Unified Quiz 2F

March 5, 2004

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don’t just write equations.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Calculators and a 2-sided sheet of paper are allowed.
- Box your final answers.

Exam Scoring

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1. (30 %) A thin airfoil of chord $c$ is rotating with steady rate $\omega$ so that its angle of attack is increasing in time.

$$\alpha = \omega t$$

As a result, the airfoil trails a vortex sheet of constant strength $\gamma$. The rotation rate $\omega$ is slow enough so that the instantaneous flow and lift very nearly correspond to the instantaneous $\alpha$. Both $\Gamma$ and $\gamma$ are defined positive in the clockwise direction.

![Diagram](image)

a) Determine the circulation $\Gamma(t)$ about the smaller dotted-line circuit containing just the airfoil.

b) The larger solid-line circuit contains both the airfoil and some part of the wake. Apply Kelvin’s theorem to this circuit at times $t$ and $t + \Delta t$, and thus determine the magnitude and sign of $\gamma$.

c) The flow is inviscid. What drag force $D'$ do you expect?

i) $D' < 0$

ii) $D' = 0$

iii) $D' > 0$

Explain your reasoning.
2. (35 %) A sail-like airfoil of chord $c$ consists of a membrane stretched between two thin rods at the leading and trailing edges. The membrane billows to a parabolic camberline shape whose height $h$ is proportional to the lift per span

$$h = L'/K$$

where $K$ is some effective stiffness of the membrane.

![Diagram of airfoil](image)

a) Use thin airfoil theory results to explicitly determine $L'$ in terms of a given $\alpha$.

*Note: Starting from known results is OK – no need to derive from scratch.*

Also determine the effective lift slope $d\ell / d\alpha$ of this airfoil.

b) What is the maximum safe operating dynamic pressure $\frac{1}{2} \rho V^2$ for this airfoil? What do you expect to happen if this is exceeded?
3. (35 %) An elliptic-planform wing with span $b$ and chord $c(y) = c_o \sqrt{1 - (2y/b)}$ is in slow steady rolling flight at roll rate $p$ and velocity $V_\infty$. The wing has no geometric twist or camber,

$$\alpha_{\text{geom}} = 0 \quad \alpha_{L=0} = 0$$

and the center chord line is lined up with the velocity vector (i.e. $\alpha = 0$).

a) Draw a velocity triangle seen by the wing airfoil at typical spanwise station $y$ and determine the local $c_\ell$. Use small-angle approximations.

b) The circulation distribution for this wing is known to be

$$\frac{1}{2} V_\infty c c_\ell \equiv \Gamma = 2bV_\infty A_2 \sin 2\theta$$

Combine this with your $c_\ell$ result from a), and determine the constant $A_2$ in terms of the known parameters.

c) In which direction is the rolling moment?

The following identities may be useful:

$$\sin 2\theta = 2 \sin \theta \cos \theta$$
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$
Problem #3 (continued)