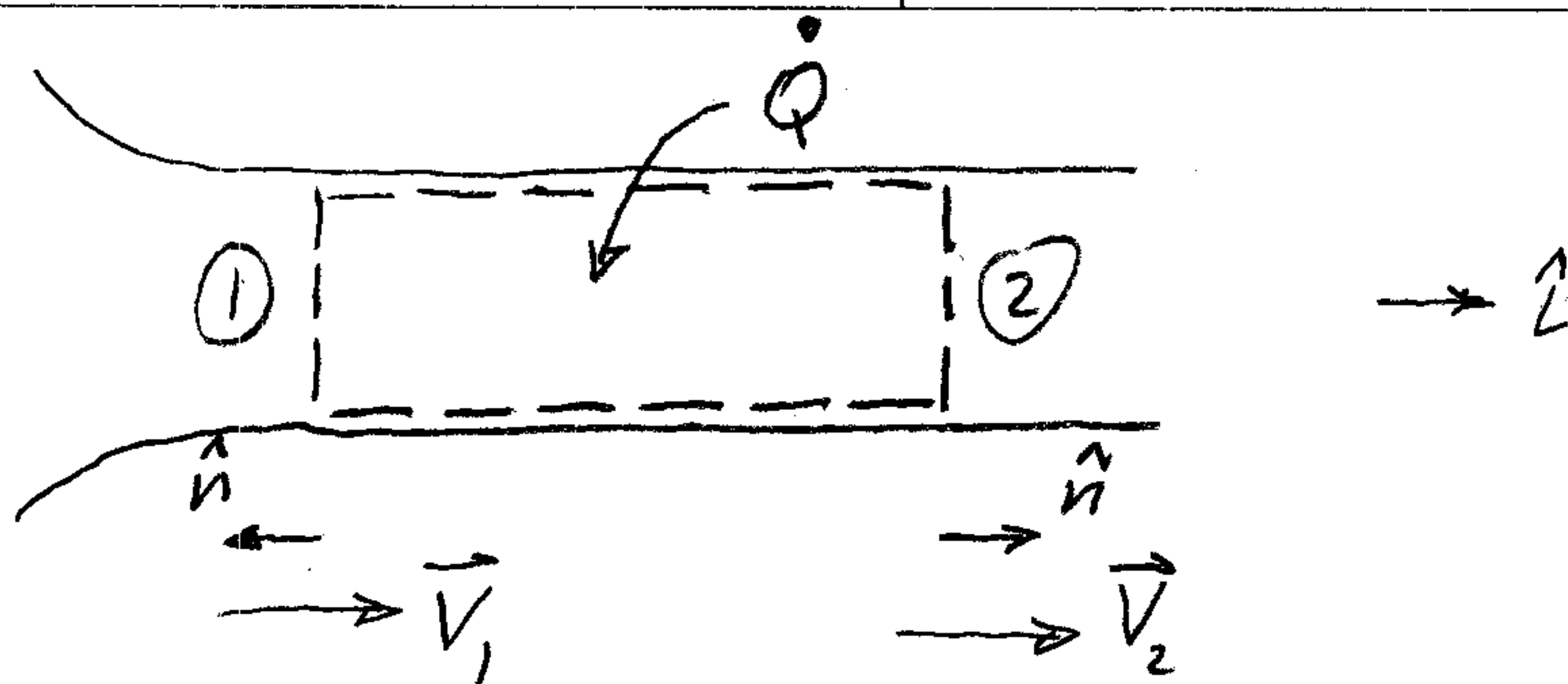


Control Volume



Mass:  $\oint \rho \vec{V} \cdot \hat{n} dA = 0 \Rightarrow -\rho_1 V_1 A + \rho_2 V_2 A = 0$

or  $\boxed{\rho_1 V_1 = \rho_2 V_2} \quad (1)$

Momentum:  $\oint \rho (\vec{V} \cdot \hat{n}) \vec{V} dA + \oint p \hat{n} dA + \vec{R} = 0$  ↙ force on radiator

$\Rightarrow -\rho_1 V_1 A V_1 \hat{z} + \rho_2 V_2 A V_2 \hat{z} - p_1 A \hat{z} + p_2 A \hat{z} + R \hat{z} = 0$

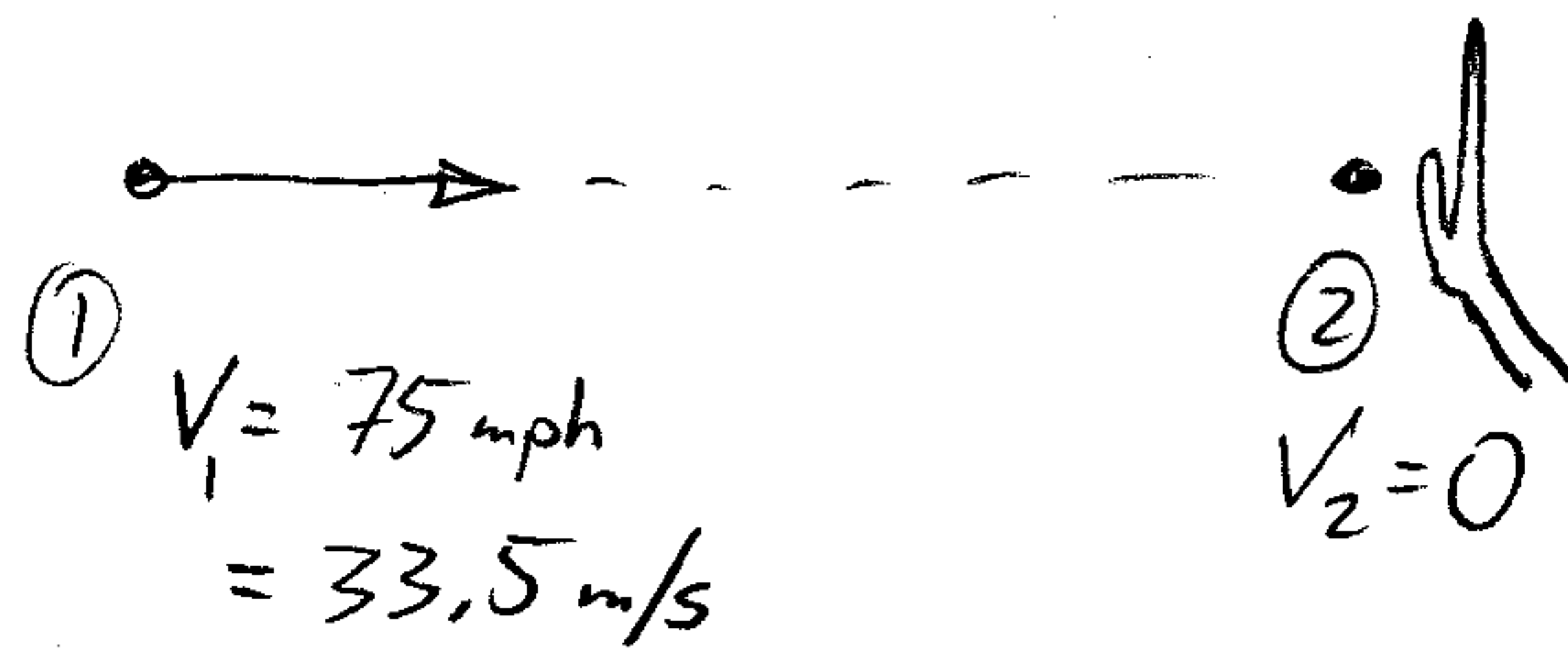
or  $\boxed{\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2 + R}$  ↗ neglect as given (2)

Energy:  $\oint \rho (\vec{V} \cdot \hat{n}) h_0 dA = \iiint \rho \dot{q} dV + \iiint \rho \vec{g} \cdot \vec{V} dV$

$-\rho_1 V_1 A h_{01} + \rho_2 V_2 A h_{02} = \dot{Q}$

or  $\boxed{\rho_1 V_1 h_{01} + \dot{Q}/A = \rho_2 V_2 h_{02}} \quad (3)$

Adiabatic + Reversible process  $\rightarrow$  Isentropic  
(no heat) (frictionless)



$$h_2 = h_{02} = h_{01} = c_p T_1 + \frac{1}{2} V_1^2 = 1004 \text{ J/kg} \cdot \text{K} \cdot 300 \text{ K} + \frac{1}{2} 33.5^2 \text{ m}^2/\text{s}^2$$

$$h_2 = 301761.1 \text{ J/kg}$$

$$T_2 = h_2 / c_p = 300.56 \text{ K}^\circ$$

$$\Delta T = 0.56 \text{ K}^\circ$$

$$P_2 = P_{02} = P_{01} = P_1 \left[ 1 - \frac{V_1^2}{2h_{01}} \right]^{-3.5} = P_1 \cdot 1.00654$$

$$P_2 = 1.00654 \times 10^5 \text{ Pa}$$

$$\Delta P = 654 \text{ Pa} \approx \frac{1}{2} \rho V^2 \text{ (low speed)}$$

OK to use Bernoulli here

$$\rho_2 = \rho_{02} = \rho_{01} = \rho_1 \left[ 1 - \frac{V_1^2}{2h_{01}} \right]^{-2.5} = \rho_1 \cdot 1.00466$$

$$\rho_2 = 1.2056 \text{ kg/m}^3$$

Note: Data as given doesn't exactly satisfy state equation. Some numerical differences will occur if the state equation is used instead of one of the adiabatic or isentropic relations.

