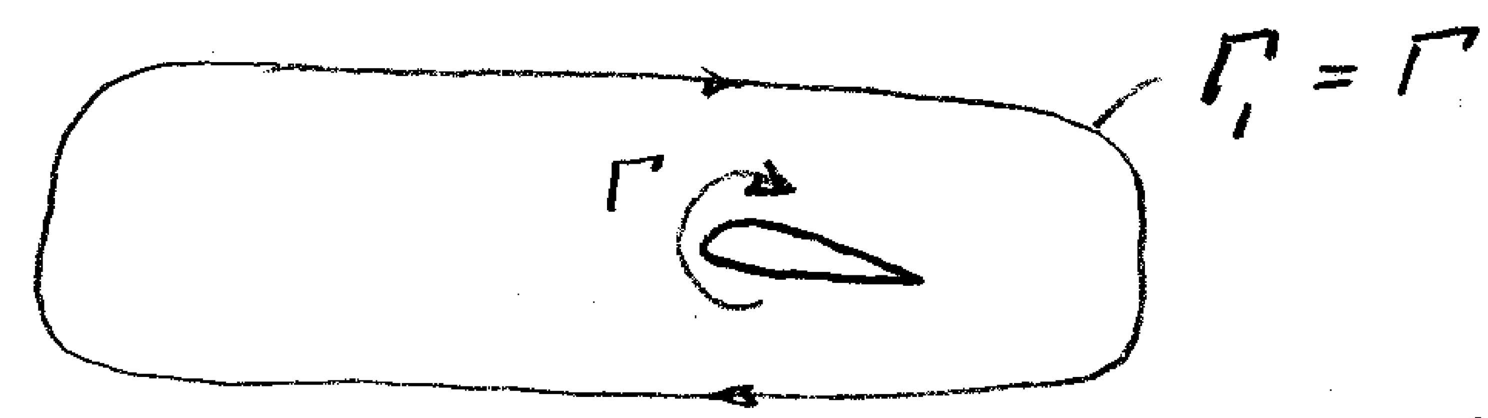
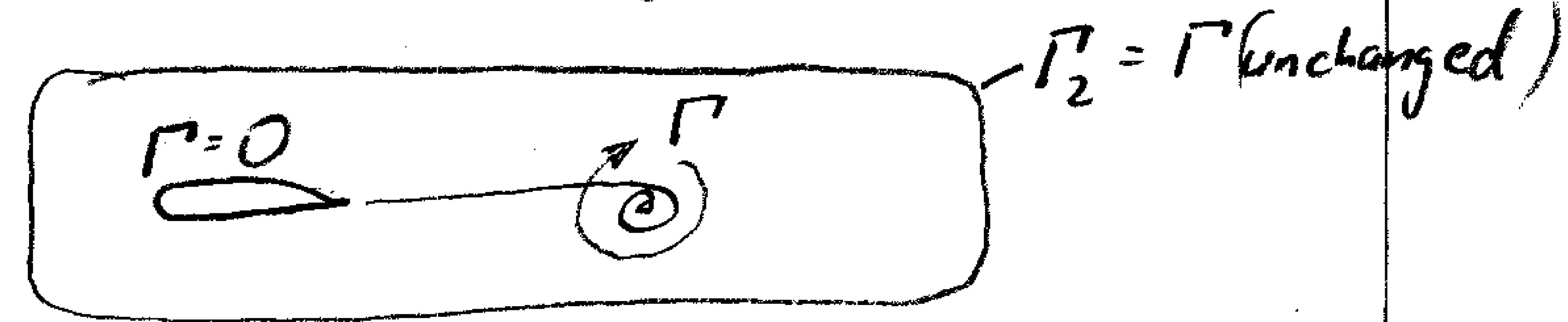
UE Fluids | F1 Solution | Spring '04

F1a. Before touchdown:

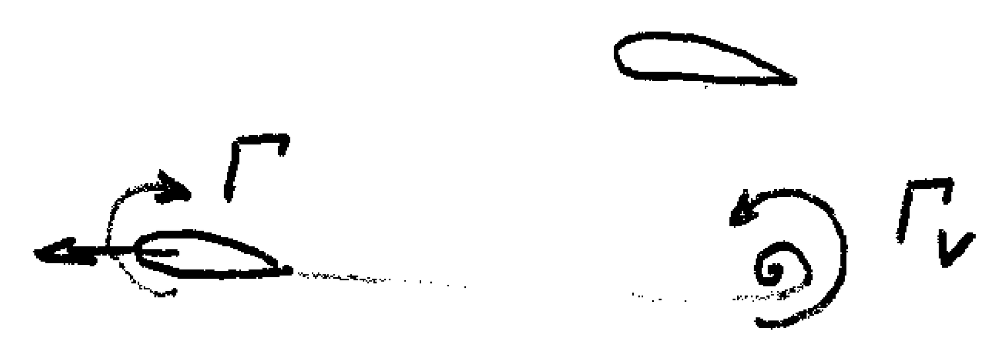


After touchdown:

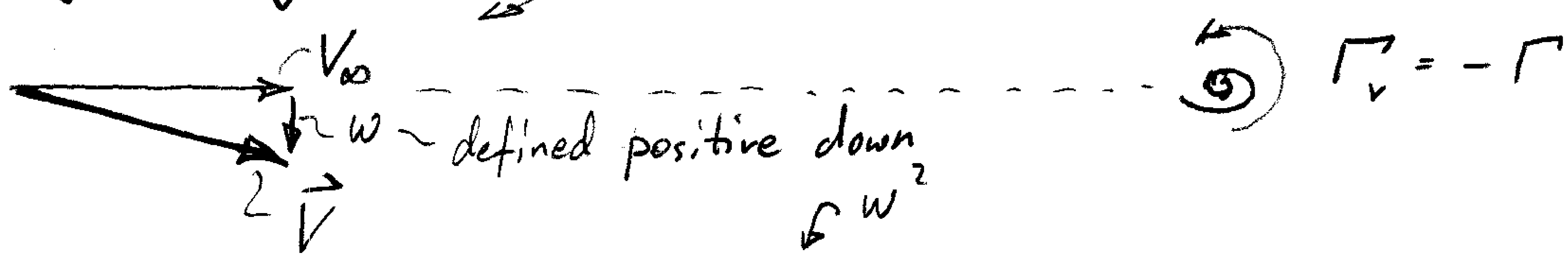


Airfoil sheds a vortex which contains all the airfoil's initial circulation. Airfoil is left with zero circulation.

