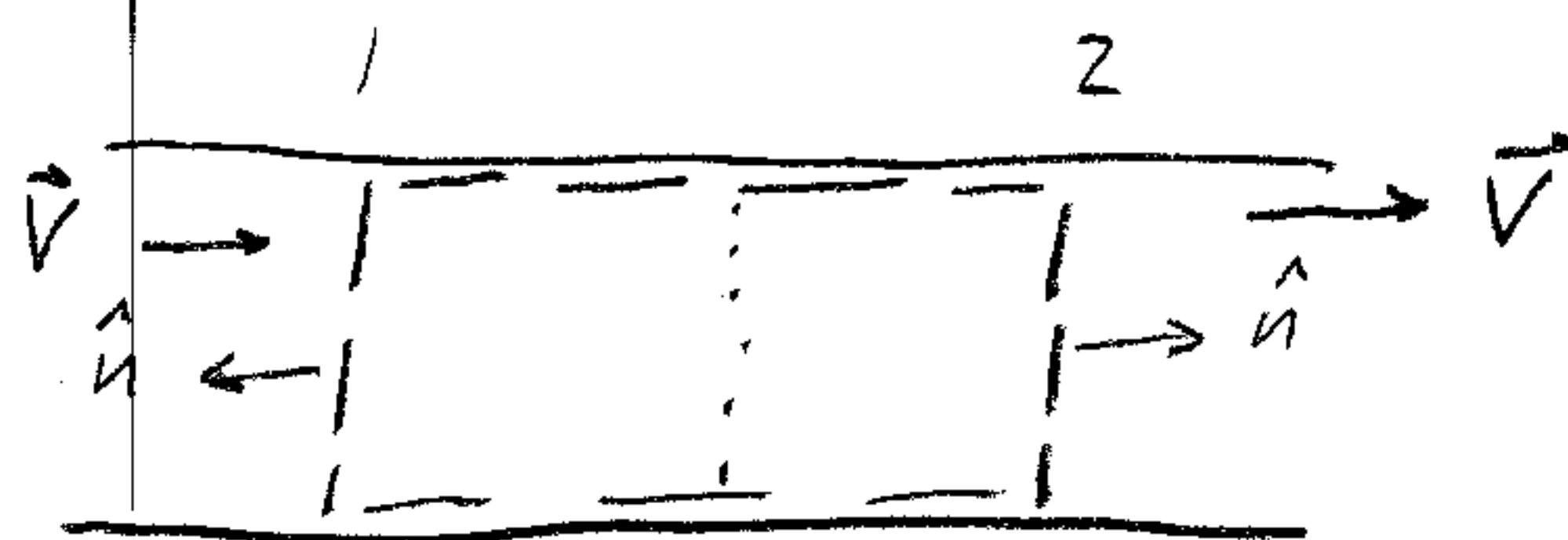


1.



$$a) \oint \rho \hat{V} \cdot n h_0 dA = \iiint \rho g dV$$

$$\rho_2 V_2 A h_{02} - \rho_1 V_1 A h_{01} = \dot{Q}$$

$$\text{or } h_{02} - h_{01} = \frac{\dot{Q}}{\dot{m}}$$

$$\text{or } h_2 + \frac{1}{2} V_2^2 - h_1 - \frac{1}{2} V_1^2 = \frac{\dot{Q}}{\dot{m}}$$

$$b) \quad h_2 - h_1 \approx \frac{\dot{Q}}{\dot{m}} \quad \text{neglect } V^2, \quad h = c_p T$$

$$T_2 - T_1 = \frac{1}{c_p} \frac{\dot{Q}}{\dot{m}}, \quad \dot{m} = \rho_1 V_1 A = 1 \cdot 1 \cdot 0.1 = 0.1 \text{ kg/s}$$

$$\boxed{T_2 = T_1 + \frac{1}{c_p} \frac{\dot{Q}}{\dot{m}} = 250 \text{ K} + \frac{1}{1000} \frac{5000}{0.1} = 300 \text{ K}}$$

$$c) \left[ \frac{\rho_2}{\rho_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_1}{T_2} \approx \frac{T_1}{T_2} = \frac{250}{300} = \frac{5}{6} = 0.8333 \right]$$

