### Unified Engineering

#### Spring 2005

Problem Set #1

Due Date: Tuesday, February 8, 2005 at 5pm

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Name: ____________________
An aircraft wing is flying at a constant velocity $V$, with a gradually-increasing angle of attack, as might occur at the start of a high-g maneuver. Let's assume that this causes the wing airfoil's circulation to increase linearly in time,

$$\Gamma(t) = \Gamma_0 + \dot{\Gamma} t$$

with $\Gamma_0$ and $\dot{\Gamma}$ some constants.

a) Describe what happens in the flow behind the wing.
Hint: Consider a succession of snapshots at $t = 0, \Delta t, 2\Delta t, 3\Delta t, \ldots$, and apply Kelvin's Theorem to any two successive snapshots.

b) In reality, the flow changes continuously rather than in the discrete jumps assumed in a). In this case, how would you describe the line streaming from the trailing edge? Describe it quantitatively if possible.
a) Use Xfoil to compute the “exact” inviscid \( c_\ell(\alpha) \) and \( c_{m_{c/4}}(\alpha) \) curves for the following airfoils, over the range \( \alpha = -3^\circ \ldots 10^\circ \):
1. NACA 2520
2. NACA 2510
3. NACA 2502
Plot all three \( c_\ell \) curves superimposed, and the three \( c_{m_{c/4}} \) curves superimposed. Note that these all have the same camberlines. Comment on the accuracy of the following Thin Airfoil Theory predictions:

\[
\frac{dc_\ell}{d\alpha} = 2\pi, \quad c_{m_{c/4}} = \text{const.} \quad \text{for all airfoils.}
\]

b) The pitching moment about some \( x \) location is given by:

\[
c_m_x = c_{m_{c/4}} + \left( \frac{x}{c} - \frac{1}{4} \right) c_\ell
\]

The Aerodynamic Center, or \( x_{ac} \), is defined as the location about which the pitching moment doesn’t change with angle of attack:

\[
\frac{dc_m_x}{d\alpha} = 0 \quad \text{at} \quad x = x_{ac}
\]

Using the \( c_{m_{c/4}} \) output of Xfoil, determine \( x_{ac} \) for the NACA 2520,2510 airfoils. Compare with the \( x_{ac} \) predicted by Thin Airfoil Theory.
This problem serves as a mechanism to review the key concepts and governing equations in general elasticity.

(a) Write out fully, in tensor notation, the governing independent equations of elasticity for a solid body. Identify the key assumptions associated with each set of these equations and the underlying fundamental(s) upon which these are based.

(b) There are also several other equations, known as “compatibility equations”. Describe what they are and from where they come. Why aren’t these also independent equations?

(c) If engineering notation is used, do these equations change? How? Indicate this either through careful and complete description or by writing out the equations that change. In either case, highlight any key differences.