Home Work 12

The problems in this problem set cover lectures C16

1.

a. Using truth tables, show that $\overline{A} \langle \, \overline{B} = \overline{(A+B)}$

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \langle \, \overline{B}$	$A+B$	$\overline{(A+B)}$
0	0	1	1	1	0	1
0	1	1	0	0	1	0
1	0	0	1	0	1	0
1	1	0	0	0	1	0

b. Using K-Maps, simplify the following expression:

$$\overline{A} \langle \, \overline{B} \langle \, \overline{C} + \overline{A} \langle \, \overline{B} \langle \, C + A \langle \, \overline{B} \langle \, C + A \langle \, \overline{B} \langle \, \overline{C}$$

A	B	C	Minterm
0	0	0	$\overline{A} \langle \, \overline{B} \langle \, \overline{C}$
0	0	1	$\overline{A} \langle \, \overline{B} \langle \, C$
0	1	0	$\overline{A} \langle \, B \langle \, \overline{C}$
0	1	1	$\overline{A} \langle \, B \langle \, C$
1	0	0	$A \langle \, \overline{B} \langle \, \overline{C}$
1	0	1	$A \langle \, \overline{B} \langle \, C$
1	1	0	$A \langle \, B \langle \, \overline{C}$
1	1	1	$A \langle \, B \langle \, C$

C/ AB	00	01	11	10
0	1			1
1	1			1

$$\overline{A} \langle \, \overline{B} \langle \, \overline{C} + \overline{A} \langle \, \overline{B} \langle \, C + A \langle \, \overline{B} \langle \, C + A \langle \, \overline{B} \langle \, \overline{C} = \overline{B}$$

c. Using K-Maps, simplify the following expression:

$$A\langle\, B\langle\, D+\overline{B}\langle\, C\langle\, D+\overline{A}\langle\, B\langle\, C\langle\, D+\overline{C}\langle\, D$$

A	B	C	D	Minterm
0	0	0	0	$\overline{A}\langle\, \overline{B}\langle\, \overline{C}\langle\, \overline{D}$
0	0	0	1	$\overline{A}\langle\, \overline{B}\langle\, \overline{C}\langle\, D$
0	0	1	0	$\overline{A}\langle\, \overline{B}\langle\, C\langle\, \overline{D}$
0	0	1	1	$\overline{A}\langle\, \overline{B}\langle\, C\langle\, D$
0	1	0	0	$\overline{A}\langle\, B\langle\, \overline{C}\langle\, \overline{D}$
0	1	0	1	$\overline{A}\langle\, B\langle\, \overline{C}\langle\, D$
0	1	1	0	$\overline{A}\langle\, B\langle\, C\langle\, \overline{D}$
0	1	1	1	$\overline{A}\langle\, B\langle\, C\langle\, D$
1	0	0	0	$A\langle\, \overline{B}\langle\, \overline{C}\langle\, \overline{D}$
1	0	0	1	$A\langle\, \overline{B}\langle\, \overline{C}\langle\, D$
1	0	1	0	$A\langle\, \overline{B}\langle\, C\langle\, \overline{D}$
1	0	1	1	$A\langle\, \overline{B}\langle\, C\langle\, D$
1	1	0	0	$A\langle\, B\langle\, \overline{C}\langle\, \overline{D}$
1	1	0	1	$A\langle\, B\langle\, \overline{C}\langle\, D$
1	1	1	0	$A\langle\, B\langle\, C\langle\, \overline{D}$
1	1	1	1	$A\langle\, B\langle\, C\langle\, D$

CD/ AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$$A\langle\, B\langle\, D+\overline{B}\langle\, C\langle\, D+\overline{A}\langle\, B\langle\, C\langle\, D+\overline{C}\langle\, D=D$$

d. Simplify the same expression using the rules of simplification.

