

**Massachusetts Institute of Technology**  
**Department of Aeronautics and**  
**Astronautics**  
**Cambridge, MA 02139**

---

**16.03/16.04 Unified Engineering III, IV**  
**Spring 2004**

**Problem Set 10**

Name: \_\_\_\_\_

Due Date: 4/21/04

	<b>Time Spent (min)</b>
<b>CP11_12</b>	
<b>P3</b>	
<b>P4</b>	
<b>S10</b>	
<b>S11</b>	
<b>S12</b>	
<b>Study Time</b>	

---

Announcements: Q5S will be on Friday, April 23 in 35-225

---

---

## CP11-12

The problems in this problem set cover lectures C11 and C12

1.
  - a. Define a recursive binary search algorithm.
  - b. Implement your algorithm as an Ada95 program.
  - c. What is the recurrence equation that represents the computation time of your algorithm?
  - d. What is the Big-O complexity of your algorithm? Show all the steps in the computation based on your algorithm.

Turn in a hard copy of your algorithm, recurrence equation, and Big-O analysis, and code listing, and an electronic copy of your code.

2. What is the Big-O complexity of:
  - a. Heapify function
  - b. Build\_Heap function
  - c. Heap\_Sort

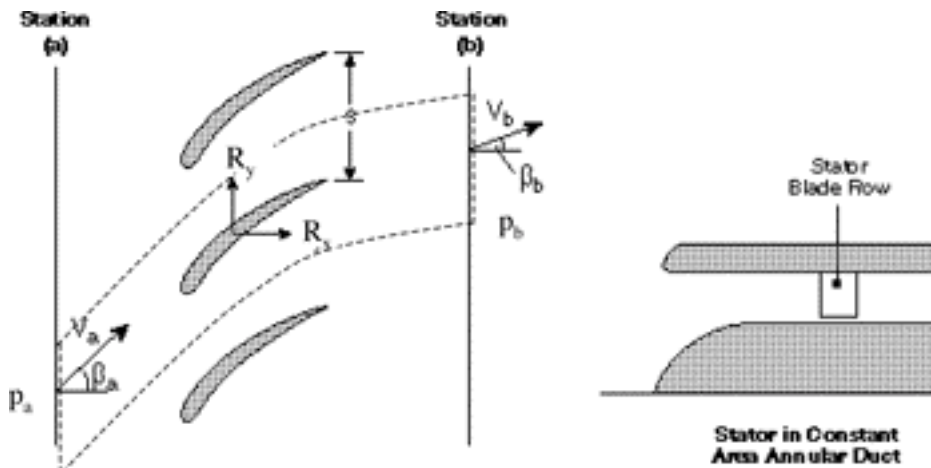
Show all the steps in the computation of the Big-O complexity.

Note: the code for heap\_sort, build\_heap and heapify was shown in lecture C11 and has been distributed via email.

## nified Engineering Spring Term 2004

### Problem P3. (Propulsion) (LO A & B)

An incompressible fluid flows steadily through a two-dimensional infinite row of fixed airfoils (e.g. a stator blade row). The blade row is contained in a constant area annulus, as shown on the right side of the figure below. The spacing between the airfoils is  $s$ . Assume that the velocities and pressures  $V_a$ ,  $V_b$ ,  $p_a$ ,  $p_b$ , are constant at stations (a) and (b), and that the flow angles are given by  $\beta_a$  and  $\beta_b$ .



- Does the magnitude of the flow velocity increase/stay the same/decrease across the stator and why?
- Using the control volume shown above (the upper and lower surfaces are streamlines), apply conservation of mass and momentum to determine the forces  $R_x$  and  $R_y$  that must be applied to the fluid (these are equal and opposite to the forces needed to keep each vane in place).

## **Unified Engineering Spring Term 2004**

### **Problem P4. (Propulsion) (L.O. D)**

For this problem, use the aerodynamic, structural and battery parameters you have determined for your Dragonfly.

- a) Calculate the speed that you would fly the Dragonfly if you wanted to maximize range with no additional payload.
  
- b) Calculate the speed that you would fly the Dragonfly if you wanted to maximize endurance with no additional payload.
  
- c) Assume the overall efficiency of the battery-motor-propeller combination is 0.15. As a function of the number of eggs carried (0 through 5) tabulate the velocity for maximum endurance, the power required, and the endurance.