F1b. Since initial circulation is zero, must have $\Gamma_v = -\Gamma$



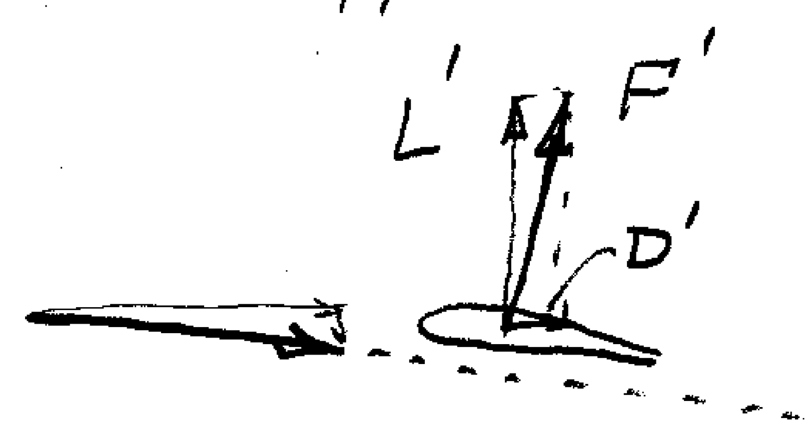
Velocity seen by airfoil



$$w = \frac{\Gamma}{2\pi d} \quad , \quad |\vec{V}|^2 = V_\infty^2 + \left(\frac{\Gamma}{2\pi d}\right)^2 \approx V_\infty^2 \quad \text{if} \quad w \ll V_\infty$$

Net lift force/span is perpendicular to apparent velocity.

$$F' = \rho |\vec{V}| \Gamma \approx \rho V_\infty \Gamma$$



force & velocity triangles are congruent.

Take components $\perp$ and $\parallel$ to $\vec{V}_\infty$

$$L' = F' \frac{V_\infty}{|\vec{V}|} \approx F' = \rho V_\infty \Gamma \longrightarrow c_l = \frac{L'}{\frac{1}{2}\rho V_\infty^2 c} = \frac{2\Gamma}{c V_\infty}$$

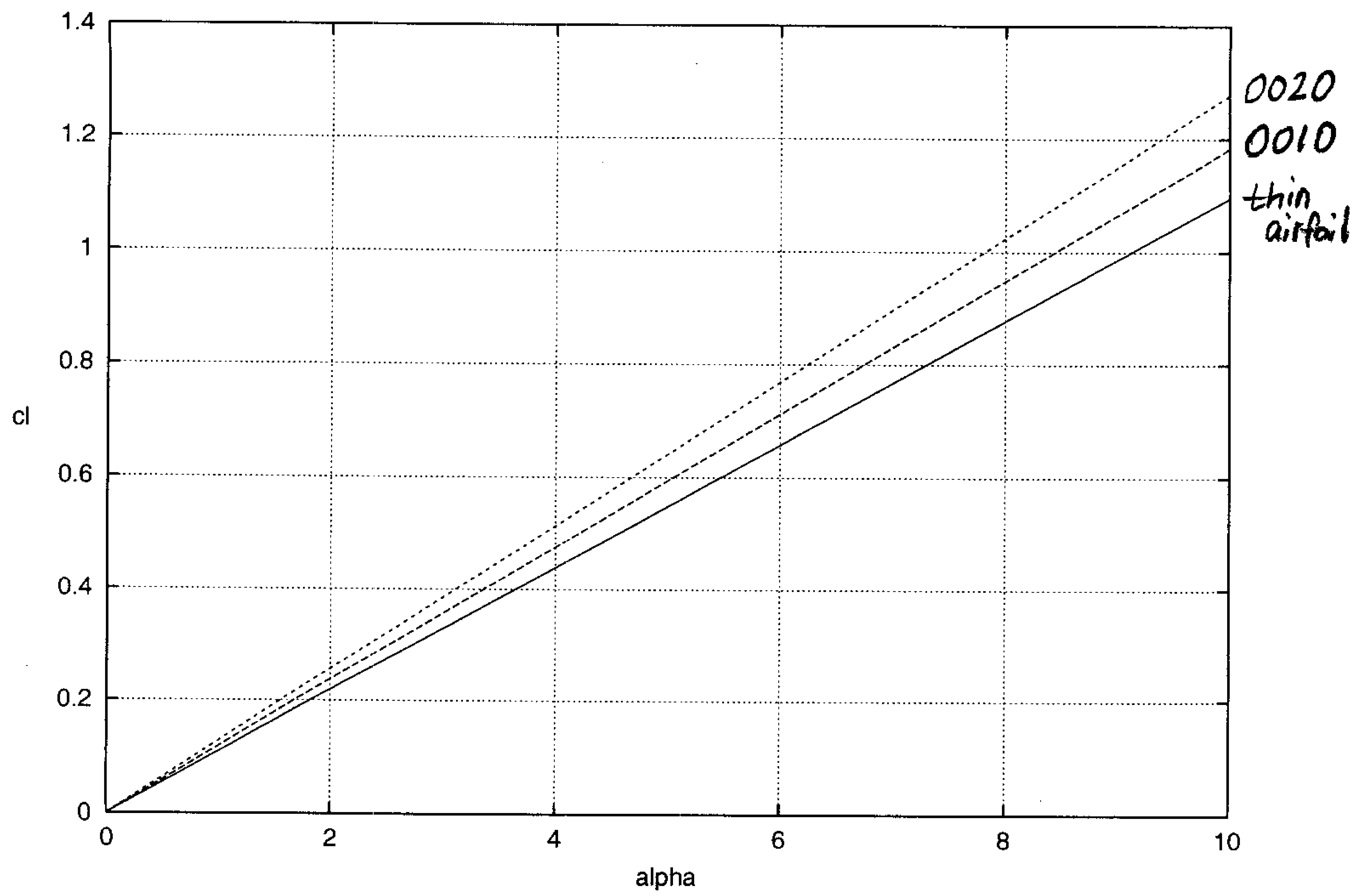
$$D' = F' \frac{w}{|\vec{V}|} \approx F' \frac{w}{V_\infty} = \rho w \Gamma \longrightarrow c_d = \frac{D'}{\frac{1}{2}\rho V_\infty^2 c} = \frac{2\Gamma}{c V_\infty}\frac{w}{V_\infty} = c_l \frac{w}{V_\infty}$$