$$a) M_{\infty} = \frac{V_{\infty}}{a_{\infty}} = \frac{V_{\infty}}{\sqrt{\gamma RT_{\infty}}}, \text{ but } RT = \frac{P}{\rho}, \text{ so } a_{\infty} = \sqrt{\frac{\gamma P_{\infty}}{\rho_{\infty}}}$$

$$M_{\infty} = V_{\infty} \sqrt{\frac{\rho_{\infty}}{\gamma P_{\infty}}}$$

$$b) P_0 = P_{\infty} \left[ 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right]^{\frac{\gamma}{\gamma-1}} \quad \text{exact}$$

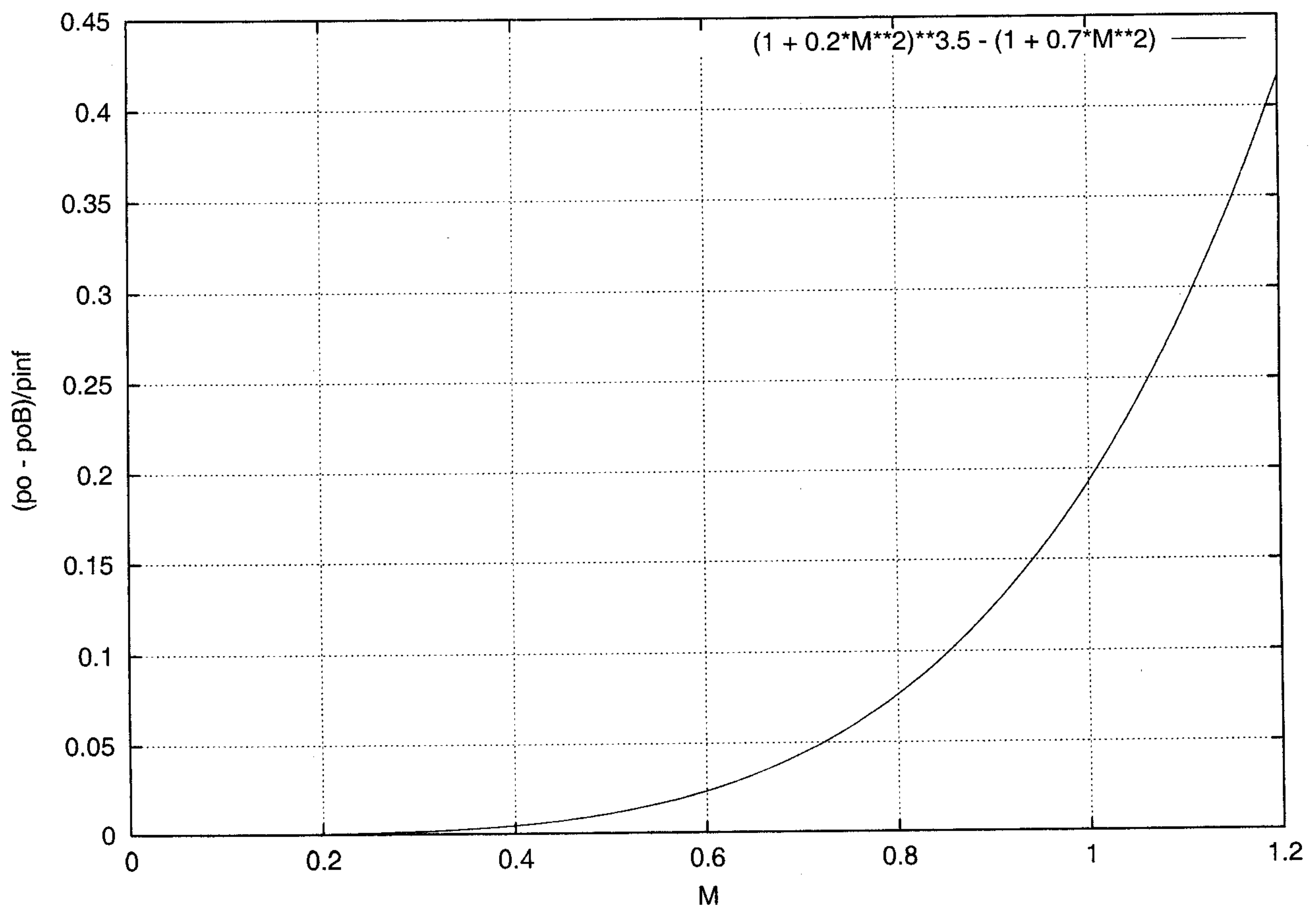
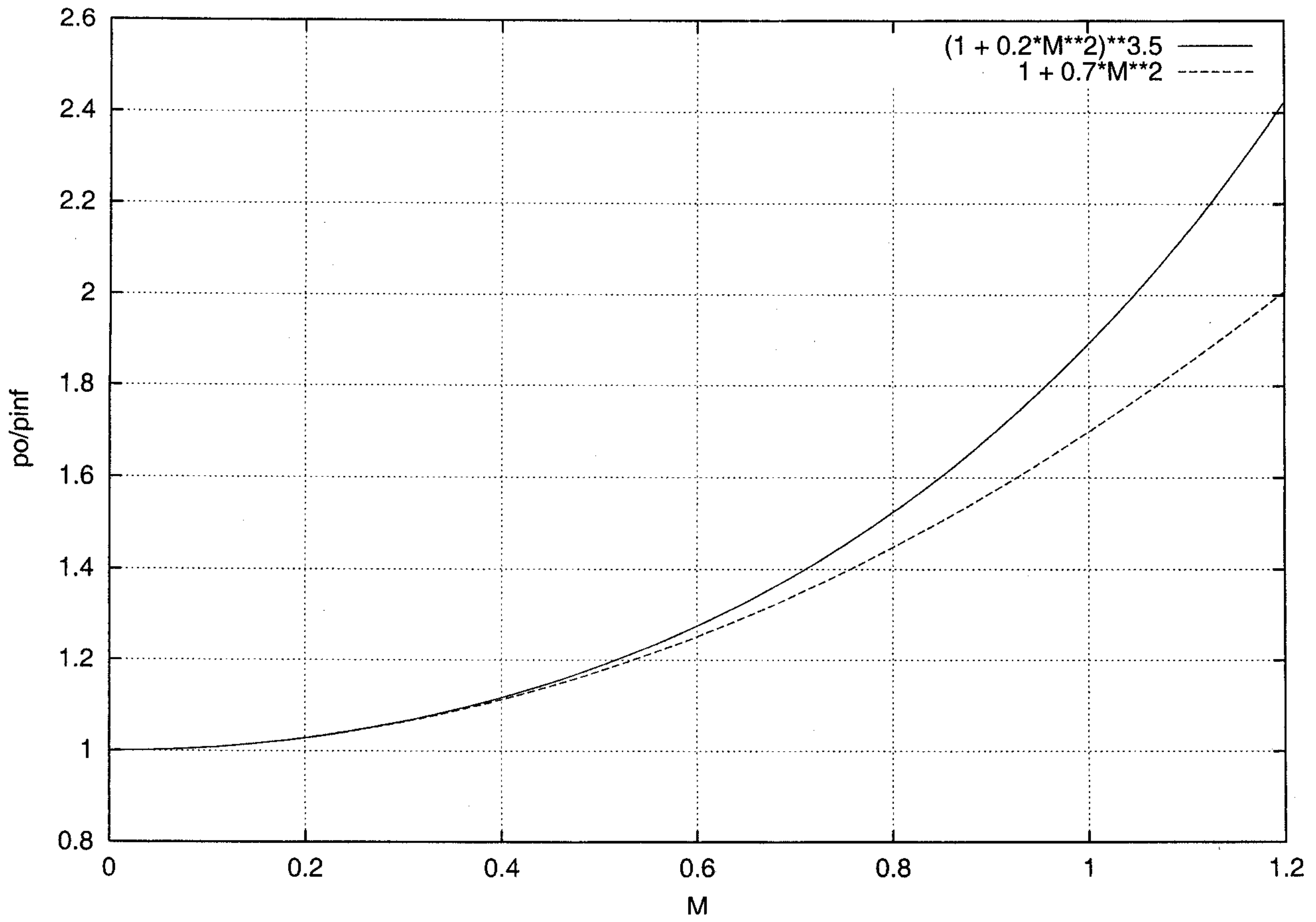
$$P_0 = P_{\infty} + \frac{1}{2} \rho_{\infty} V_{\infty}^2$$

$$= P_{\infty} + \frac{1}{2} \rho_{\infty} M_{\infty}^2 \cdot \frac{\gamma P_{\infty}}{\rho_{\infty}}$$

$$P_0 = P_{\infty} \left[ 1 + \frac{\gamma}{2} M_{\infty}^2 \right] \quad \text{Bernoulli}$$

Plot  $\left( 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right)^{\frac{\gamma}{\gamma-1}}$  and  $1 + \frac{\gamma}{2} M_{\infty}^2$  attached

Plot  $\left( 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right)^{\frac{\gamma}{\gamma-1}} - \left( 1 + \frac{\gamma}{2} M_{\infty}^2 \right)$  attached



For any taper  $r$ :  $c(y) = c_{avg} \frac{2}{1+r} \left[ 1 - (1-r) \frac{2y}{b} \right]$

a) Assuming  $q_{nc}$ :  $q(y) = q_{avg} \frac{2}{1+r} \left[ 1 - (1-r) \frac{2y}{b} \right]$

Total lift on half span:  $F = q_{avg} \cdot \frac{b}{2} = 10 N \rightarrow q_{avg} = \frac{10 N}{1 m} = 10 N/m$

$S(y) = \int_{b/2}^y q(y) dy = q_{avg} \frac{2}{1+r} \left[ y - (1-r) \frac{y^2}{b} \right]_{b/2}^y = q_{avg} \frac{2}{1+r} \left[ y - \frac{b}{2} + \frac{1-r}{b} \left( \frac{b^2}{4} - y^2 \right) \right]$