$A\langle\ B\langle\ D+\overline{B}\langle\ C\langle\ D+\overline{A}\langle\ B\langle\ C\langle\ D+\overline{C}\langle\ D$

$B\langle\ D(A+\overline{A}C)+D(\overline{B}\langle\ C+\overline{C})$ [Distributive Property]

$B\langle\ D\langle\ (A+C)+D(\overline{B}+\overline{C})$ [Two Value Theorem]

$A\langle\ B\langle\ D+B\langle\ C\langle\ D+D\langle\ \overline{B}+D\langle\ \overline{C}$ [Distributive Property]

$D(AB+\overline{B})+D(BC+\overline{C})$ [Distributive Property]

$D(A+\overline{B})+D(B+\overline{C})$ [Two Value Theorem]

$D\langle\ A+D\langle\ \overline{B}+D\langle\ B+D\langle\ \overline{C}$ [Distributive Property]

$D\langle\ A+D(B\langle\ \overline{B})+D\langle\ \overline{C}$ [Distributive Property]

$D\langle\ A+D\langle\ 1+D\langle\ \overline{C}$ [Single Value Theorem]

$(D\langle\ A+D)+D\langle\ \overline{C}$ [Two Value Theorem]

$D+D\langle\ \overline{C}$ [Single Value Theorem]

D [Single Value Theorem]

2. Convert the following expression into product of sum form:

$$\overline{A}\langle\ \overline{B}\langle\ \overline{C}+\overline{A}\langle\ B\langle\ C+A\langle\ B\langle\ \overline{C}+A\langle\ \overline{B}\langle\ C$$

A	B	C	Minterm
0	0	0	$\overline{A}\langle\ \overline{B}\langle\ \overline{C}$
0	0	1	$\overline{A}\langle\ \overline{B}\langle\ C$
0	1	0	$\overline{A}\langle\ B\langle\ \overline{C}$
0	1	1	$\overline{A}\langle\ B\langle\ C$
1	0	0	$A\langle\ \overline{B}\langle\ \overline{C}$
1	0	1	$A\langle\ \overline{B}\langle\ C$
1	1	0	$A\langle\ B\langle\ \overline{C}$
1	1	1	$A\langle\ B\langle\ C$

$$\overline{A}\langle\,\overline{B}\langle\,\overline{C}+\overline{A}\langle\,B\langle\,C+A\langle\,B\langle\,\overline{C}+A\langle\,\overline{B}\langle\,C$$

C/ AB	00	01	11	10
0	1	0	1	0
1	0	1	0	1

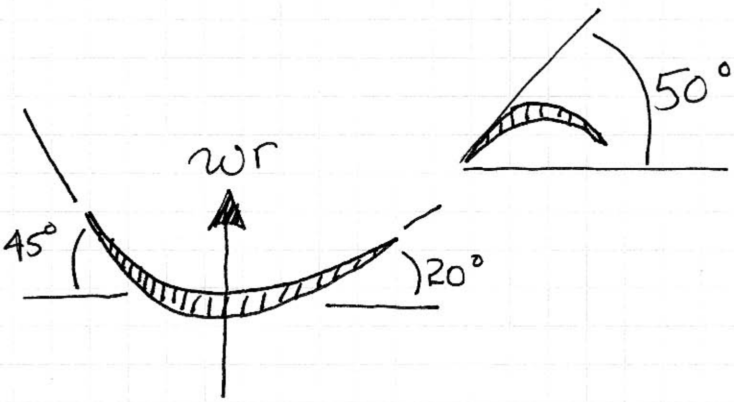
$$=\overline{\overline{(\overline{A}\langle\,\overline{B}\langle\,C+\overline{A}\langle\,B\langle\,\overline{C}+A\langle\,B\langle\,C+A\langle\,\overline{B}\langle\,\overline{C})}}$$

$$=(A+B+\overline{C})\langle(A+\overline{B}+C)\langle\,(\overline{A}+\overline{B}+\overline{C})\langle\,(\overline{A}+B+C)$$

UNIFIED PROPULSION P8 SOLUTIONS — WAITZ 2001

THE MOST CONVENIENT WAY TO OBTAIN THE BLADE ANGLES IS TO SIGHT ALONG THE BLADE (THROUGH THE PLEXIGLASS).