$$\text{or} \quad c_d = \frac{2\Gamma}{c V_\infty} \cdot \frac{\Gamma}{2\pi d V_\infty} = \frac{1}{4\pi}\frac{c}{d} c_l^2$$

since $w \sim \frac{1}{d} \sim \frac{1}{\text{time}}$

C_d decreases as $\frac{1}{\text{time}}$

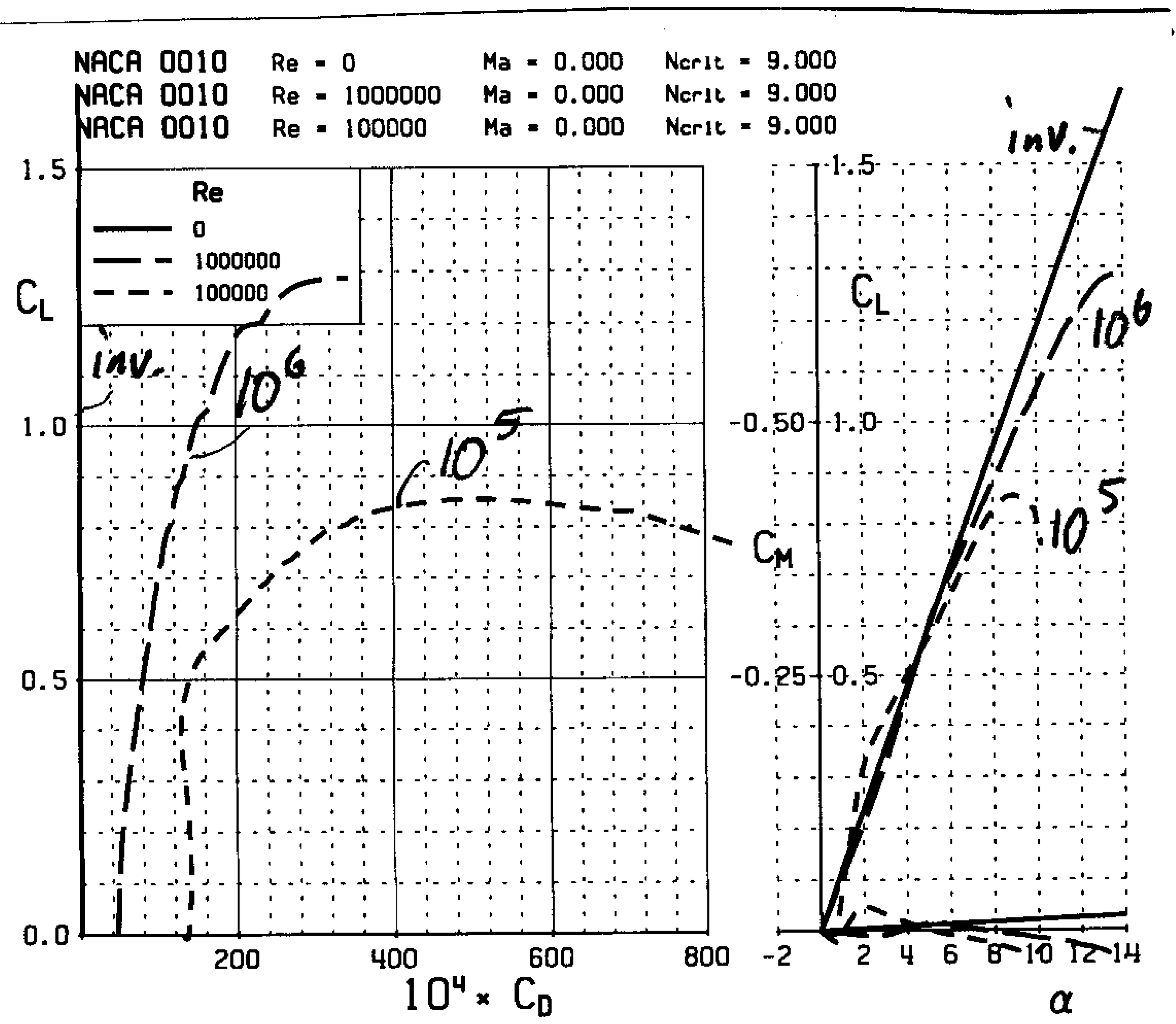
F2a.



$\leftarrow C_l = 2\pi\alpha$
$= 2\pi\alpha^\circ \cdot \frac{\pi}{180}$

As airfoil thickness goes to zero, the panel results approach the result from thin airfoil theory (assumptions become more valid).

F2b.



Viscous results approach inviscid results as Re increases (as viscosity gets smaller)

Unified problems M1 – M3
Solutions

M1 Minimum mass structure with a strength requirement, need to maximize σ_f/ρ

∴ compute σ_f/ρ for available materials

	σ_f/MPa	ρ/kg/m³	kPa/kg/m³
Steel	220	7900	27.8
Al	350	2800	125
Ti	850	4500	188
CFRP	700	1500	467
Wood	30	600	50
SiC	300	3000	100

CFRP works best for bars in truss.