$M(y) = \int_{b/2}^y S(y) dy = q_{avg} \frac{2}{1+r} \left[ \frac{1}{2} y^2 - \frac{b}{2} y + (1-r) \left( \frac{b^2}{4} y - \frac{1}{3} y^3 \right) \right]_{b/2}^y$

$M(y) = q_{avg} \frac{2}{1+r} \left[ \frac{1}{2} \left( y^2 - \frac{b^2}{4} \right) + \frac{b}{2} \left( \frac{b}{2} - y \right) + \left( \frac{1-r}{b} \right) \left( \frac{b^2}{4} \left( y - \frac{b}{2} \right) + \frac{1}{3} \left( \frac{b^3}{8} - y^3 \right) \right) \right]$

could simplify this I suppose. Plots attached

b)  $M = Ph = Pc \tau \rightarrow P(y) = \frac{M(y)}{c(y) \tau}$ , Plots attached.

c)  $P = A\sigma \rightarrow A_{min}(y) = \frac{P(y)}{\sigma_{max}}$  same plot as  $P(y)$ , aside from scale

Area is roughly parabolic.  $Vol = \int_0^{b/2} A(y) dy \approx \frac{1}{3} A(0) \cdot \frac{b}{2}$

$Vol \approx \frac{1}{3} \frac{P(0)}{\sigma_{max}} \cdot \frac{b}{2} = \frac{b}{6} \frac{1}{\sigma_{max}} \frac{M(0)}{c(0) \tau}$  (one cap for half-wing)

we have  $M(0) = q_{avg} \frac{2}{1+r} \left[ -\frac{b^2}{8} + \frac{b^2}{4} + \frac{1-r}{b} \left( -\frac{b^3}{8} + \frac{b^3}{24} \right) \right] = q_{avg} \frac{2}{1+r} \left[ \frac{b^2}{8} - (1-r) \frac{b^2}{12} \right]$

$c(0) = c_{avg} \frac{2}{1+r}$

$\therefore Vol = \frac{b^3}{6} \frac{1}{\sigma_{max} \tau} \frac{q_{avg}}{c_{avg}} \left( \frac{1}{8} - \frac{1-r}{12} \right) = \begin{cases} 11.9 \times 10^{-6} m^3 = 11.9 cm^3 & (r=1.0) \\ 7.9 \times 10^{-6} m^3 = 7.9 cm^3 & (r=0.5) \end{cases}$

A-cap mass  $m = \rho \cdot Vol = \begin{cases} 6.0 g & (r=1.0) \\ 4.0 g & (r=0.5) \end{cases}$

d)  $I = \frac{1}{2} A h^2 = \frac{1}{2} A c^2 \tau^2 = \frac{1}{2} \frac{M}{c \tau \sigma_{max}} c^2 \tau^2 = \frac{1}{2} \frac{M c \tau}{\sigma_{max}}$

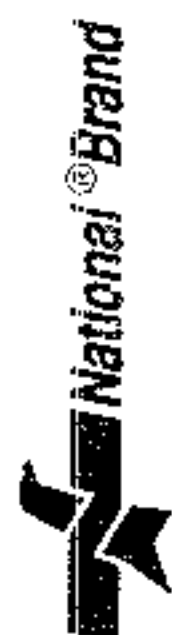
$K = \frac{M}{EI} = \frac{2 \sigma_{max}}{E} \frac{1}{c \tau}$ ,  $K(0) = 2 \frac{7 MPa}{1.36 GPa} \cdot \frac{1}{0.08} \frac{1}{c(0)} = 0.129 \cdot \frac{1+r}{2 c_{avg}} = \begin{cases} 0.52/m & r=1.0 \\ 0.39/m & r=0.5 \end{cases}$

$\delta = \frac{1}{2} K (b/2)^2 = \begin{cases} 0.258 m & r=1.0 \\ 0.193 m & r=0.5 \end{cases}$

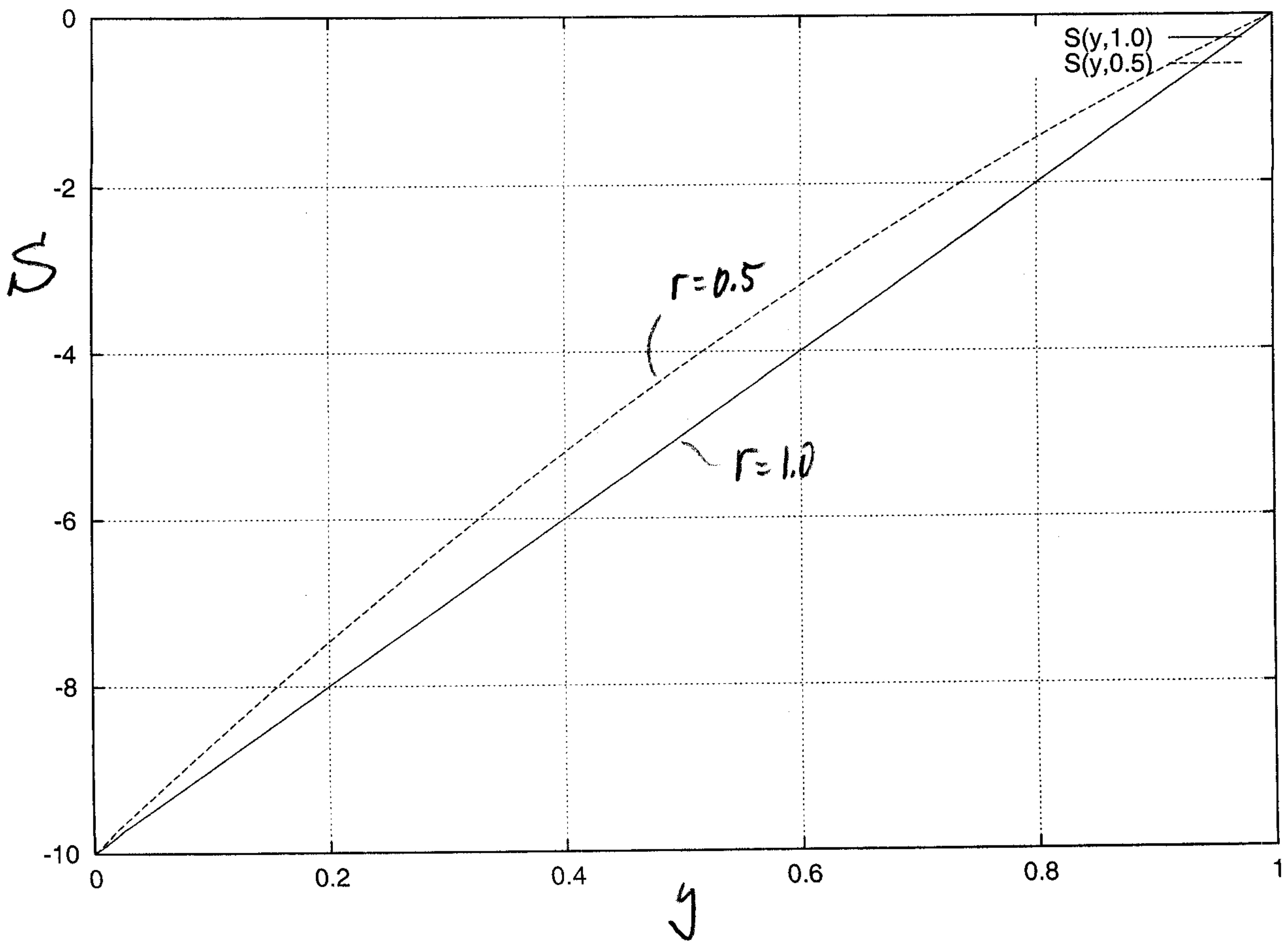
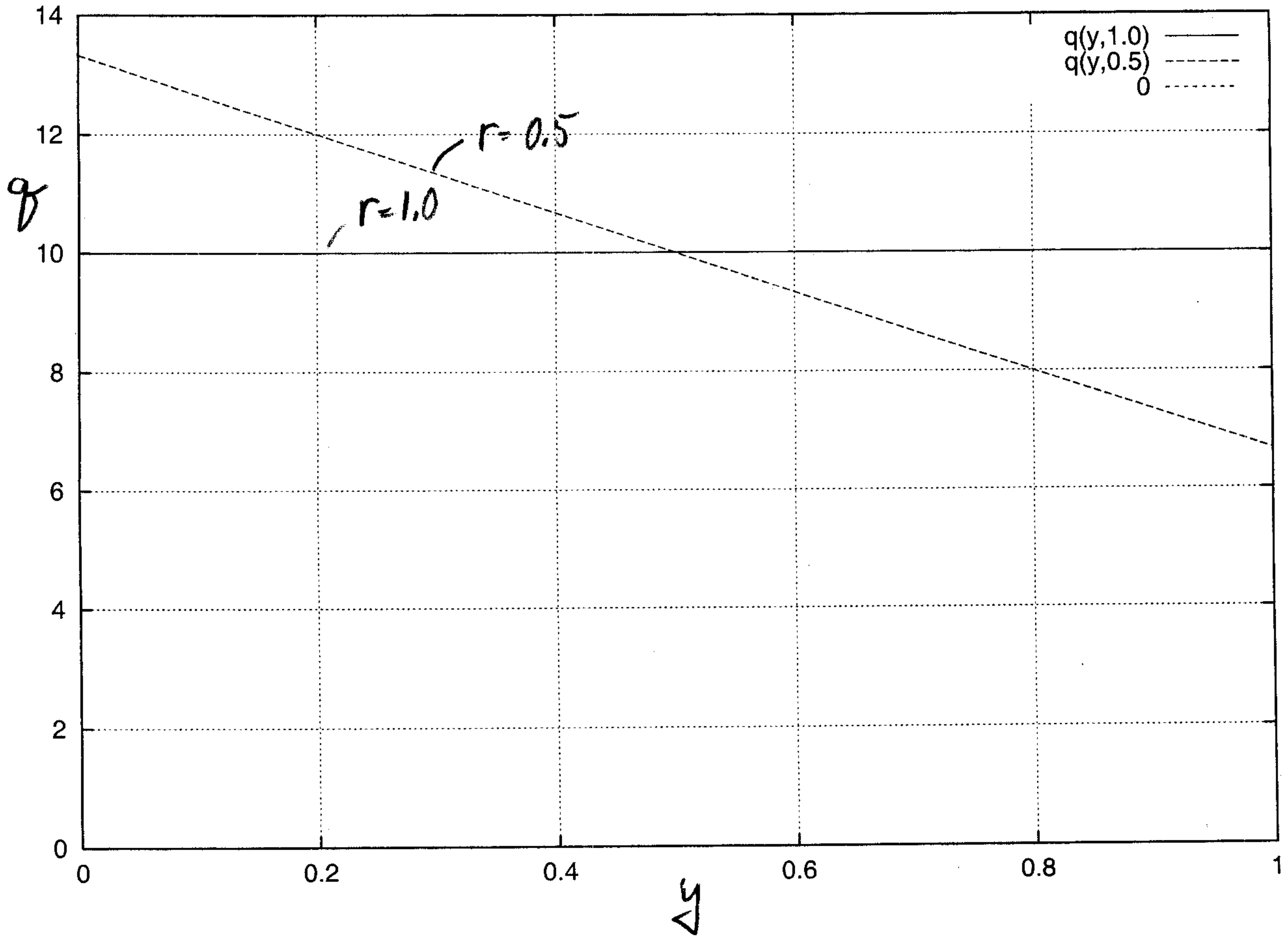
The tapered wing seems better in all respects.

