UE Fluids | Quiz 1 Solution

1. a) $C_\ell = 2\pi(\alpha(t) - \alpha_{L=0})$ instantaneous lift corresponds to instantaneous α, as given

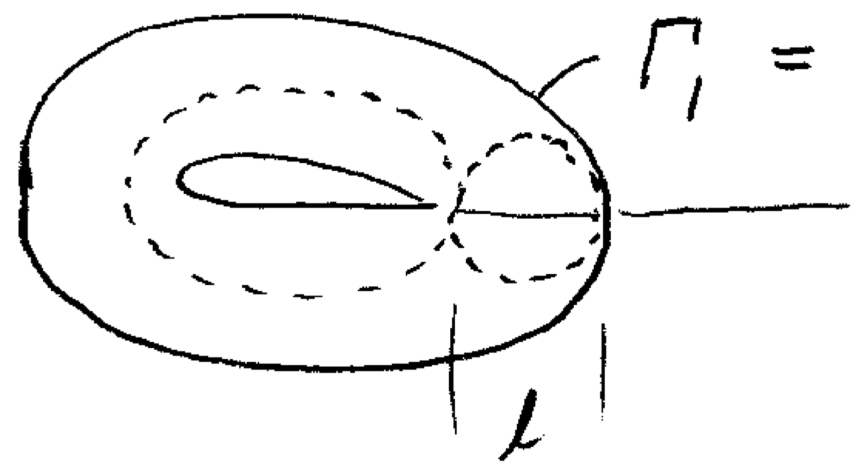
$$\Gamma = \frac{1}{2} V_\infty c \, c_\ell$$

$$\therefore \quad \Gamma(t) = \frac{1}{2} V_\infty c \, 2\pi(\omega t - \alpha_{L=0}) \qquad (\alpha_{L=0} = 0 \text{ if airfoil is symmetric})$$

b) Let ℓ be the length of wake inside larger circuit at time t.

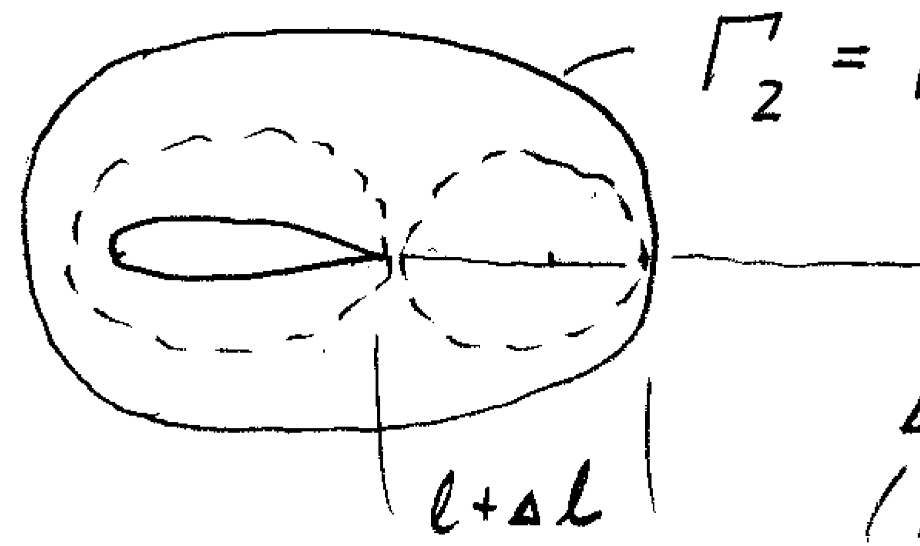
At time t:

$$\Gamma_1 = \Gamma(t) + \gamma \ell$$



At time $t + \Delta t$:
(circuit moves with flow)

$$\Gamma_2 = \Gamma(t+\Delta t) + \gamma \cdot [\ell + \Delta\ell]$$



$\Delta\ell = V_\infty \Delta t$
(moves with flow)