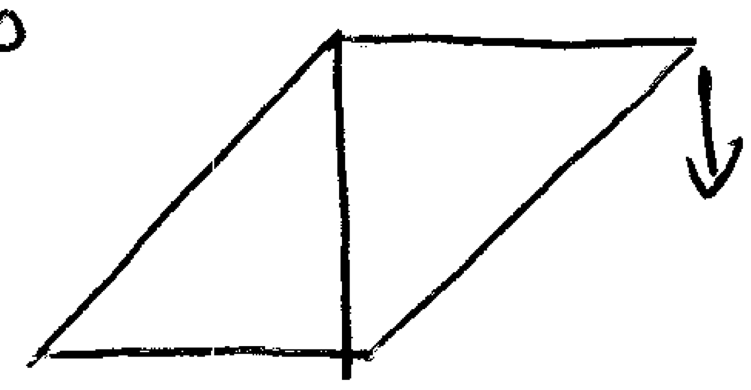
Now decide on truss configuration

Design considerations:
Minimize number of bars (simplicity is good.

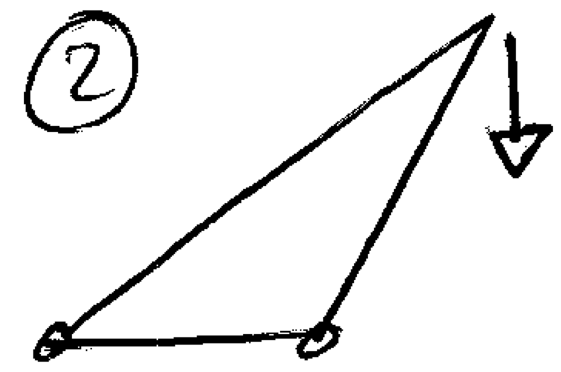
Aim to have all bars carrying similar loads

Candidates

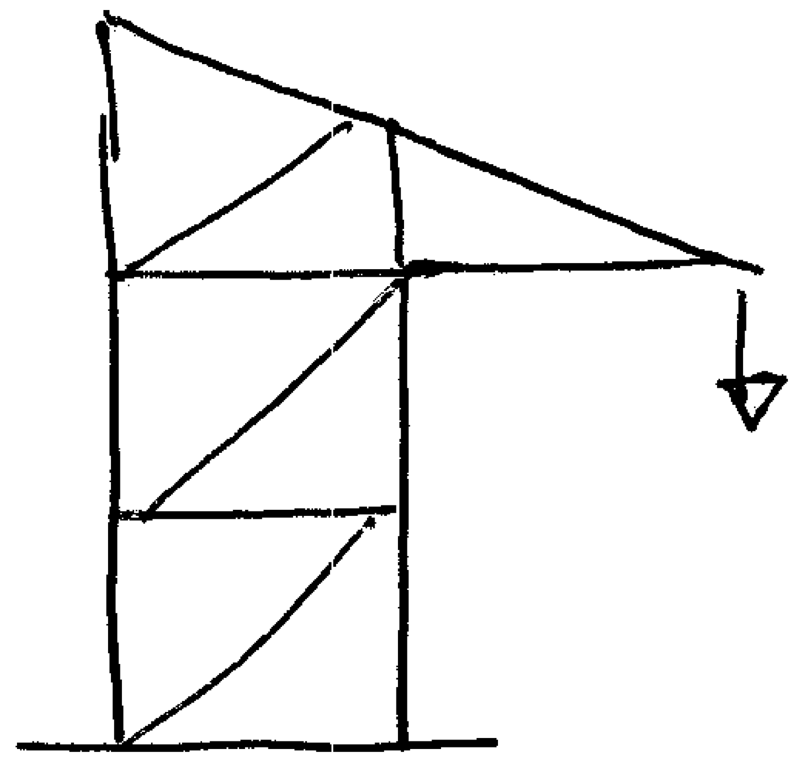
(1)



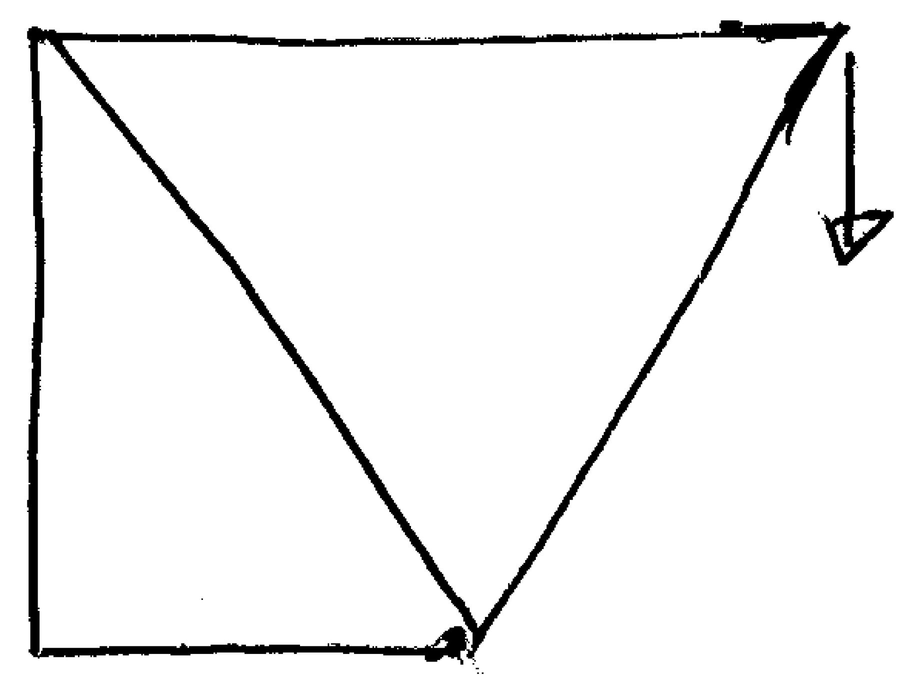
or (2)