- better  $e$  (lower  $C_{Dc}$ )
  - lighter spar
  - smaller  $\delta$
- Also, balsa caps are very light. Look attractive.

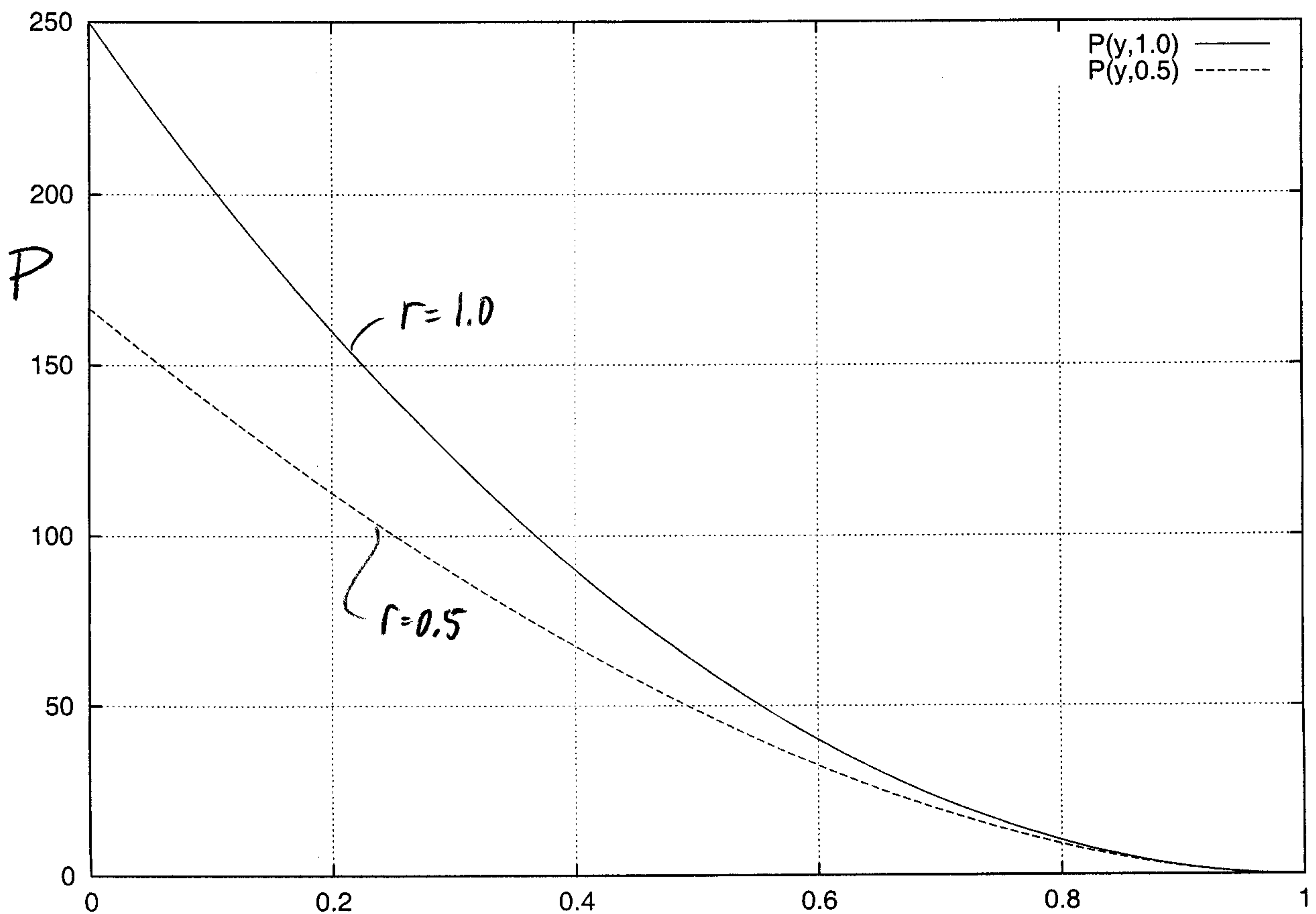
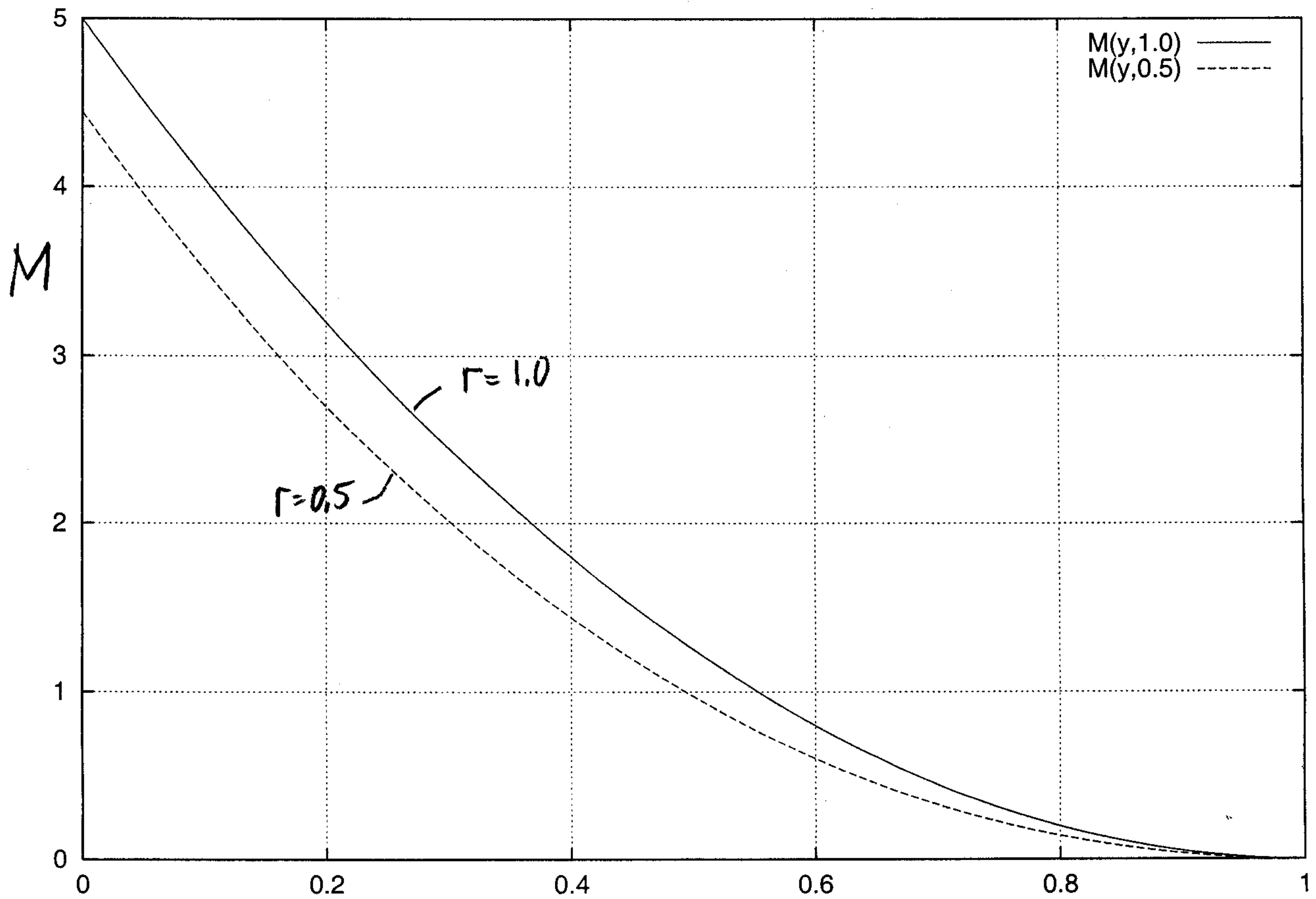
13-782  
50 SHEETS, FILLER, 5 SQUARE  
50 SHEETS, EYE-EASER, 5 SQUARE  
100 SHEETS, EYE-EASER, 5 SQUARE  
200 SHEETS, EYE-EASER, 5 SQUARE  
42-389  
42-392  
42-399  
200 RECYCLED WHITE, 5 SQUARE  
Made in U.S.A.