THIS IS WHAT I CAME UP WITH:



FAN

FIRST STATOR IN BOOSTER

NOTE:
- THE RADIUS IS ABOUT 16" AT ENTRANCE TO THE BOOSTER
- THE TIP RADIUS IS 30"

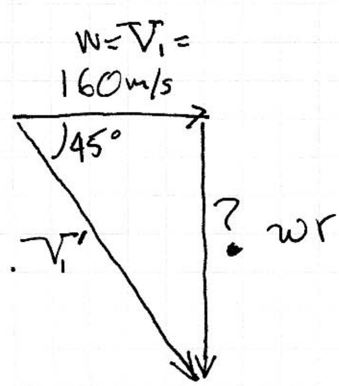
THERE ARE TWO WAYS TO ESTIMATE THE BLADE SPEED:

1) FLOW SHOULD BE ROUGHLY ALIGNED WITH FAN BLADE LEADING EDGE (OR A SMALL + ANGLE OF ATTACK) — IF NOT, FLOW WILL SEPARATE

2) FLOW WILL LEAVE FAN TRAILING EDGE AT METAL ANGLE AND MUST ROUGHLY LINE UP WITH STATOR BLADE LEADING EDGE ANGLE (OR A SMALL + ANGLE OF ATTACK)

FOR ESTIMATE 1):

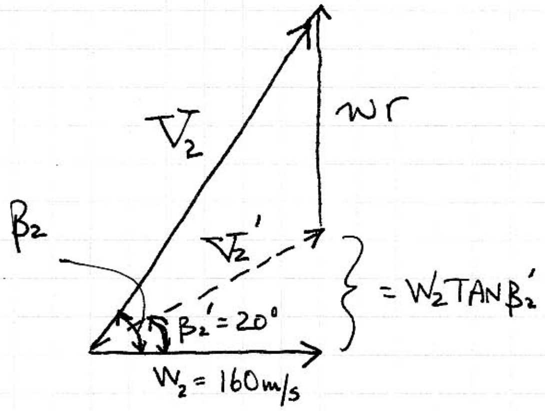
AXIAL VELOCITY → $M = 0.5 \approx 160$ m/s



SO WHAT ωr WILL GIVE ROUGHLY A 45° FLOW ANGLE INTO THE FAN?

$\omega r = 160 \tan 45° = \underline{160 \text{ m/s}}$

FOR ESTIMATE 2):



WHAT ωr GIVES A β_2 OF ABOUT 50°?

$$\frac{\omega r + 160 \tan 20°}{W_2} = \tan \beta_2$$

$$160 \tan 50° - 160 \tan 20° = \omega r = 132 \text{ m/s}$$

SINCE $r \approx 0.4$ m THEN $\omega \approx 394$ rad/s (ESTIMATE 1)

$\omega \approx 325$ rad/s (ESTIMATE 2)

ω rad/s → CONVERT TO RPM

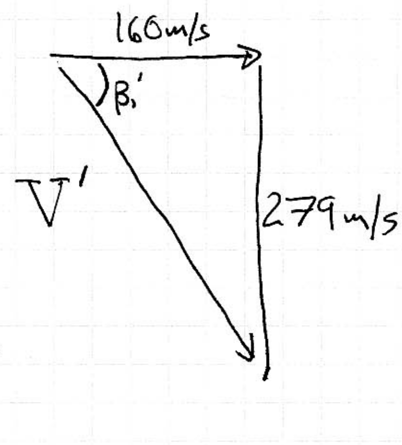
$$394 \frac{rad}{s} \cdot \frac{60 s}{min} \cdot \frac{rev}{2\pi \, rad} = 3760 \text{ RPM}$$

$$325 \frac{rad}{s} \cdot \frac{60 s}{min} \cdot \frac{rev}{2\pi \, rad} = 3100 \text{ RPM}$$