or



(3) or

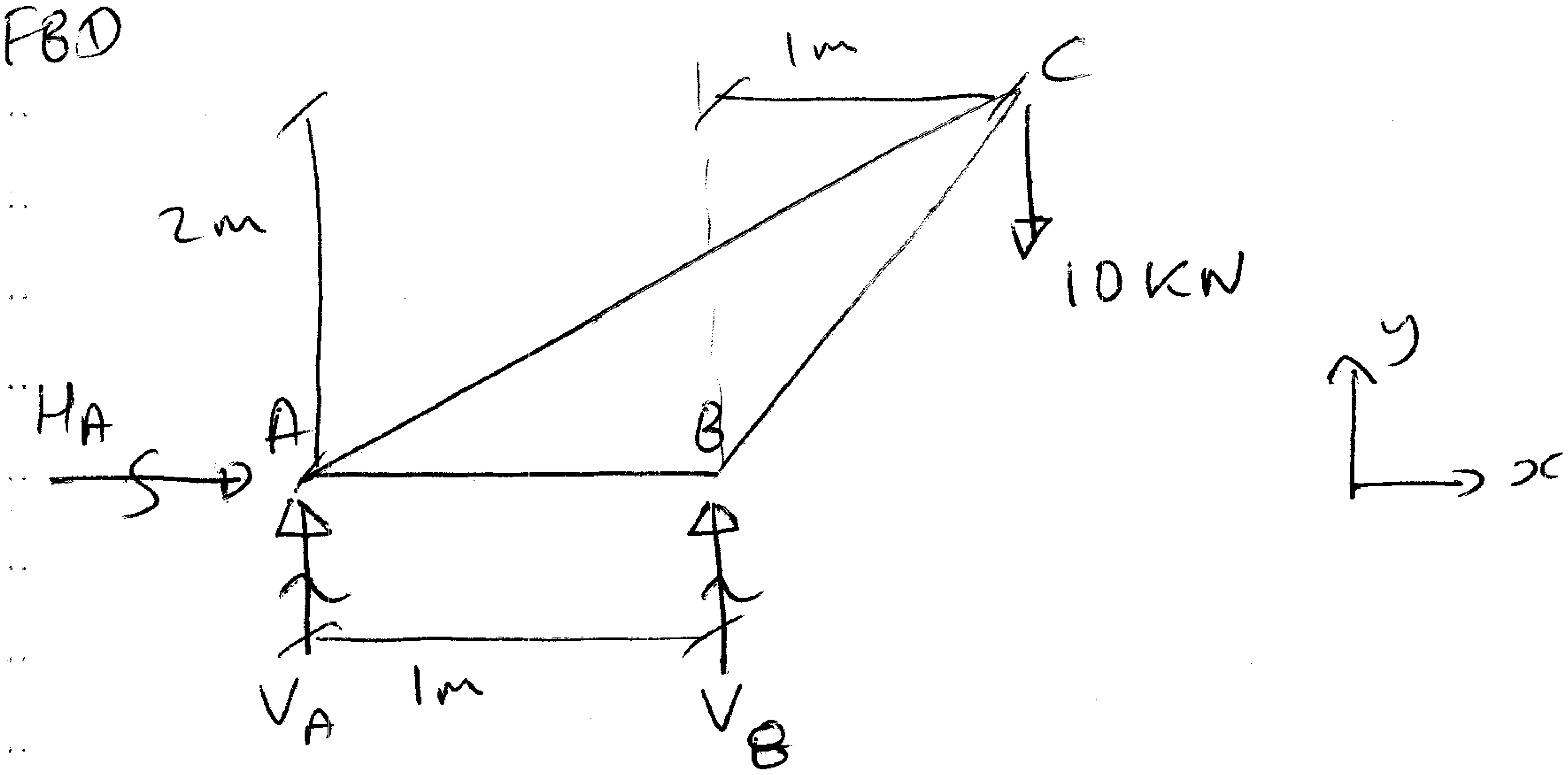


(2) looks simplest – smallest number of bars ∴ probably lowest mass

suspect (1) may have more bars at same stress – ∴ more efficient

In any case go with (2)

FBD



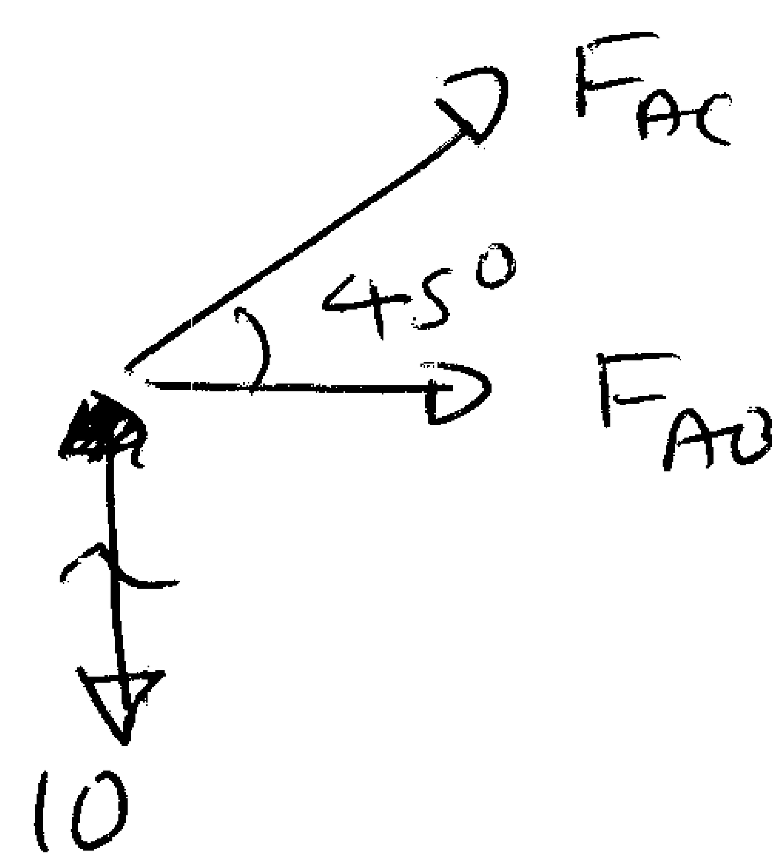
$\rightarrow \sum F_x = 0 : \; H_A + 0 = 0 \;\Rightarrow\; H_A = 0$