$$\boxed{\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{6}{5} = 1.2}$$

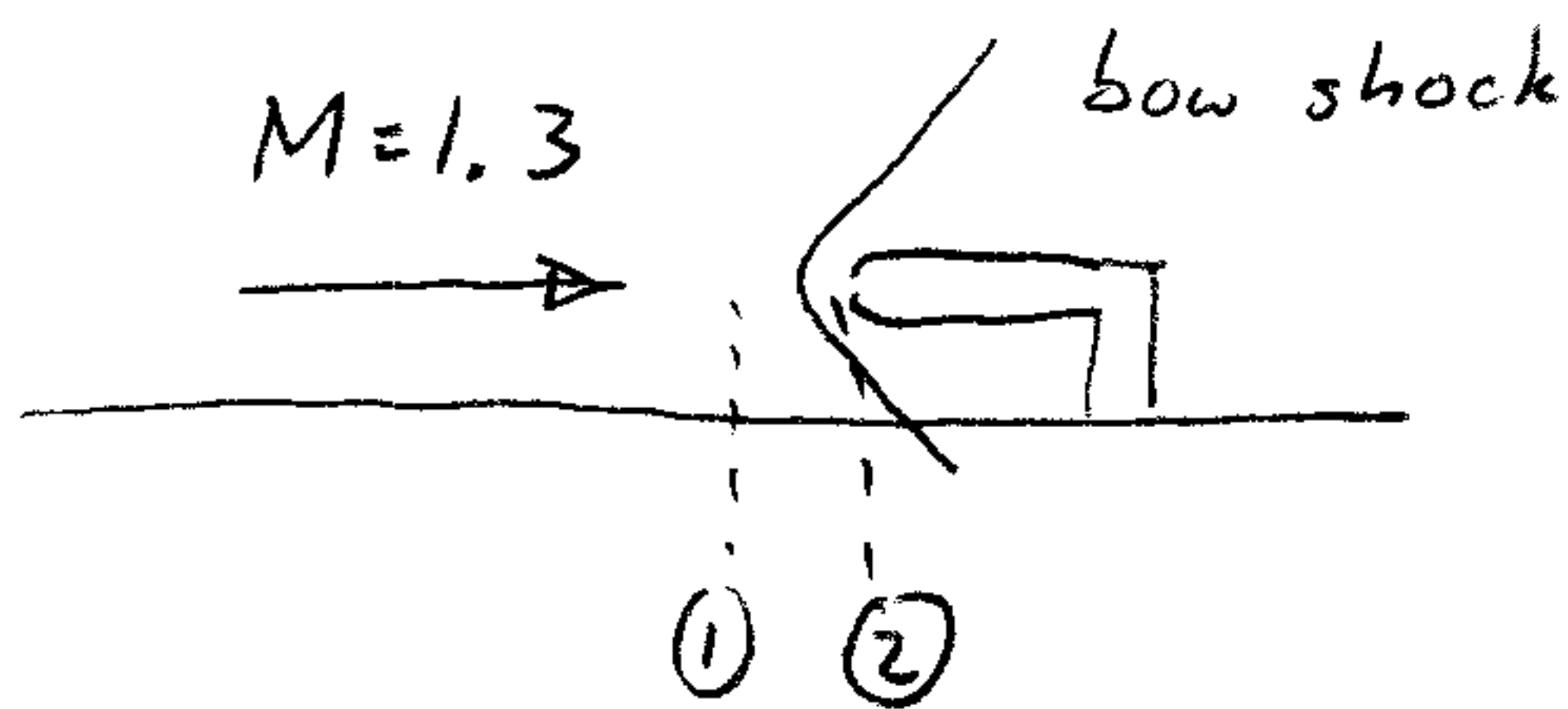
$$d) \text{ Momentum eqn: } p_2 + \rho_2 V_2^2 - p_1 - \rho_1 V_1^2 = 0$$

$$p_2 - p_1 = \rho_1 V_1^2 - \rho_2 V_2^2 = \rho_1 V_1 (V_1 - V_2) \quad \text{since } \rho_1 V_1 = \rho_2 V_2$$

$$\boxed{p_2 - p_1 = 1 \cdot 1 \cdot (1 - 1.2) = -0.2 \text{ Pa}}$$

2.

a)



Pitot senses total pressure  $P_{02}$  behind bow shock.