b) IF WE TAKE IT AS 3500 RPM, $\omega = 366.5$ rad/s

TIP RADIUS = 0.76 m SO TIP SPEED IS 279 m/s

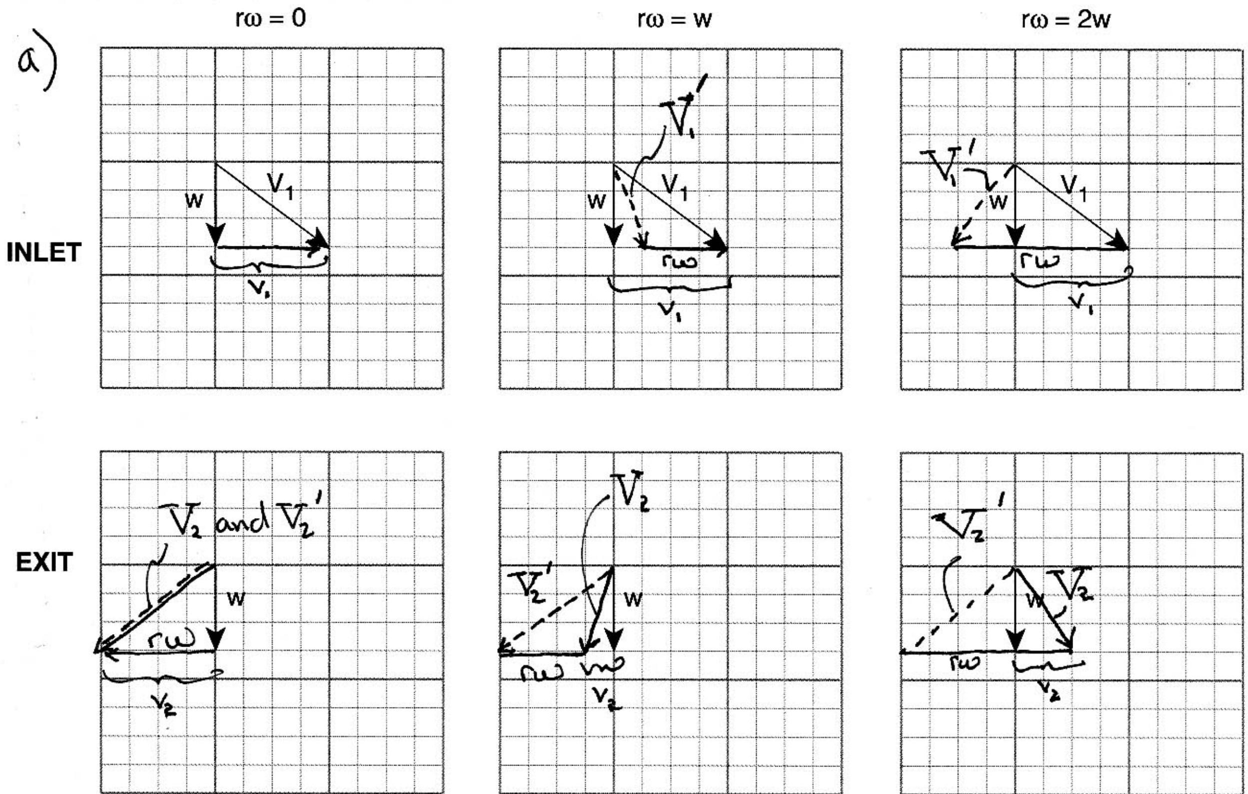
(NOTE, THIS IS WHY THE BLADES ARE TWISTED, SINCE β' CHANGES WITH RADIUS)



$$V' = \sqrt{160^2 + 279^2} = 322 \text{ m/s}$$

ABOUT $M \approx 1$ RELATIVE TO THE FAN

a)



b) $r\omega = W$ EXTRACTS THE MOST POWER. IT LEAVES THE LEAST SWIRLING KINETIC ENERGY IN THE FLOW $(\sim v_2^2)$ (OF THE 3 CASES SHOWN ABOVE)

c) ARGUMENT 1: IF $r\omega = \frac{4}{3}W$ ALL SWIRLING KINETIC ENERGY IS EXTRACTED (i.e. $v_2 = 0$). CAN SEE THIS FROM LOOKING AT THE GRAPHS.

ARGUMENT 2: TAKE DERIVATIVE OF EULER EQUATION w.r.t. $r\omega$ & SET $= 0$

$$\frac{d}{d(r\omega)}\left[(\omega r)W\tan\beta_1 + (\omega r)W\tan\beta_2' - (\omega r)^2\right] = 0 \quad \text{WITH } \beta_1 = \beta_2'$$