$\uparrow \sum F_y = 0 : \; V_A + V_B - 10 = 0$

$\sum \circlearrowright M_A = 0 : \; 1 \cdot V_B - 10 \cdot 2 = 0 : \; V_B = +20 \text{ kN}$

$\therefore \; V_A = -10 \text{ kN}$

MoJ @ A



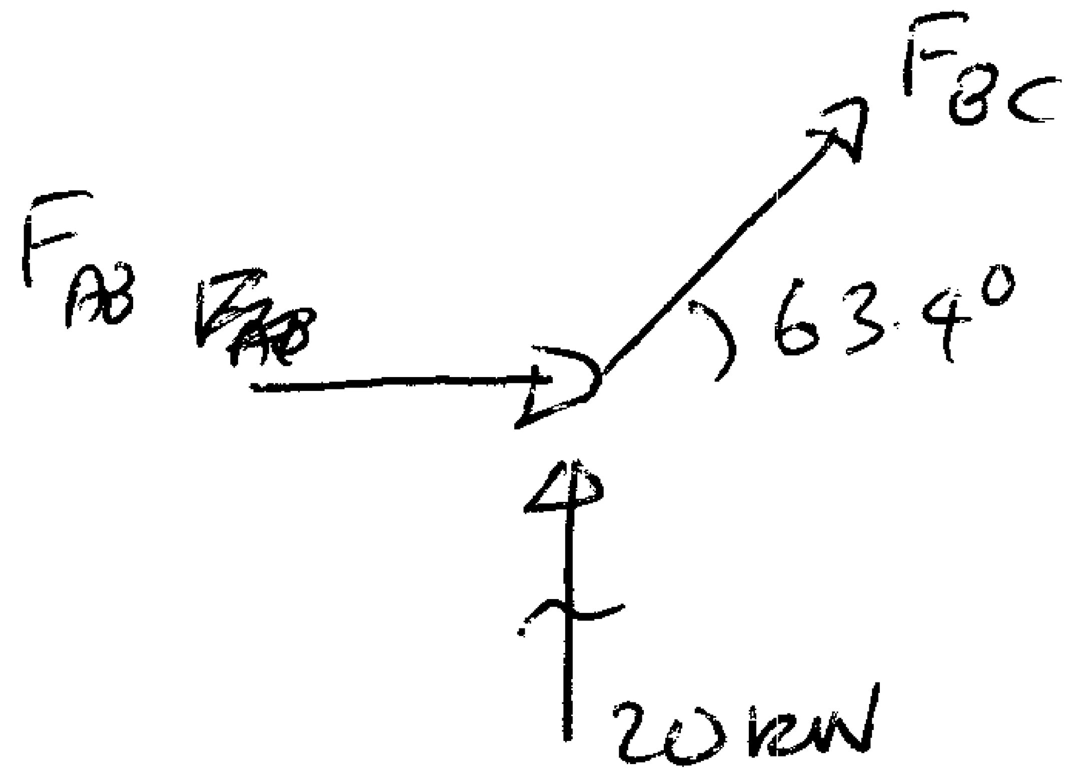
$\sum F_y \uparrow = 0 : \; F_{AC} \sin 45° - 10 = 0$

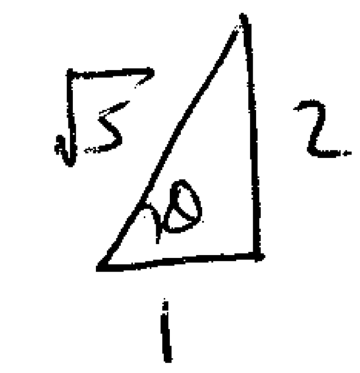
$F_{AC} = +10\sqrt{2} \text{ kN}$

$\sum \overset{\Rightarrow}{F_x} = 0 : \; F_A \cos 45° + F_{AB} = 0$

$F_{AB} = -10 \text{ kN}$

MoJ @ B





$\sin\theta = \frac{2}{\sqrt{5}}$

$\Sigma F_y \uparrow = 0 \qquad F_{BC} \sin\theta + 20 = 0$

$$F_{BC} = \frac{-20 \cdot \sqrt{5}}{2} = -22.4 \text{ kN}$$

Critical bar is BC - carries highest load
∴ this determines cross-section

$$\frac{22.4 \times 10^3}{\sigma_f} = A_{crit}$$

$$\frac{22.4 \times 10^3}{700 \times 10^6} = 31.9 \times 10^{-6} \text{ m}^2 = 31.9 \text{ mm}^2 \Leftarrow$$