M10



M10



M11 Torsion of circular cross-section shafts

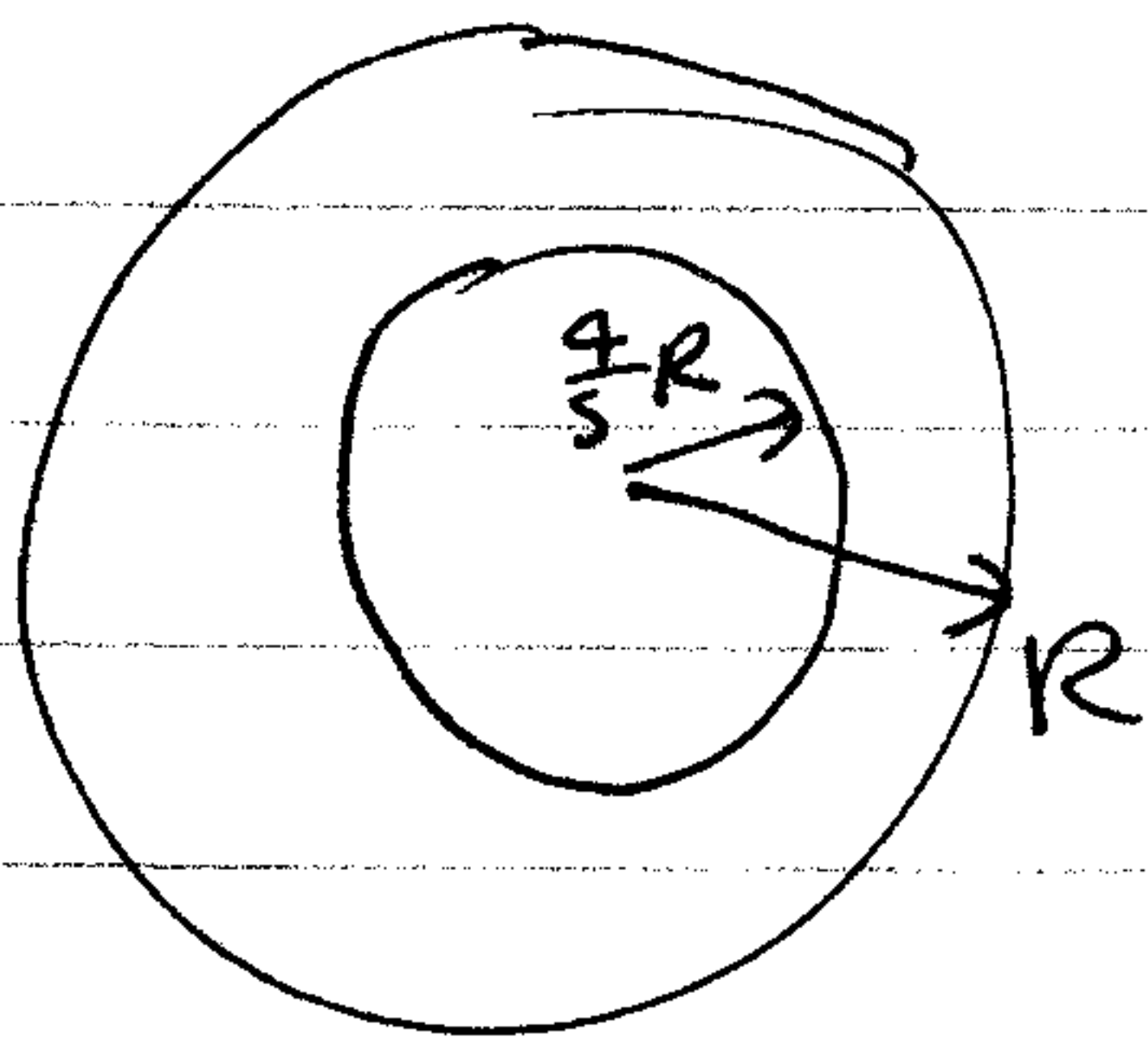
a) Torque shear stress relation:  $\hat{\tau} = \frac{Tr}{J}$

for solid circular cross section  $J = \frac{\pi R^4}{2}$

$$\therefore R^3 = \frac{2TR}{\pi \hat{\tau}_y} \Rightarrow R = \sqrt[3]{\frac{2T}{\pi \hat{\tau}_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 200 \times 10^6}}$$

$$= 0.086 \text{ m} \Rightarrow R \text{ diameter} = 0.172 \text{ m} \Leftarrow$$

b) Hollow shaft



$$J = \frac{\pi R^4}{2} - \frac{\pi \left(\frac{4}{5}\right)^4 R^4}{2}$$

$$= \frac{\pi R^4}{2} \left(1 - \left(\frac{4}{5}\right)^4\right)$$

$$= \frac{\pi R^4}{2} \left(1 - \frac{256}{625}\right)$$

$$\therefore \frac{\pi R^4}{2} (0.5904) = \frac{TR}{\hat{\tau}_y}$$

$$R = \sqrt[3]{\frac{2T}{\pi(0.5904)\hat{\tau}_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 0.5904 \times 200 \times 10^6}} = 0.103 \text{ m}$$

$$\therefore \text{diameter} = 0.205 \text{ m} \Leftarrow$$



$$\text{Fractional weight} = \frac{\pi (0.086^2 - (0.103)^2) \left(1 - \left(\frac{4}{5}\right)^2\right)}{\pi (0.086)^2}$$

$$= 1 - \left(\frac{0.103}{0.086}\right)^2 \left(1 - \left(\frac{4}{5}\right)^2\right)$$

$$= 0.484 = 48.4\% \text{ weight saving.}$$

c) Twist angle. from  $T = GJ \frac{d\phi}{ds}$

Since Torque is constant, twist angle =  $L \frac{d\phi}{ds}$

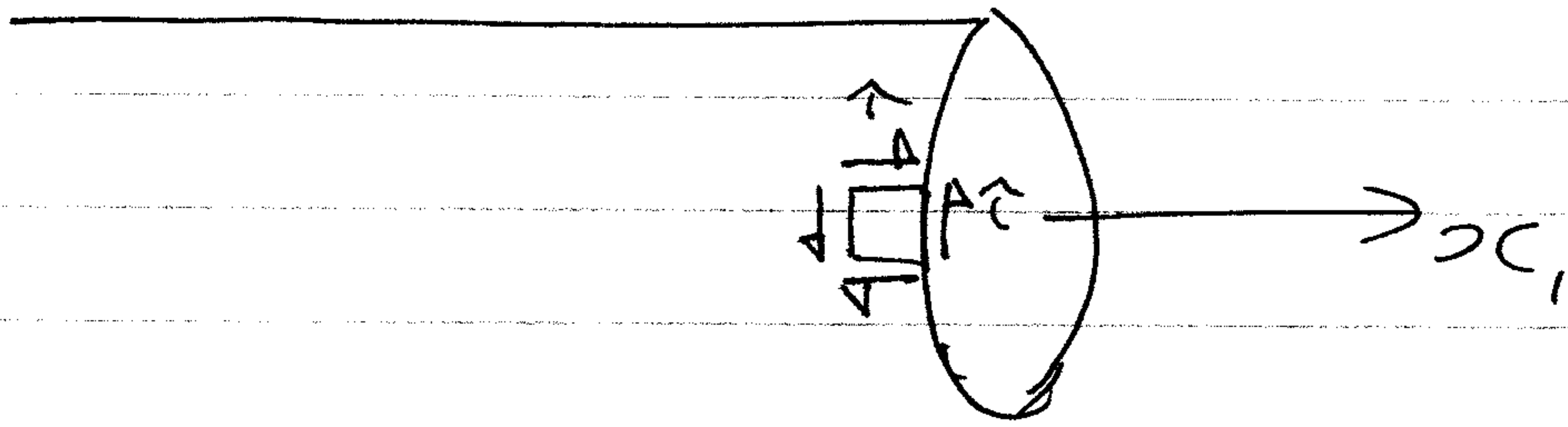
$\therefore$  ratio of twist angles is ratio of  $J$