$$2W\tan\beta_1 = 2\omega r \quad \therefore \quad \omega r = W\tan\beta_1$$

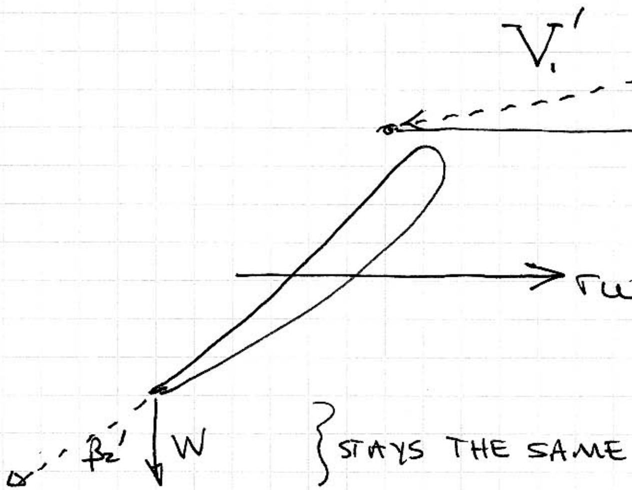
$$= W\frac{v_1}{W} = v_1$$

$$= \frac{4}{3}W \checkmark$$

d) IT BEGINS TO ACT LIKE A COMPRESSOR WHEN IT PUTS MORE SWIRL KINETIC ENERGY INTO FLOW ($\sim V_2^2$) THAN IT STARTED WITH ($\sim V_1^2$).

THIS HAPPENS (GRAPHICALLY) FOR $r\omega > \frac{8}{3} W$, WHICH IS ALSO WHEN THE EULER TURBINE EQUATION STARTS GIVING NEGATIVE VALUES OF $T_{T_1} - T_{T_2}$, IMPLYING AN ENTHALPY RISE NOT AN ENTHALPY DROP.

REGARDING THE AERODYNAMICS FOR THIS SITUATION, CONSIDER THE RELATIVE FRAME VELOCITIES



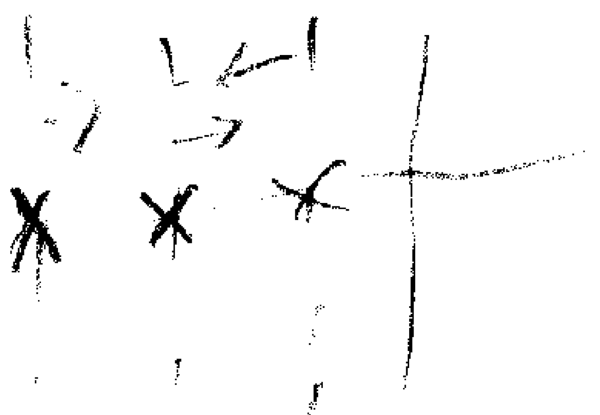
NEGATIVE ANGLE OF ATTACK! (USUALLY DOESN'T WORK WELL)

5/4

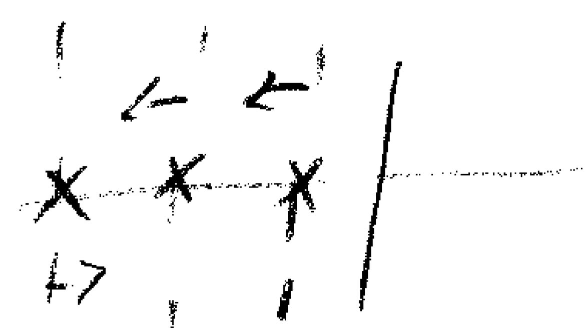
1. $e^{-2t}u(t) - 2e^{2t}u(-t) + 3\delta(t)$

2. $-e^{-2t}u(-t) - 2e^{2t}u(-t) + 3\delta(t)$

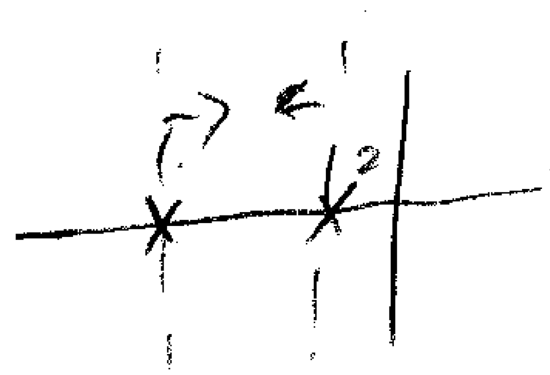
3. $e^{-3t}u(t) + 2e^{-2t}u(t) - 3e^{-t}u(-t)$