Total length of bars

$= L_{AC} + L_{AB} + L_{BC}$

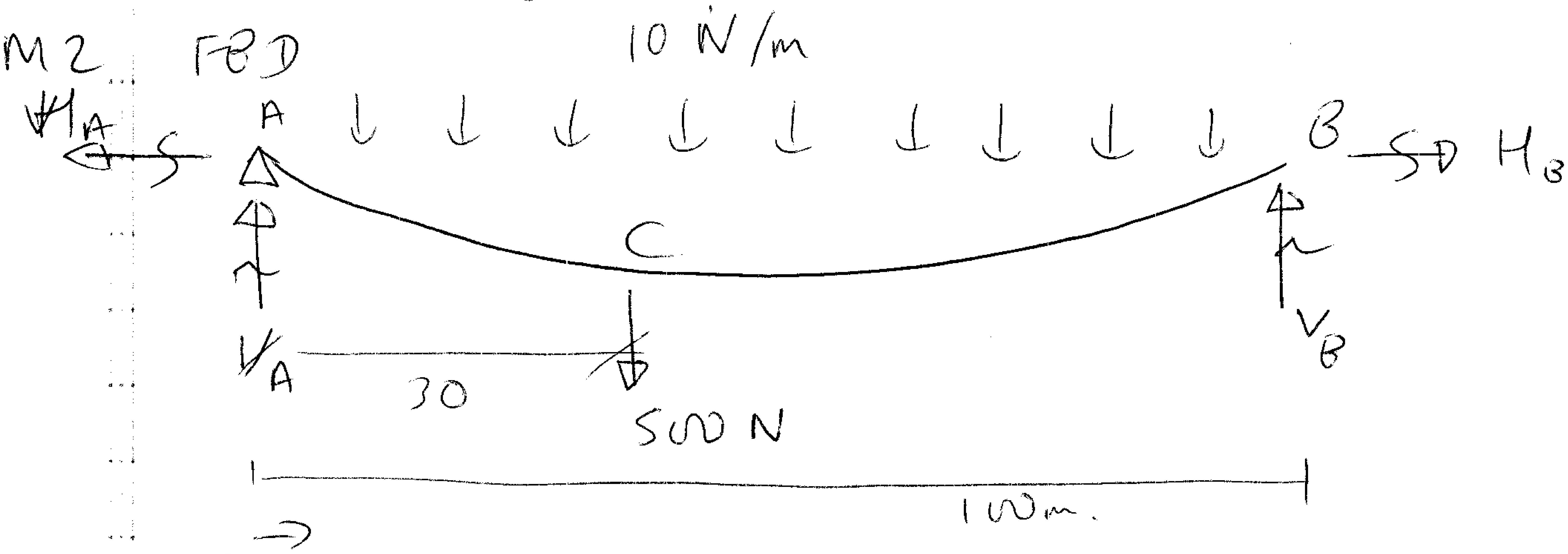
$2\sqrt{2} + 1 + \sqrt{5} = 6.06 \text{ m}$

∴ Total mass $= 1500 \times 6.06 \times 31.9 \times 10^{-6} = 0.29$ kg

ie. $\frac{1}{2}$ a 16! seems light!

Assume weight of cable is constant per horizontal length.

M2 FBD



$$\sum \vec{F_x} = 0 \qquad -H_A + H_B = 0 \qquad H_A = H_B$$

$$\sum F_y \uparrow = 0 \qquad V_A + V_B - \cancel{1}00 \times 10 - 500 = 0$$

$$\underline{-1500}$$

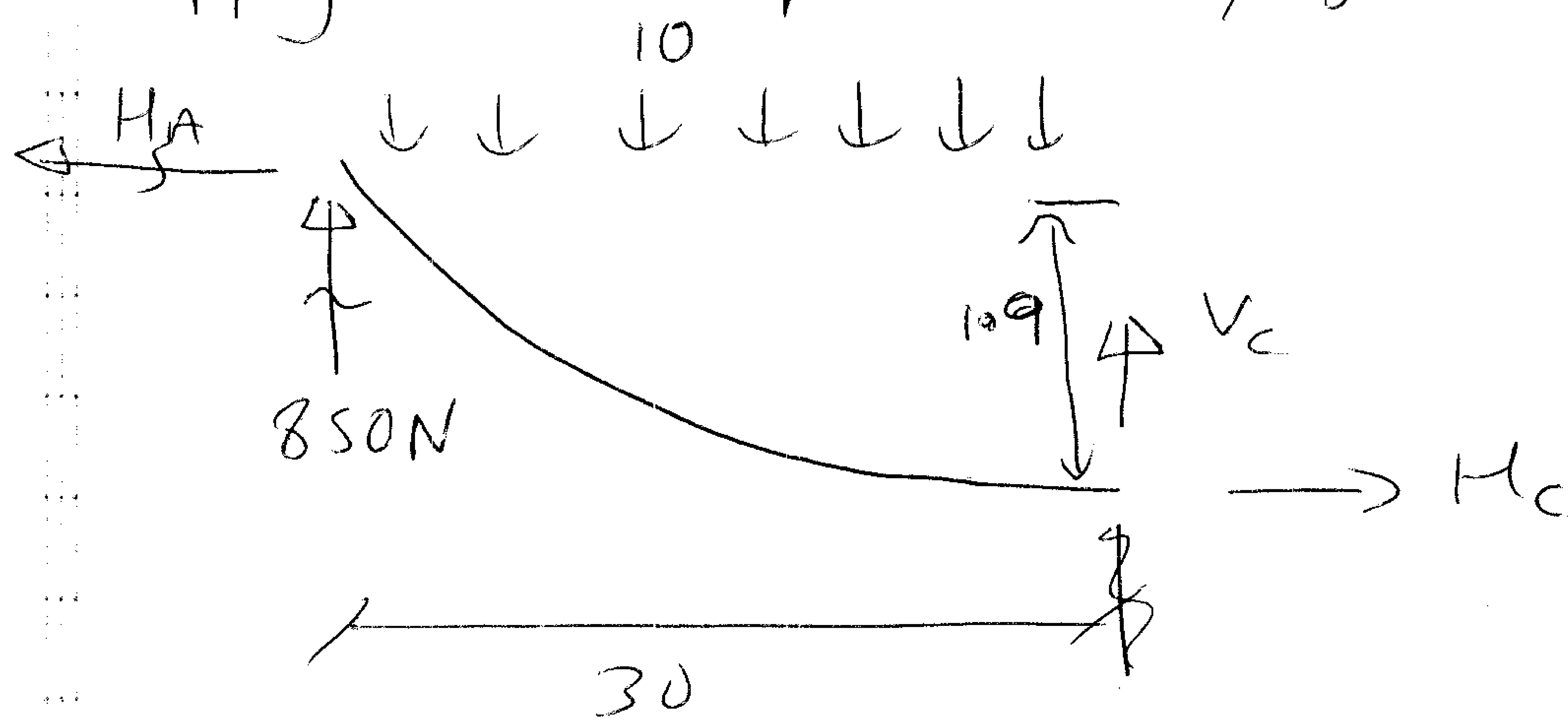
$$\circlearrowleft \sum M_A = 0 \qquad V_B\, 100 - 500 * 30 - 1000 \times 50 = 0$$

$$V_B = \cancel{610}\ 650\,N$$

$$V_A = 1500 - 650 = 850\,N$$

~~Ap~~ Structure is apparently statically indeterminate

Apply method of sections, just to left of C



$\Sigma \vec{F_x} = 0: \; -H_A + H_c = 0$ (tension in cable constant)