$$= \frac{\pi (0.103)^4 \left(1 - \frac{256}{625}\right)}{\pi (0.086)^4}$$

$$= \frac{(0.103)^4 \left(1 - \frac{256}{625}\right)}{(0.086)^4} = 1.215$$

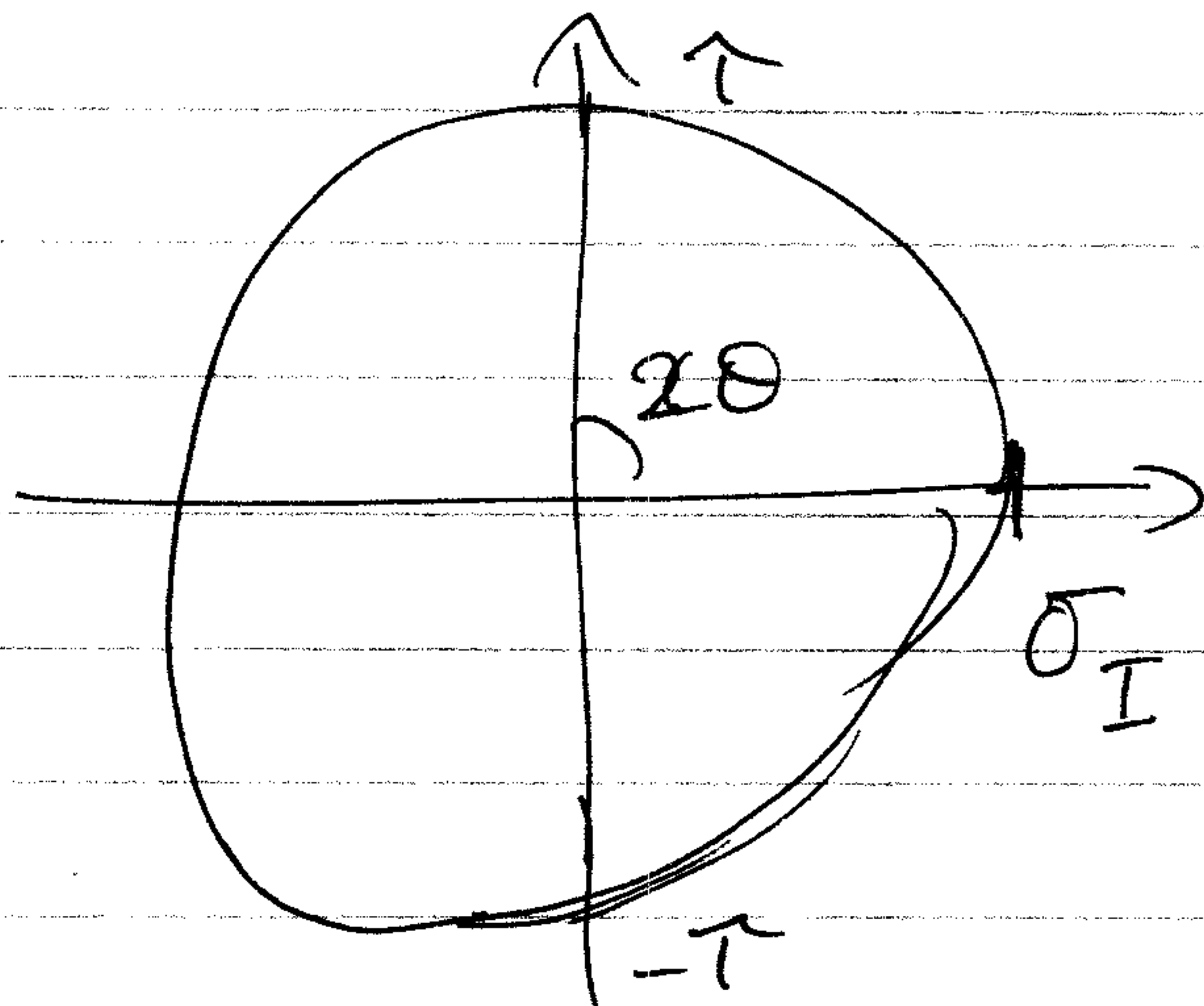
$\therefore$  shaft (a) will twist 21.5% more than shaft (b).

d.

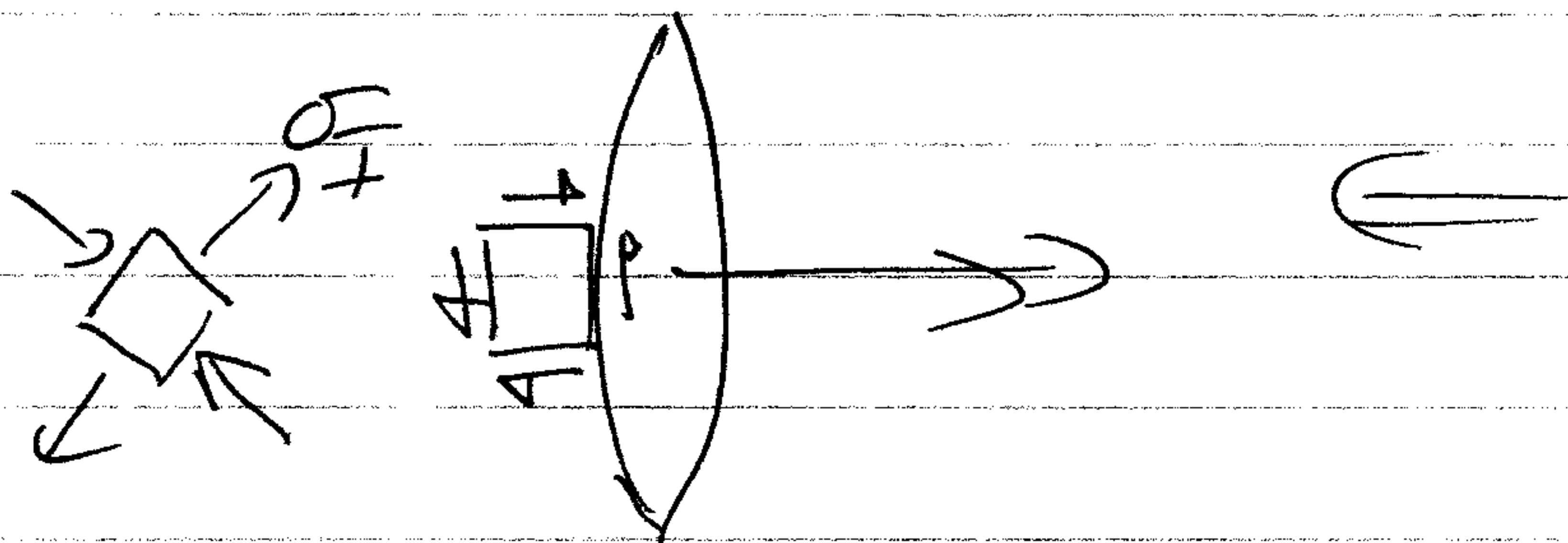


in case b (or a) shear stress  $\tau$  acts in plane of section.