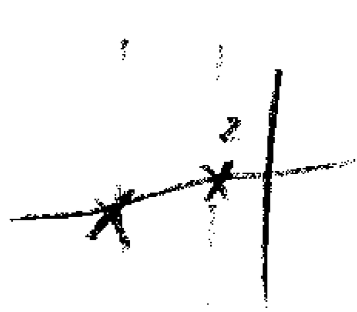
4. $e^{-3t}u(t) - 2e^{-2t}u(-t) - 3e^{-t}u(-t)$



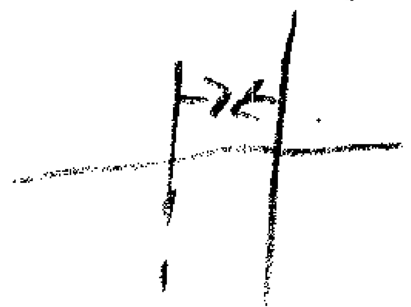
5. $3e^{-2t}u(t) - e^{-t}u(-t) - 2te^{-t}u(-t)$



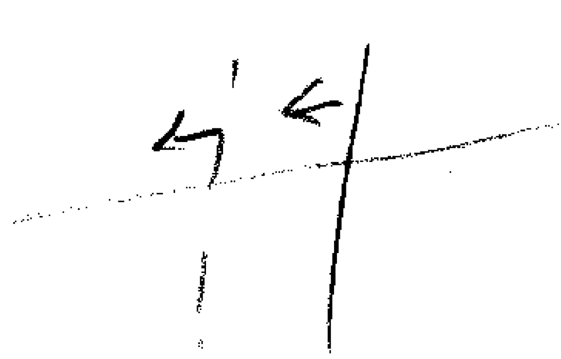
6. $-3e^{-2t}u(-t) - e^{-t}u(-t) - 2te^{-t}u(-t)$



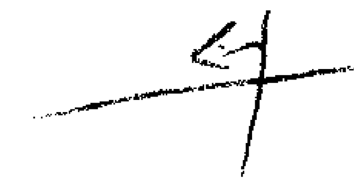
7. $-u(-t) - 2tu(-t) + 3e^{-t}u(t) + 4te^{-t}u(t)$



8. $-u(-t) - 2tu(-t) - 3e^{-t}u(-t) - 4te^{-t}u(-t)$



9. $-\cos(2t)u(-t) - \sin(3t)u(-t)$



1. $$G(j\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-T)\, e^{-j\omega t}\, dt$$

$$= e^{-j\omega T} \quad \text{(Using the "sifting property")}$$

$\therefore$,

$$\boxed{G(j\omega) = e^{-j\omega T}}$$

2. $$G(j\omega) = \int_{-T}^{T} 1 \cdot e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega}\, e^{-j\omega t} \Big|_{t=-T}^{T}$$

$$= \frac{-1}{j\omega}\left[e^{-j\omega T} - e^{+j\omega T}\right]$$

$$= \frac{1}{j\omega}\left[e^{+j\omega T} - e^{-j\omega T}\right]$$

$G(j\omega)$ can be simplified by application of Euler's formula, or by inspection. The result is

$$\boxed{G(j\omega) = \frac{2}{\omega} \sin \omega T}$$

3. $G(j\omega) = \int_{-\infty}^{\infty} \frac{1}{t^2+T^2} e^{-j\omega t} dt$

But, I don't know how to do this integral.
Use duality:

If $\mathcal{F}[g(t)] = f(\omega)$, then

$$\mathcal{F}[f(t)] = 2\pi g(-\omega)$$

$g(-\omega)$ is given by

$$g(-\omega) = \frac{1}{(-\omega)^2 + T^2} = \frac{1}{\omega^2+T^2}$$

$$= \frac{1}{-s^2+T^2} = \frac{-1}{(s+T)(s-T)}$$

$$= \frac{1/2T}{s+T} - \frac{1/2T}{s-T}$$

$$= \frac{1}{2T}\left[\frac{1}{j\omega+T} - \frac{1}{j\omega-T}\right]$$

Therefore,

$$f(t) = 2\pi \mathcal{F}^{-1}\left[g(-\omega)\right]$$

$$= 2\pi \frac{1}{2T}\left[e^{-tT}\sigma(t) + e^{+tT}\sigma(-t)\right]$$

$$= \frac{\pi}{T} e^{-|t|T}$$

$\therefore$

$$G(j\omega) = f(\omega) = \frac{\pi}{T} e^{-|\omega| T}$$

4. $$g(t) = \frac{\sin \pi t/T}{\pi t/T}$$

Use duality:

$$\mathcal{F}[g(t)] = G(j\omega) = f(\omega)$$

$$\mathcal{F}[f(t)] = 2\pi\, g(-\omega)$$

In this case,

$$g(-\omega) = \frac{\sin(-\pi\omega/T)}{-\pi\omega/T} = \frac{\sin \pi\omega/T}{\pi\omega/T}$$

