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Cambridge, MA 02139

16.03/16.04 Unified Engineering III, IV
Spring 2004

Problem Set 8

Name: _____

Due Date: 4/6/04

	Time Spent (min)
CP5_6	
S4	
S5	
S6	
S7	
Study Time	

Announcements:

Q4F is on Wednesday, 3/31 at 9am in WALKER.

CP5_6

The problems in this problem set cover lectures C5 and C6.

1.

a. What are doubly linked lists? What is the record declaration for a node in a doubly linked list? (hint: see Feldman page 672, problem 8)

b. Write an algorithm to insert a node into a sorted doubly linked list. Use a diagram to show the sequence of operations that have to be performed to carry out the insertion step.

Hint: Extend the approach used in class/ notes for singly linked lists.

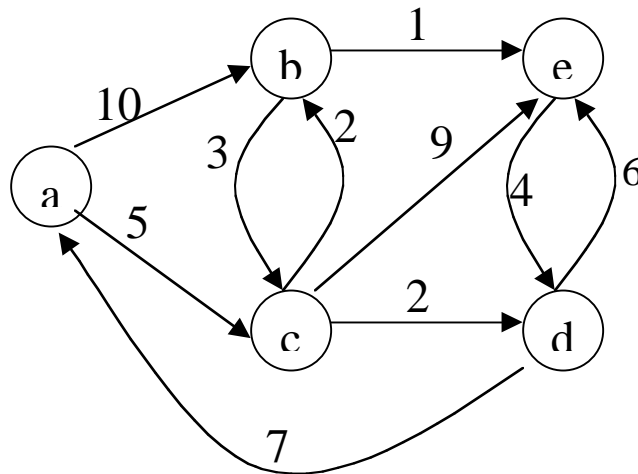
c. Implement your algorithm as an Ada95 program.

Assume:

- i. The data field consists of a single integer.

Turn in a hard copy of your algorithm and code listing, and an electronic copy of your code.

2. What is the Shortest Path through the graph shown below using Dijkstra's algorithm? Show all the steps in the computation of the shortest path.



3. Define the following terms (as applied to graphs):

- a. Walk
- b. Path
- c. Eulerian Path
- d. Cycle
- e. Degree of a vertex

Problem S4 (Signals and Systems)

Note: This problem is similar to one given a couple years ago. Please try to do this one without looking at bibles — the solution is instructive.

One of the benefits of the approach of using the superposition integral is that you don't have to guess the particular solution — it pops right out of the integral, automatically. In some cases, the particular solution can be hard to guess, but easy to find using the convolution integral. To see this, consider the system described by the differential equation

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = u(t)$$

1. Find the step response of the system.
2. Take the derivative of the step response to find the impulse response.
3. Now assume that the input is given by

$$u(t) = e^{-2t}\sigma(t)$$

Before doing part (4), try to find the particular solution by the usual method, that is, by intelligent guessing. Be careful — it may not be what you expect!

4. Now find $y(t)$ using the superposition integral. Is the particular solution what you expected?

Problem S5 (Signals and Systems)

Do problem 2.22 from *Oppenheim and Willsky*. Note that O&H use the notation $x(t)$ for the input signal, $h(t)$ for the impulse response, and $u(t)$ for the unit step.

Problem S6 (Signals and Systems)

Note: This problem is similar to one given a couple years ago. Please try to do this one without looking at bibles — the solution is instructive.

This problem shows, in a somewhat simplified way, how a radar system determines the distance to an aircraft. More basically, it will give you practice doing convolution using the “flip and slide” method.

The radar sends out a signal, $u(t)$, that reflects off the aircraft and returns to the radar system. The time it takes the signal to return is twice the distance to the aircraft, divided by the speed of light. The received signal is $u(t - T)$, where T is the round trip travel time of the signal.

It may not seem obvious, but it’s not easy to determine T by measuring the returned signal. In principal, one could simply display the returned signal on an oscilloscope and measure the time of some key feature of the signal, such as the leading edge of the signal, or the peak of the signal, or whatever. However, the signal received by the radar is relatively weak, because most of the energy radiated by the radar does not strike the aircraft, and most of the reflected energy is not received by the radar. Often the noise in the received signal is as large or larger than the signal itself.

Instead, a better approach is to try to identify the whole received signal instead of a single key feature. How can we do this? Convolution! The received signal is run through an LTI system G with impulse response $g(t)$. The impulse response is designed to accentuate the signal, so that one can measure a key feature of the output signal, $y(t)$, which is relatively insensitive to noise. Often, the best choice for $g(t)$ is a time-reversed version of $u(t)$. That is,

$$g(t) = u(-t) \tag{1}$$

This is called a “matched filter.”

1. For the signal $u(t)$ shown below, and $g(t) = u(-t)$, find the convolution

$$y(t) = g(t) * u(t)$$

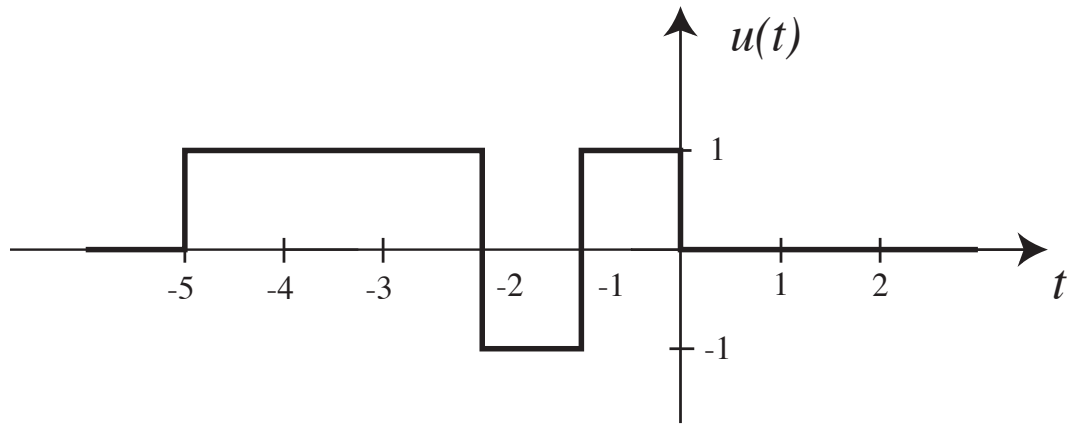
using the flip and slide method.

2. Using the results of part (1) and the LTI properties of the system G , what is the convolution

$$y(t) = g(t) * u(t - T)$$

3. What feature of $y(t)$ would you use to identify the time T ?

4. Can you explain why the right impulse response for G is a signal that has the same shape as $u(t)$, but time reversed?



Problem S7 (Signals and Systems)

For each of the functions below, find the Laplace transform of the function, as well as the region of convergence. Do not use any outside reference (table of transforms or the Unified bible) to find the answer. You may use a table of Laplace transforms to *check* your work. However, show the derivation of the result, *i.e.*, work out the Laplace integral. Make sure that you include the region of convergence.

1.

$$g(t) = \begin{cases} te^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

2.

$$g(t) = \begin{cases} t^2e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

3.

$$g(t) = \begin{cases} t^n e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

where n is a positive integer.

4.

$$f(t) = e^{-(t-a)^2/2b^2}, \quad \text{for all } t$$