Unified Quiz 2F

March 5, 2004

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don't just write equations.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Calculators and a 2-sided sheet of paper are allowed
- Box your final answers.

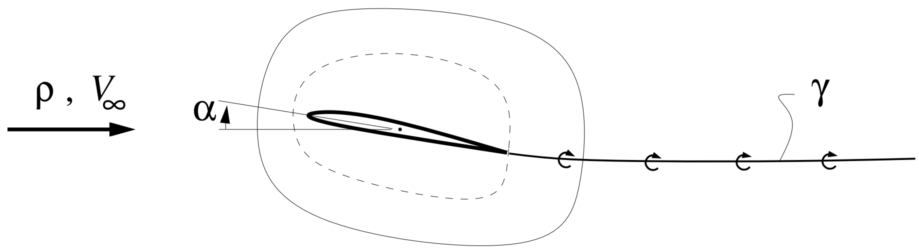
Exam Scoring

#1 (30 %)	
#2 (35%)	
#3 (35%)	
Total	

1. (30 %) A thin airfoil of chord c is rotating with steady rate ω so that its angle of attack is increasing in time.

$$\alpha = \omega t$$

As a result, the airfoil trails a vortex sheet of constant strength γ. The rotation rate ω is slow enough so that the instantaneous flow and lift very nearly correspond to the instantaneous α. Both Γ and γ are defined positive in the clockwise direction.



a) Determine the circulation $\Gamma(t)$ about the smaller dotted-line circuit containing just the airfoil.

b) The larger solid-line circuit contains both the airfoil and some part of the wake. Apply Kelvin's theorem to this circuit at times t and $t + \Delta t$, and thus determine the magnitude and sign of γ.

c) The flow is inviscid. What drag force D' do you expect?
i) $D' < 0$
ii) $D' = 0$
iii) $D' > 0$
Explain your reasoning.

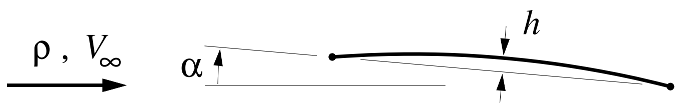
Name________________

Problem #1 (continued)

2. (35 %) A sail-like airfoil of chord c consists of a membrane stretched between two thin rods at the leading and trailing edges. The membrane billows to a parabolic camberline shape whose height h is proportional to the lift per span

$$h = L'/K$$

where K is some effective stiffness of the membrane.



a) Use thin airfoil theory results to explicitly determine L' in terms of a given α.
Note: Starting from known results is OK – no need to derive from scratch.
Also determine the effective lift slope $dc_\ell/d\alpha$ of this airfoil.

b) What is the maximum safe operating dynamic pressure $\frac{1}{2}\rho V_\infty^2$ for this airfoil? What do you expect to happen if this is exceeded?

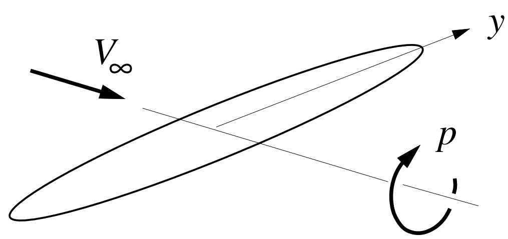
Name_________________

Problem #2 (continued)

3. (35 %) An elliptic-planform wing with span b and chord $c(y) = c_o\sqrt{1 - (2y/b)}$ is in slow steady rolling flight at roll rate p and velocity V_∞. The wing has no geometric twist or camber,

$$\alpha_{\text{geom}} = 0 \qquad\qquad \alpha_{L=0} = 0$$

and the center chord line is lined up with the velocity vector (i.e. $\alpha = 0$).



a) Draw a velocity triangle seen by the wing airfoil at typical spanwise station y and determine the local c_ℓ. Use small-angle approximations.

b) The circulation distribution for this wing is known to be

$$\frac{1}{2}V_\infty \, c \, c_\ell \;\equiv\; \Gamma \;=\; 2bV_\infty \, A_2 \, \sin 2\theta$$

Combine this with your c_ℓ result from a), and determine the constant A_2 in terms of the known parameters.

c) In which direction is the rolling moment?

The following identities may be useful:

$$\begin{aligned} \sin 2\theta &= 2 \sin\theta \cos\theta \\ \cos 2\theta &= \cos^2\theta \,-\, \sin^2\theta \end{aligned}$$

Name________________

Problem #3 (continued)