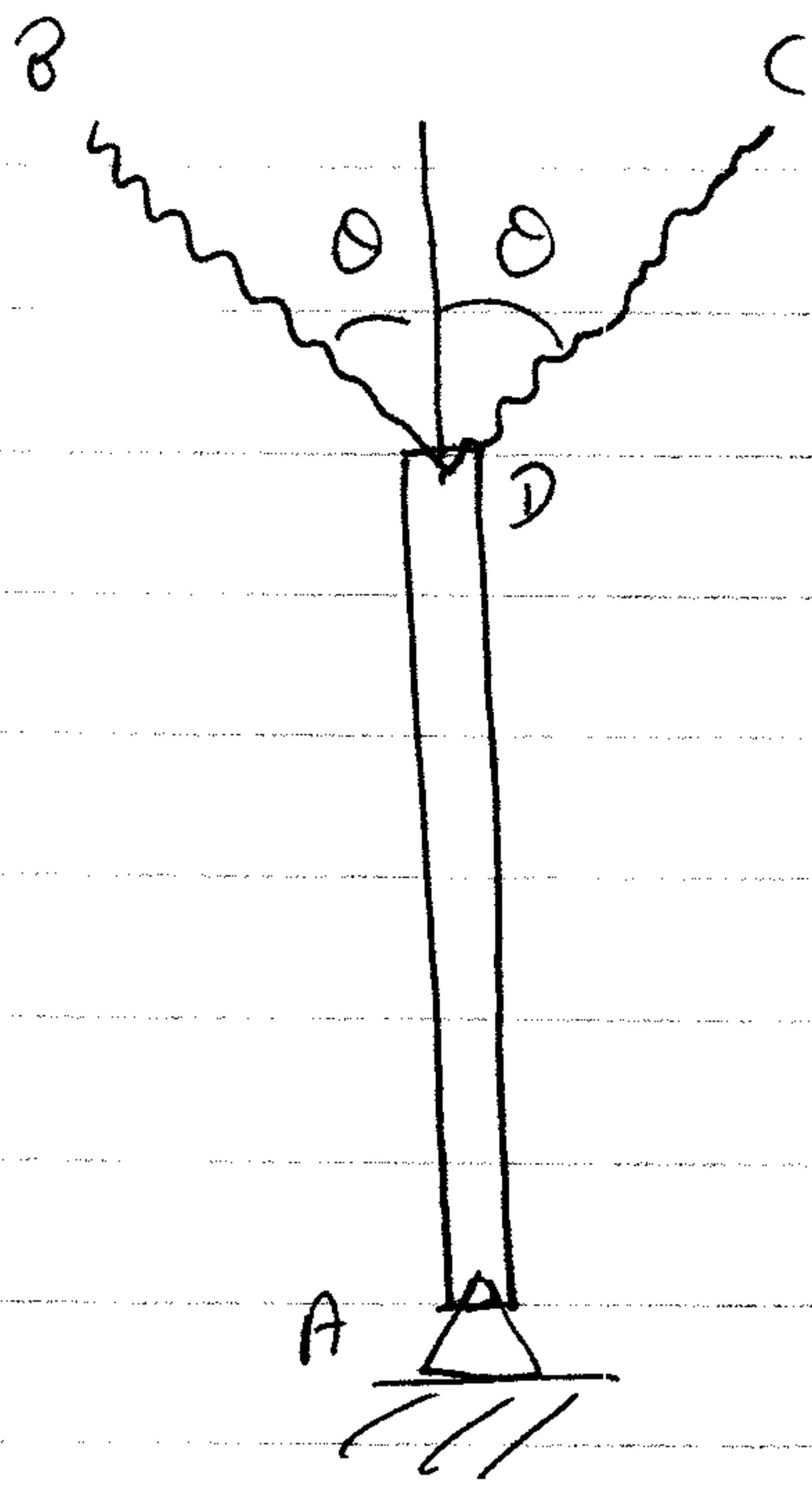
Drawing Mohr's circle



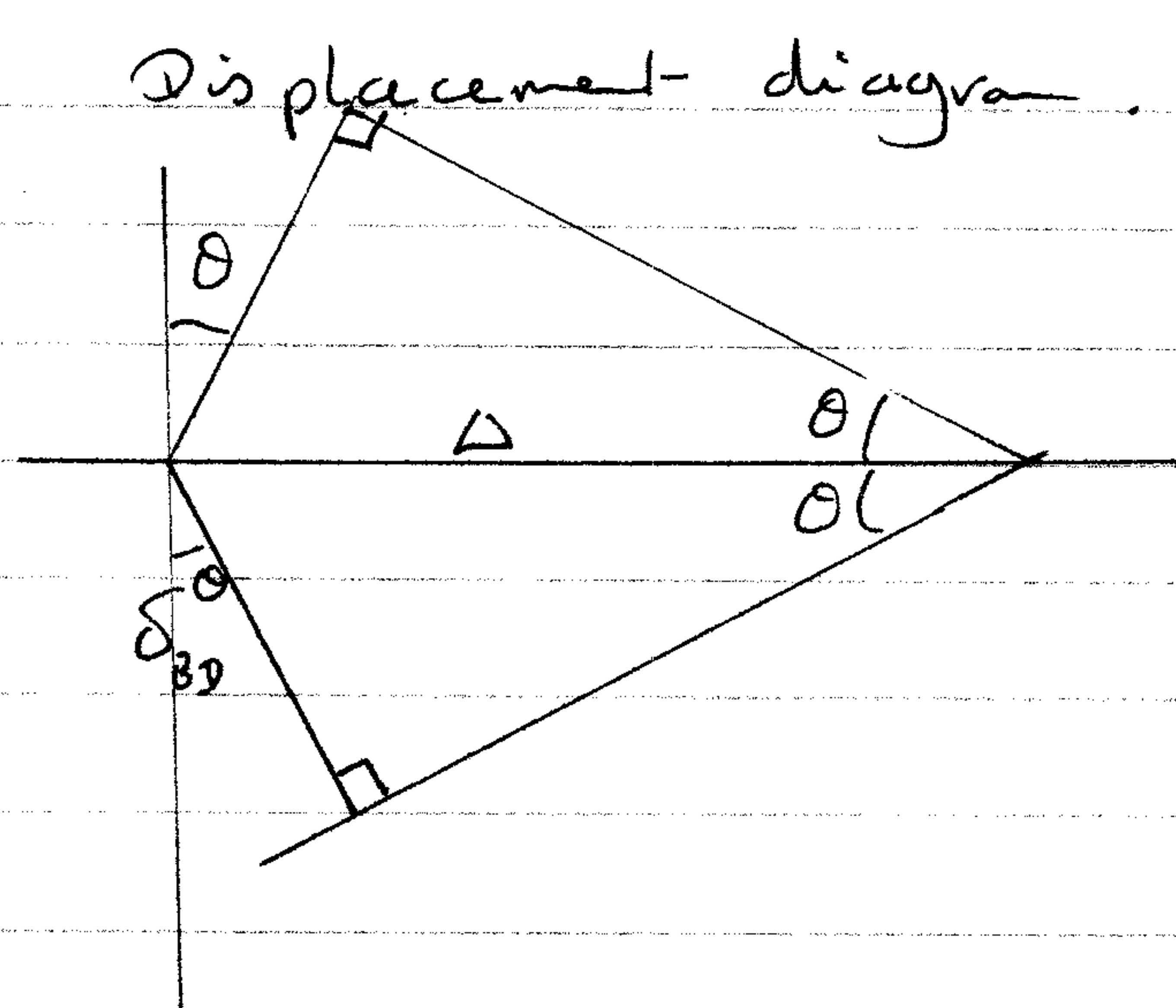
max tensile stress,  $\sigma_I$  will act at  $45^\circ$  to max shear i.e.



M12

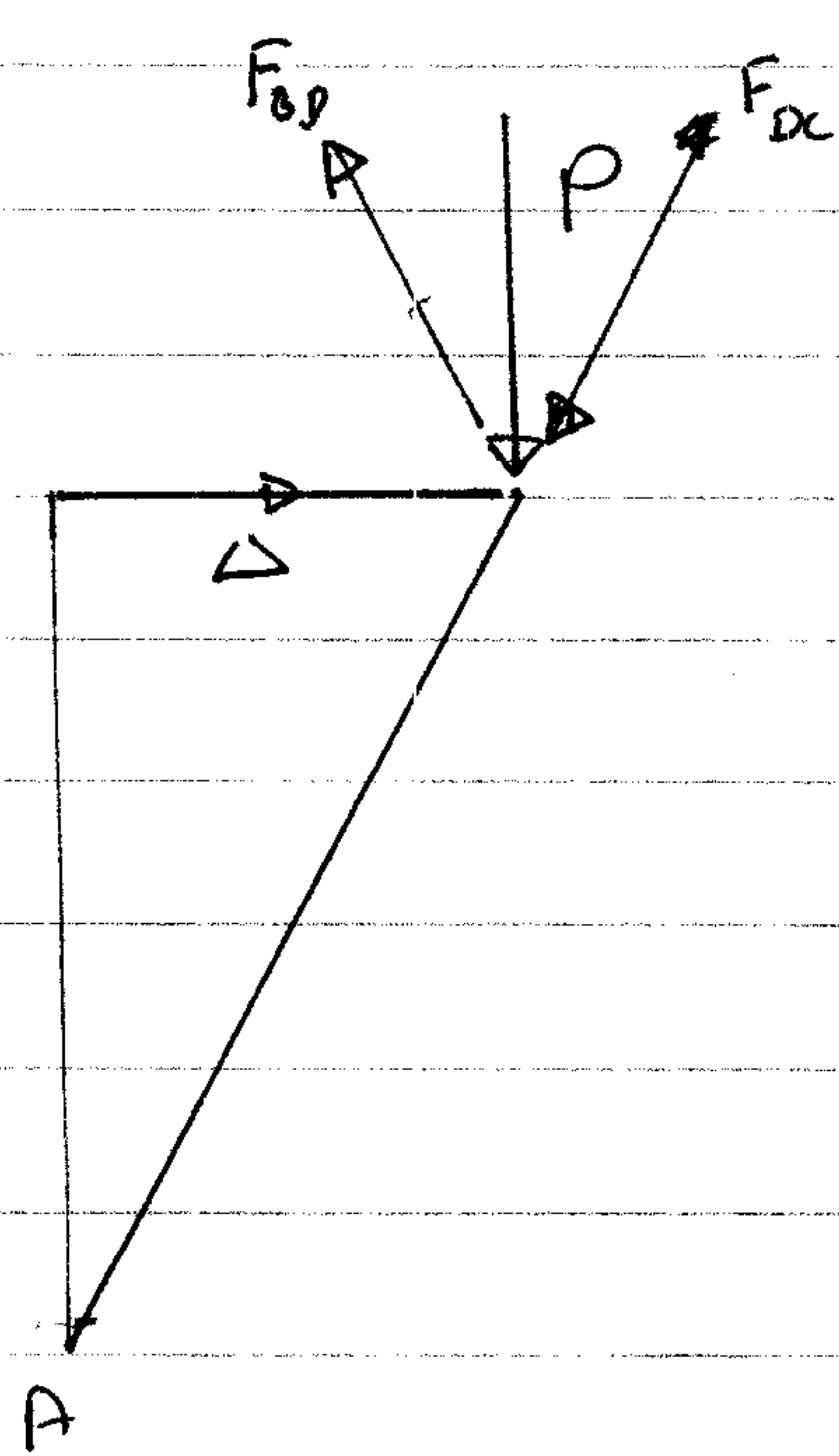


Displace D horizontally by  $\Delta$   
 BD + DC extend by  $\delta$  and rotate



$\therefore$  compatibility :  $\Delta \sin \theta = \delta_{BD} = \delta_{DC} = \delta$

BD extending, DC shortening  $\therefore$  FBD



$$F_{BD} = k \delta_{BD} = k \delta$$

$$F_{DC} = k \delta_{DC} = k \delta$$

$\therefore$  Vertical comp: Net vertical force = 0

Horizontal component =  $2 F_{BD} = 2k \delta$   
 $= 2k \Delta \sin \theta \leftarrow$

∴ equilibrium of moments about A

$$\Rightarrow \left( \begin{array}{l} M = 0 \\ \curvearrowright_A \end{array} : +2kL \sin \theta \cdot L - PL = 0 \right)$$

$$P = 2kL \sin \theta$$

if  $P > 2kL \sin \theta$  then collapse occurs  $\Leftarrow$

M13 Need to consider possibility of buckling in compressive members of truss

### Material selection

achieve (certain (required) buckling load while minimizing mass

$$P_{\text{crit}} = \frac{\pi^2 EI}{L^2}$$

$$\text{mass} = \pi R^2 L \rho$$

assume circular cross-section  $\therefore I = \frac{\pi R^4}{4}$

$$\text{also rewrite } R = \sqrt{\frac{M}{\pi L \rho}} \quad \text{or } R^4 = \left(\frac{M}{\pi L \rho}\right)^2$$

$$\therefore P_{\text{crit}} = \frac{\pi^2 E}{L^2} \cdot \frac{\pi}{4} \cdot \left(\frac{M}{\pi L \rho}\right)^2 = \frac{M^2}{F} \cdot \frac{\pi}{4L^4} \cdot \frac{E}{\rho^2}$$

F                  G                  M

maximize  $E/\rho^2$  for highest buckling load for given mass.