From normal-shock table: for  $M=1.3$ ,  $P_{02}/P_1 = 2.714$

since  $P_1 = P_{\infty}$ ,  $P_{02} = 2.714 P_{\infty}$

Can also use table's  $P_{02}/P_{01} = 0.9794$

since  $P_{01} = P_{\infty} \left[ 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right]^{\frac{\gamma}{\gamma-1}} = 2.771 P_{\infty}$

$\rightarrow P_{02} = 2.771 P_{\infty} \cdot 0.9794 = 2.714$  same result

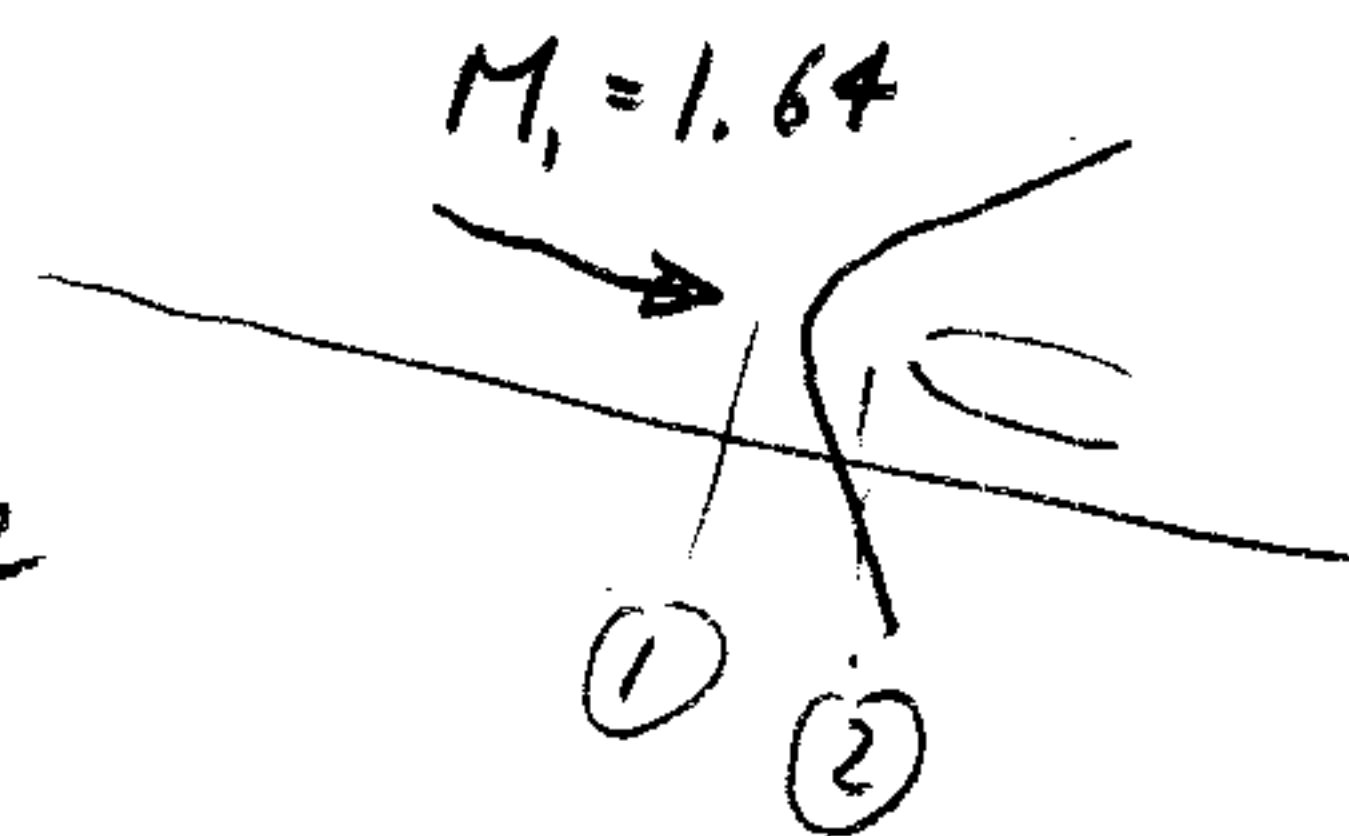
b) Now  $P_1$  is behind expansion fan, with  $\theta = 10^\circ$

$\nu(M_1) - \nu(M_{\infty}) = 10^\circ$ ,  $\nu(M_{\infty}) = \nu(1.3) = 6.17^\circ$