Since circuit is defined to move with flow, Kelvin's Thm applies:

$$\Gamma_1 = \Gamma_2$$

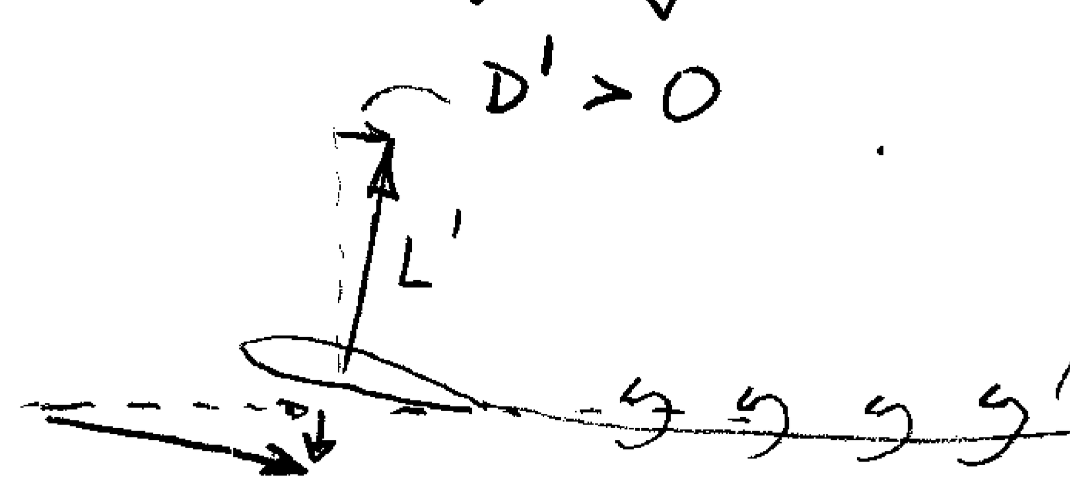
$$\frac{1}{2} V_\infty c \, 2\pi(\omega t - \alpha_{L=0}) + \gamma \ell = \frac{1}{2} V_\infty c \, 2\pi(\omega \cdot (t + \Delta t) - \alpha_{L=0}) + \gamma[\ell + V \Delta t]$$

after cancelling left & right terms:

$$0 = \frac{1}{2} V c \, 2\pi \, \omega \Delta t + \gamma V \Delta t$$

$$\boxed{\gamma = -\pi c \omega}$$

c) Vortex sheet causes downwash at airfoil.
Lift vector will tilt aft, giving $D' > 0$



2. a) T.A.T. Result: $c_\ell = 2\pi(\alpha + 2\epsilon)$, $\epsilon = \frac{h}{c}$

Here, $h = L'/K$ or $\epsilon = \frac{L'}{cK}$

$$c_\ell = 2\pi\left(\alpha + 2\frac{L'}{cK}\right)$$

But $L' = \frac{1}{2}\rho V_\infty^2 c\, c_\ell$

$$\text{so} \quad L' = \overbrace{\tfrac{1}{2}\rho V_\infty^2}^{\equiv q_\infty} c\, 2\pi\left(\alpha + 2\frac{L'}{cK}\right)$$

$$L'\left(1 - 4\pi\frac{q_\infty}{K}\right) = q_\infty c\, 2\pi\alpha$$

$$\boxed{L' = q_\infty c \frac{2\pi}{1 - 4\pi\frac{q_\infty}{K}}\,\alpha}$$

$$c_\ell = \frac{L'}{q_\infty c} = \frac{2\pi}{1 - 4\pi\frac{q_\infty}{K}}\,\alpha$$

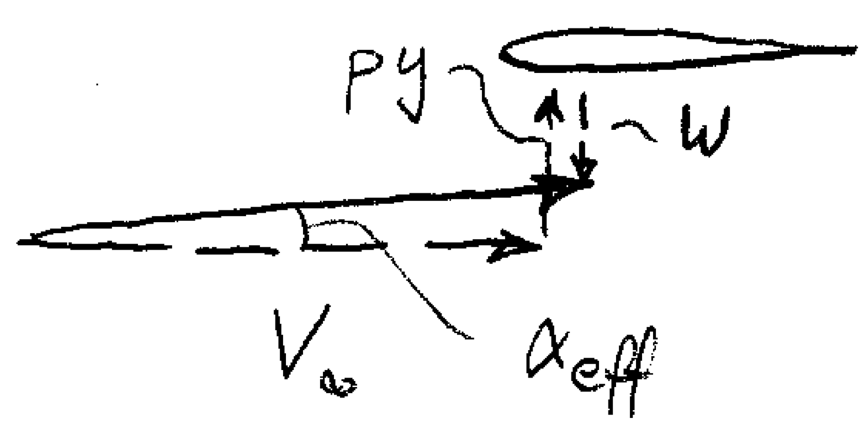
$$\boxed{\frac{dc_\ell}{d\alpha} = \frac{2\pi}{1 - 4\pi\frac{q_\infty}{K}}}$$