If we let $T' = \pi/T$, this becomes

$$g(-\omega) = \frac{\sin \omega T'}{\omega T'}$$

The inverse FT (From part 1) is

$$\mathcal{F}^{-1}[g(-\omega)] = \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega T'}\right)$$

$$= \frac{1}{2T'} \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega/2}\right)$$

$$= \begin{cases} 1/2T', & |t| \leq T' \\ 0, & \text{else} \end{cases}$$

$$= f(t)/2\pi$$

Therefore,

$$f(t) = \begin{cases} \pi/T', & |t| \leq T' \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow \quad G(j\omega) = f(\omega)$$

$$= \begin{cases} \pi/T', & |\omega| \leq T' \\ 0, & \text{else} \end{cases}$$

$$= \begin{cases} T, & |\omega| \leq \pi/T \\ 0, & \text{else} \end{cases}$$

5. Let $F(j\omega) = \dfrac{\sin \omega T}{\omega T}$

Then $G(j\omega) = [F(j\omega)]^2$

$\Rightarrow \quad g(t) = f(t) * f(t)$ (convolution property)

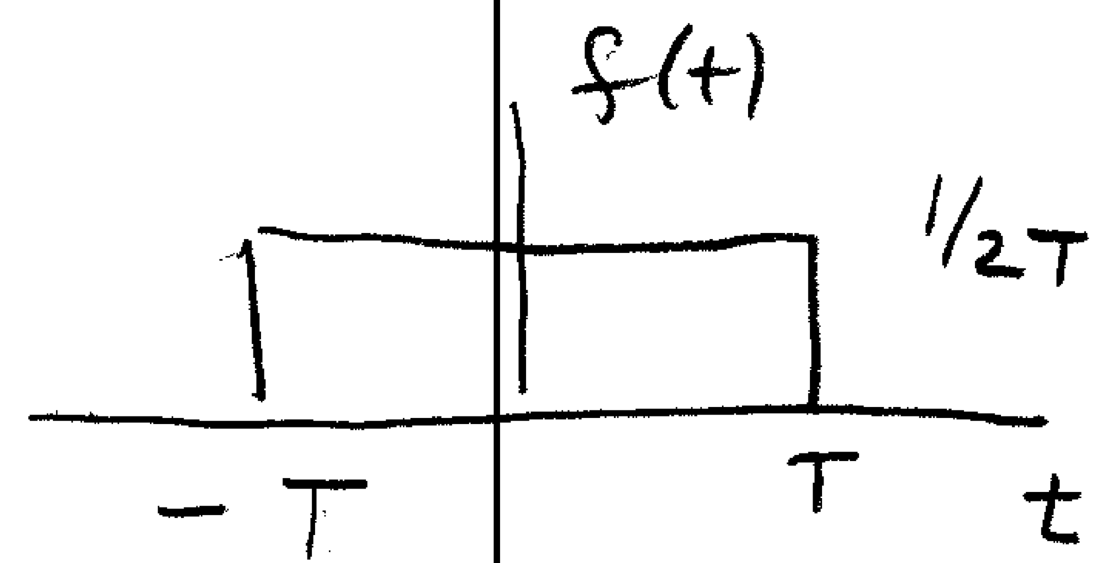
Using the results of part (1),

$$f(t) = \mathcal{F}^{-1}\left[\frac{\sin \omega T}{\omega T}\right]$$

$$= \frac{1}{2T}\,\mathcal{F}^{-1}\left[\frac{\sin \omega T}{\omega/2}\right]$$

$$= \begin{cases} 1/2T, & |t| \le T \\ 0, & \text{else} \end{cases}$$

=



g(t) is the convolution of f(t) with f(t), which is