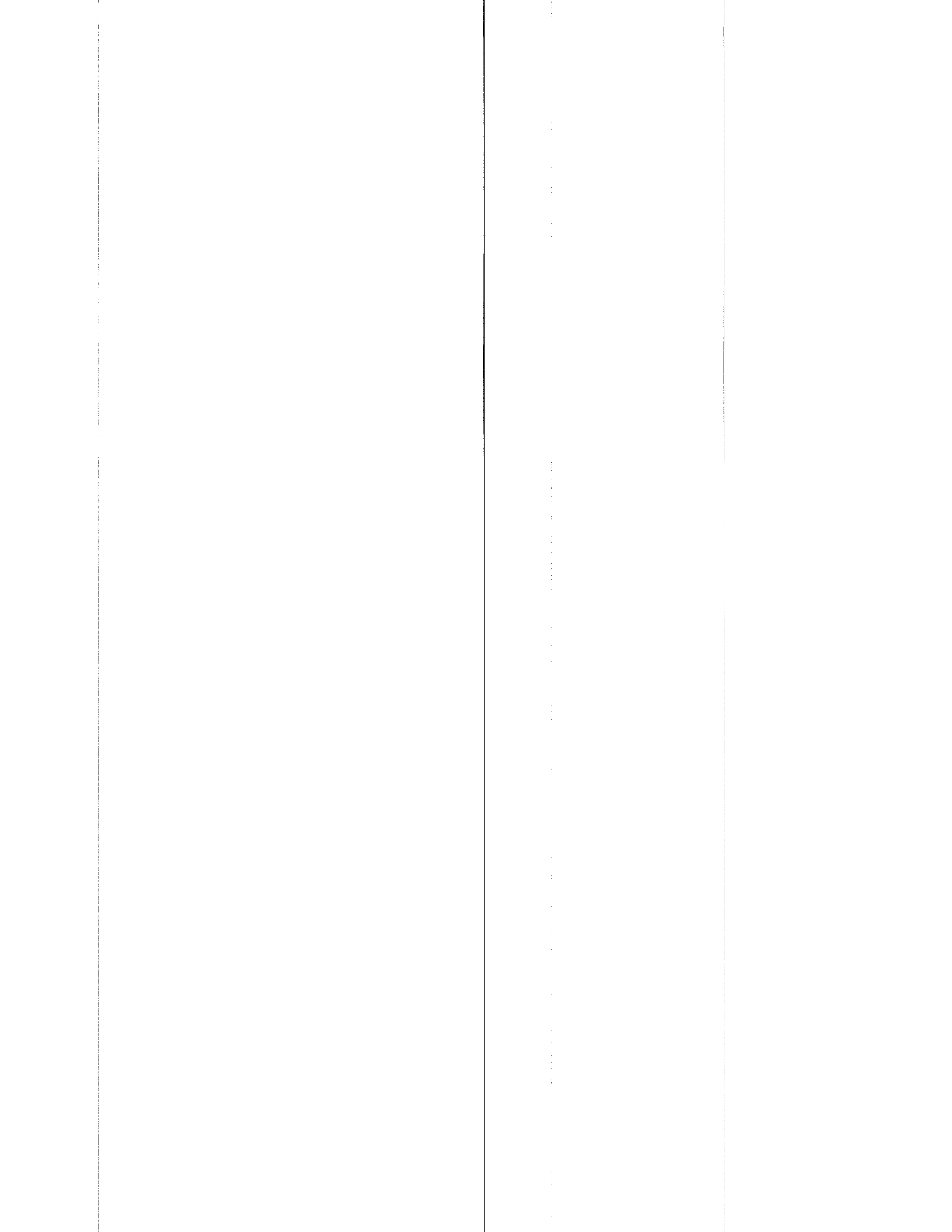
$\nu(M_1) = 16.17^\circ \rightarrow M_1 \approx 1.64$

Now,  $P_{02}/P_1 = 3.97$  from normal-shock table

$\frac{P_1}{P_{\infty}} = \frac{\left( \frac{P_{01}}{P_{\infty}} \right) \left[ 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right]^{\frac{\gamma}{\gamma-1}}}{\left[ 1 + \frac{\gamma-1}{2} M_1^2 \right]^{\frac{\gamma}{\gamma-1}}} = \frac{2.771}{4.511} = 0.614$



$P_{02} = 3.97 P_1 = 3.97 \cdot 0.614 P_{\infty} = 2.438 P_{\infty}$



3.

a)  $A^* = A_{t_1}$  since flow is choked.