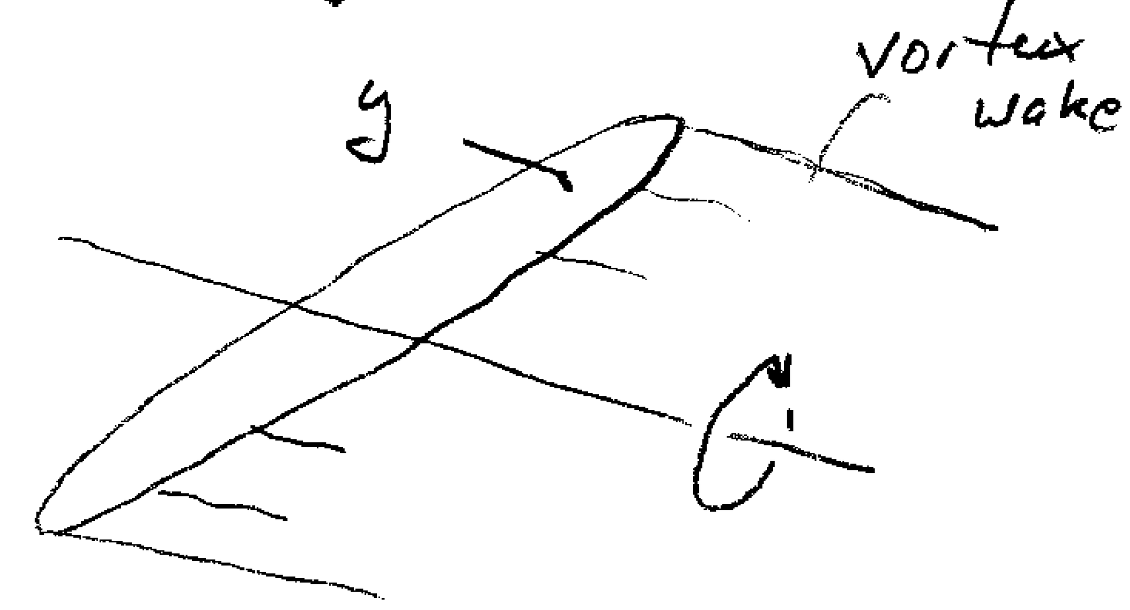
b) $\frac{dc_\ell}{d\alpha}$ tends to ∞ as $4\pi\frac{q_\infty}{K} \to 1$ or $q_\infty \to \frac{K}{4\pi}$

Airfoil billows out without limit

if q_∞ exceeds $\boxed{\frac{K}{4\pi}}$

3. a) Velocities seen by airfoil at location y:





$$\alpha_{eff} = \frac{py}{V_\infty} - \alpha_i \quad , \quad \alpha_i = \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin\theta}$$

For this case, only A_2 is nonzero: $\alpha_i = 2A_2 \dfrac{\sin 2\theta}{\sin\theta}$

$$\text{or} \quad \alpha_i = 2A_2 \frac{2\cos\theta \sin\theta}{\sin\theta} = 4A_2 \cos\theta$$

$$C_\ell = 2\pi\,\alpha_{eff} = 2\pi\left[\frac{py}{V_\infty} - 4A_2\cos\theta\right] = 2\pi\left[\frac{pb}{2V_\infty} - 4A_2\right]\cos\theta$$

since $y = \frac{b}{2}\cos\theta$

b) $\Gamma = A_2 \sin 2\theta = \frac{1}{2} V_\infty c\, C_\ell$

$$2bV_\infty A_2\, 2\sin\theta\cos\theta = \frac{1}{2} V_\infty c_0 \sin\theta \cdot 2\pi\left[\frac{pb}{2V_\infty} - 4A_2\right]\cos\theta$$

$$4bV_\infty A_2 = \pi V_\infty c_0\left[\frac{pb}{2V_\infty} - 4A_2\right]$$

$$\left[4bV_\infty + \pi V_\infty c_0 \cdot 4\right] A_2 = \pi V_\infty c_0 \frac{pb}{2V_\infty}$$

$$A_2 = \frac{\pi c_0 pb/2}{\left[4b + 4\pi c_0\right] V_\infty}$$

c) Moment is opposite to roll rate