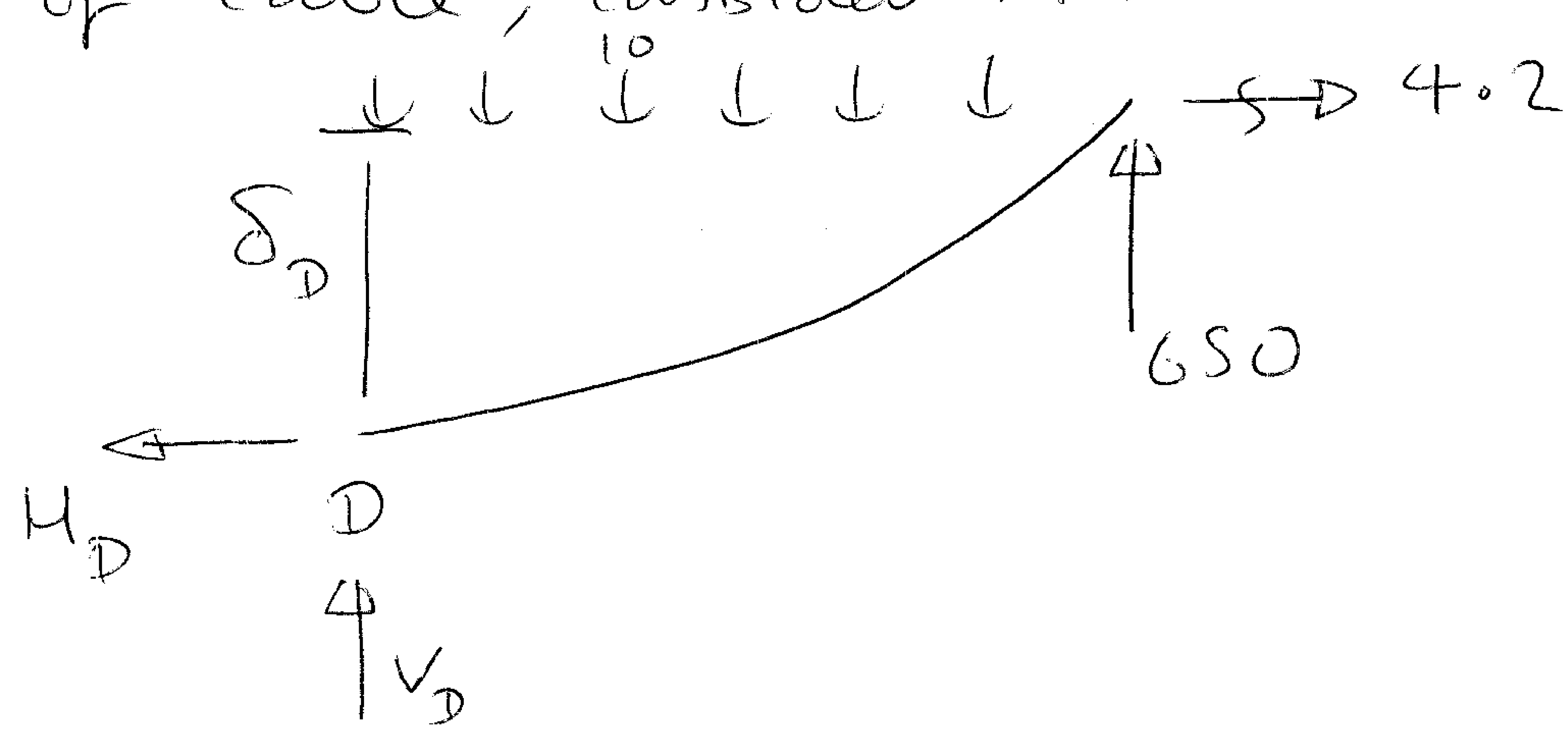
$\Sigma F_y \uparrow = 0 \quad 850 - 10 \times 30 + V_c = 0$

$$V_c = -550 \text{ N}$$

$\Sigma M_A = 0: \; -30 \times 10 \times 15 + V_c \cdot 30 + H_c \cdot \cancel{8}1.9 = 0$

$$\frac{-4500 - 550 \times 30}{-1.9} = H_c = \cancel{11.05}\; 11.0 \text{ kN} \leftarrow$$

At center of cable, consider RHS



$\cancel{\Sigma M_A}\; \Sigma F_y \uparrow = 0. \quad \cancel{V_D - 10 \times 50 + 650 = 0}$

$$\cancel{V_D = -150 \text{ N}}$$

$$\cancel{\Sigma M_D = 0: \; 650 \times 50 - 4.2 \times 10^3 \cdot \delta_D = 0}$$

$$\cancel{\delta_D = \frac{650 \times 50}{4.2 \times 10^3}} =$$

$\Sigma \curvearrowright M_D = 0: \quad 650 \times 50 - 10 \times 50 \times 25 - 11.05 \times 10^3 \delta_D = 0$

$$\delta_D = \frac{650 \times 50 - 500 \times 25}{11.05 \times 10^3} = 1.8\,\text{m} \Leftarrow$$

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b) Consider only horizontal component of tension in cable (much larger than vertical)

$H = 11\,\text{kN} \quad \therefore \quad \sigma = \dfrac{11 \times 10^3}{1000 \times 10^{-6}} = 11.0\,\text{MPa}$

Young's modulus $= 2\,\text{GPa}$

$\therefore$ Strain $= \dfrac{11 \times 10^6}{2 \times 10^9} = 5.5 \times 10^{-3} = 5500\,\mu\varepsilon \Leftarrow$

Change in length $= 5500 \times 10^{-6} \times 100 = 0.55\,\text{m}$

$= 0.55\,\text{m}$ → this is on the same order as the dip of the cable so it is likely to result in an appreciable change in geometry which would need to be accounted for.

<u>Note</u> 2 GPa is a low modulus – equivalent to Nylon or polyester rope.