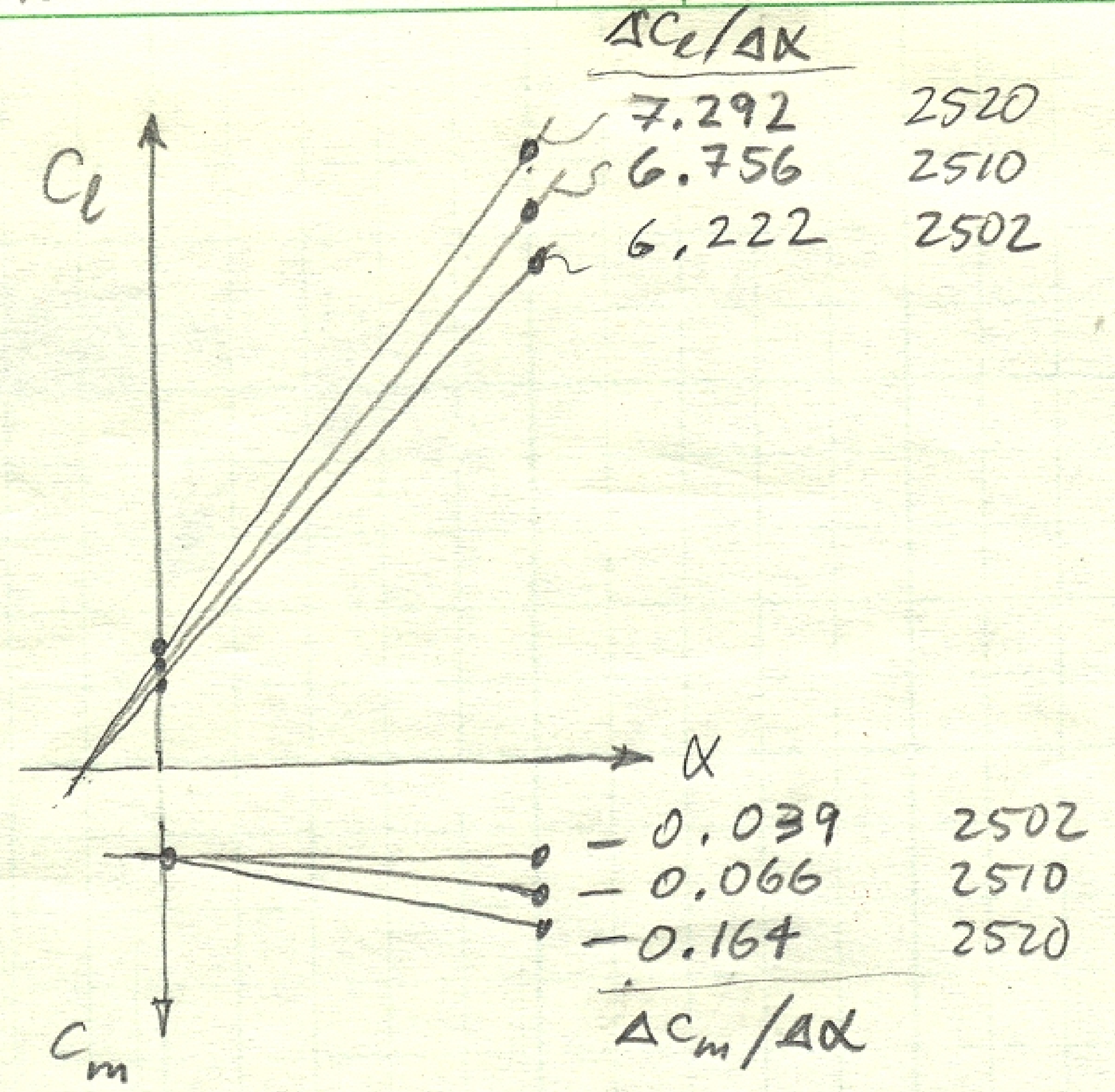
**Massachusetts Institute of Technology**
**Department of Aeronautics and Astronautics**
**Cambridge, MA 02139**