**Problem S10 (Signals and Systems)**

This problem provides lots of practice using partial fraction expansions to determine inverse Laplace transforms. Please use the coverup method — it really is superior to other methods, and more reliable. Also, please check your answer, that is, verify that your expansion really is equivalent to the  $G(s)$  given.

For each of the following Laplace transforms, find the inverse Laplace transform.

1.  $G(s) = \frac{3s^2 + 3s - 10}{s^2 - 4}, \quad \text{Re}[s] > 2$

2.  $G(s) = \frac{6s^2 + 26s + 26}{(s + 1)(s + 2)(s + 3)}, \quad \text{Re}[s] > -1$

3.  $G(s) = \frac{4s^2 + 11s + 9}{(s + 1)^2(s + 2)}, \quad \text{Re}[s] > -1$

4.  $G(s) = \frac{4s^3 + 11s^2 + 5s + 2}{s^2(s + 1)^2}, \quad \text{Re}[s] > 0$

5.  $G(s) = \frac{s^3 + 3s^2 + 9s + 12}{(s^2 + 4)(s^2 + 9)}, \quad \text{Re}[s] > 0$

**Problem S11 (Signals and Systems)**

Consider an aircraft flying in cruise at 250 knots, so that

$$v_0 = 129 \text{ m/s}$$

Assume that the aircraft has lift-to-drag ratio

$$\frac{L_0}{D_0} = 15$$

Then the transfer function from changes in thrust to changes in altitude is

$$G(s) = \frac{2g}{mv_0} \frac{1}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad (1)$$

where the *natural frequency* of the phugoid mode is

$$\omega_n = \sqrt{2} \frac{g}{v_0} \quad (2)$$

the *damping ratio* is

$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0)} \quad (3)$$

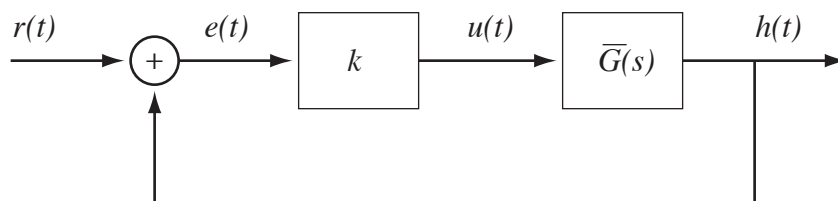
and  $g = 9.82 \text{ m/s}$  is the acceleration due to gravity. The transfer function can be normalized by the constant factor  $\frac{2g}{mv_0}$ , so that

$$\bar{G}(s) = \frac{1}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad (4)$$

is the normalized transfer function, corresponding to normalized input

$$u(t) = \frac{2g}{mv_0} \delta T$$

1. Find and plot the impulse response corresponding to the transfer function  $\bar{G}(s)$ , using partial fraction expansion and inverse Laplace techniques. Hint: The poles of the system are complex, so you will have to do complex arithmetic.
2. Suppose we try to control the altitude through a feedback loop, as shown below



That is, the control input  $u(t)$  (normalized throttle) is a gain  $k$  times the error,  $e(t)$ , which is the difference between the altitude  $h(t)$  and the altitude reference  $r(t)$ . The transfer function from  $r(t)$  to  $h(t)$  can be shown to be

$$H(s) = \frac{1}{1 + kG(s)}$$

For the gain  $k$  in the range  $[0, 0.1]$ , plot the poles of the system in the complex plane. You should find that for any positive  $k$ , the complex poles are made less stable. What gain  $k$  makes the complex poles unstable, i.e., for what gain is the damping ratio zero?

3. For the gain  $k$  in the range  $[-0.1, 0]$ , plot the poles of the system in the complex plane. You should find that for any negative  $k$ , the real pole is unstable.

Note that neither positive gain or negative gain makes the system more stable than without feedback control. It is possible to do better with a dynamic gain, but this problem should give you an idea of why the phugoid dynamics are so hard to control with throttle only.

**Problem S12 (Signals and Systems)**

For each signal below, find the bilateral Laplace transform (including the region of convergence) by directly evaluating the Laplace transform integral. If the signal does not have a transform, say so.

1.  $g(t) = \sin(at)\sigma(-t)$
2.  $g(t) = te^{at}\sigma(-t)$
3.  $g(t) = \cos(\omega_0 t) e^{-a|t|}$ , for all  $t$