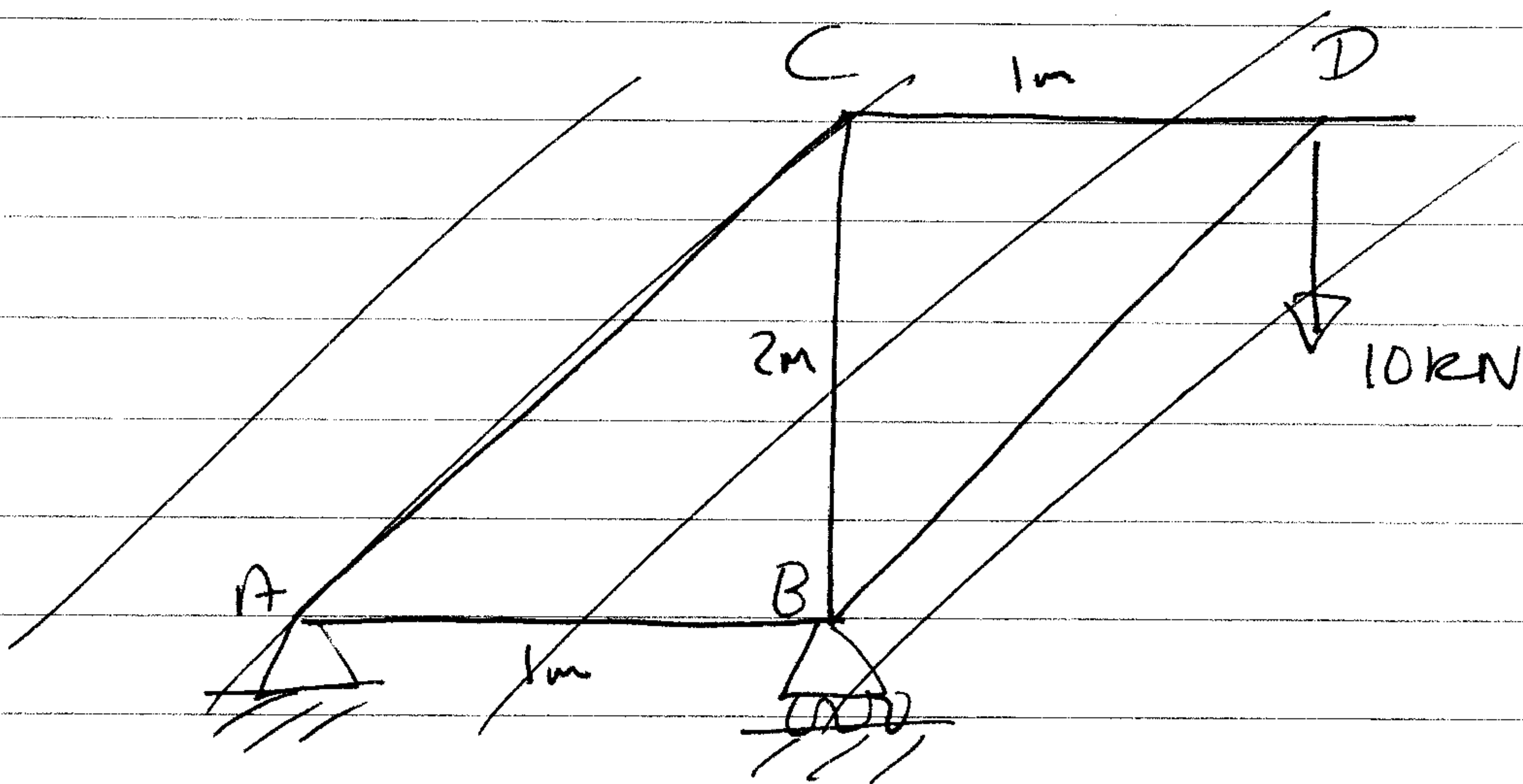
(previously maximize  $\sigma_f/\rho$ )

## Re ranking materials

	$\sigma_f / e$	$E / e^2$
Steel	$28 \times 10^3$	$3.3 \times 10^3$
Al	$125 \times 10^3$	$9.0 \times 10^3$
Ti	$188 \times 10^3$	$5.9 \times 10^3$
CFRP	$466 \times 10^3$	$31 \times 10^3$
Wood	$50 \times 10^3$	$33 \times 10^3$
SiC	$100 \times 10^3$	$45 \times 10^3$

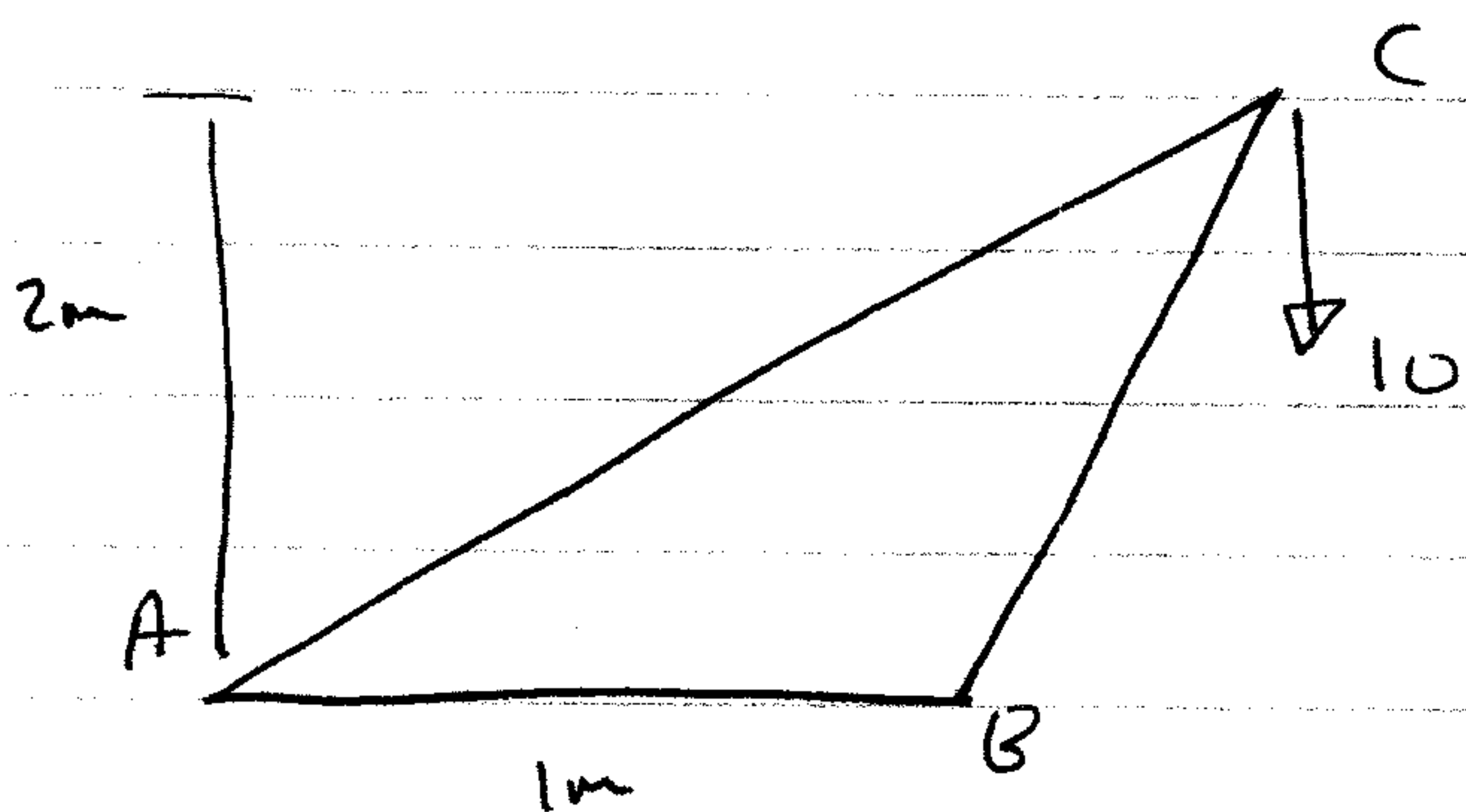
CFRP still looks very good. - high  $E/e^2$   
Wood might be better in buckling dominated design.

## Reconsider design



End

Reconsider design.



$$F_{AB} = -10 \text{ kN}$$

$$F_{BC} = -22.4 \text{ kN}$$

BC is the longest member at highest compressive force  
 $\therefore$  only need to consider this.

Assume that it is a simply supported column.

$$P_{\text{crit}} = \frac{\pi^2 EI}{L^2}$$

given circular cross-section

$$I = \frac{\pi R^4}{4}$$

$$\therefore P_{\text{crit}} = \frac{\pi^2 E \pi R^4}{4 L^2}$$

$$L = \sqrt{5} \text{ m}$$

$$R = \sqrt[4]{\frac{P_{\text{crit}} \times 4 L^2}{\pi^3 E}} = \sqrt[4]{\frac{22.4 \times 10^3 \times 4 \times 5}{\pi^3 \times 70 \times 10^9}}$$

$$= 0.021 \text{ m}$$

$$\therefore \text{area} = \pi R^2 = 0.0014 = 1430 \text{ mm}^2 \text{ (cf } 32 \text{ mm}^2 \text{ before)}$$

$$\therefore \text{mass increases by } \frac{1430}{32} = 44.7 \text{ Now weights weights}$$

$$0.29 \times 44.7 = 13.0 \text{ kg!}$$