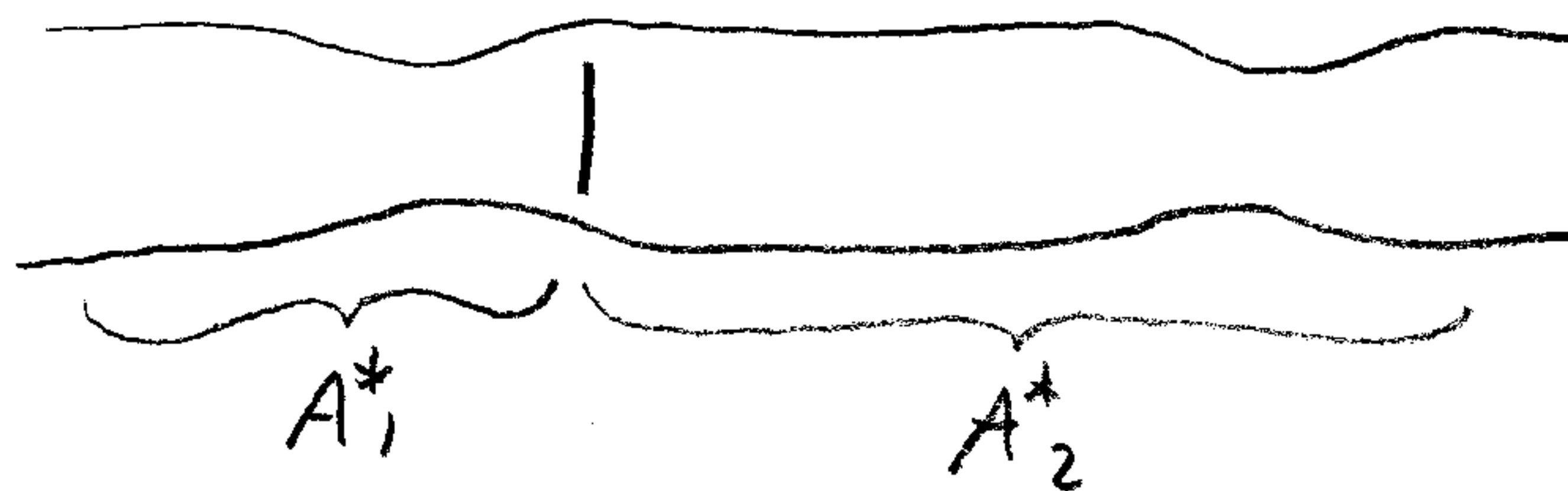
At shock location, we still have  $A^* = A_{t_1}$

from Isentropic Flow table, for  $M=1.5$  ...  $A/A^* = 1.176$

$$A = 1.176 A^* = 1.176 A_{t_1} = 1.176 \cdot 0.8 = 0.941$$

b) Behind the shock,  $h_{0_2} = h_{0_1}$ ,  $P_{0_2} = 0.9298 P_{0_1}$  (from Table)

Need to find  $A_2^*$



In general,  $\rho^* a^* A^* = \dot{m} = \text{constant}$

$$\text{so } \rho_1^* a_1^* A_1^* = \rho_2^* a_2^* A_2^* \rightarrow \frac{A_2^*}{A_1^*} = \frac{\rho_1^* a_1^*}{\rho_2^* a_2^*}$$

$$a^* = \sqrt{(\gamma-1) h_0} \left[ 1 + \frac{\gamma-1}{2} \right]^{-\frac{1}{2}} \quad \text{same for 1 and 2 since } h_{0_1} = h_{0_2}$$

$$\rho^* = \rho_0 \left[ 1 + \frac{\gamma-1}{2} \right]^{-\frac{1}{\gamma-1}}$$

$$\text{so } \frac{\rho_1^*}{\rho_2^*} = \frac{P_1^*}{P_2^*} \frac{h_2^*}{h_1^*} = \frac{P_1^*}{P_2^*} = \frac{P_{0_1}}{P_{0_2}} = \frac{1}{0.9298}$$

$$\frac{A_2^*}{A_1^*} = \frac{A_{t_2}}{A_{t_1}} = \frac{\rho_1^*}{\rho_2^*} = \frac{1}{0.9298} = 1.07$$

$$\boxed{A_{t_2} = 1.07 A_{t_1} = 1.07 \cdot 0.8 = 0.856 \text{ m}^2}$$