# Unified Engineering
# Spring 2005

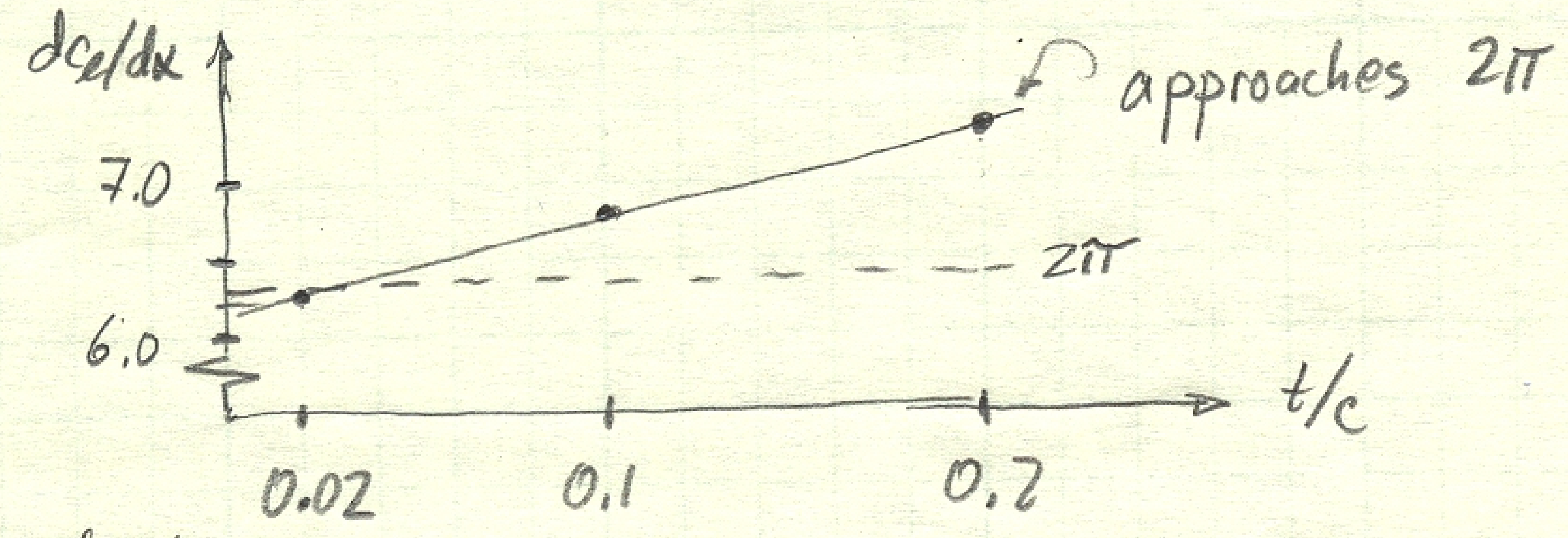
## Problem Set #1
## Solutions

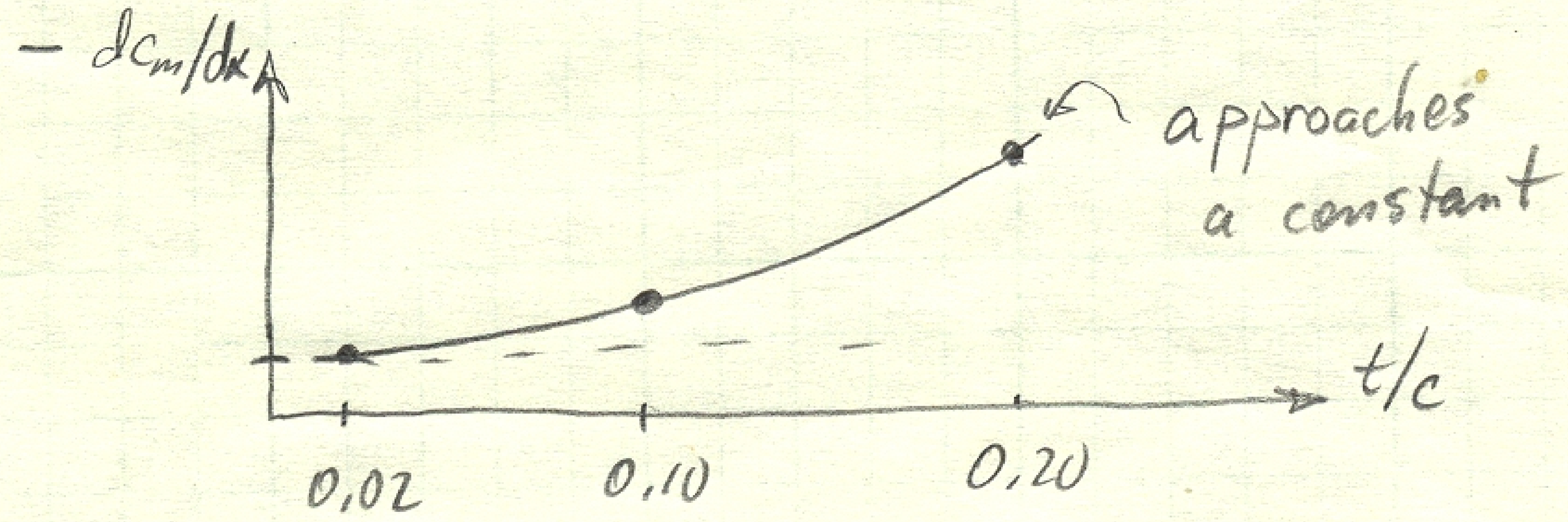
a) XFOIL results

| | $dc_l/d\alpha$ | $dc_m/d\alpha$ |
|---|---|---|
| 2520 | 7.292 | −0.164 |
| 2510 | 6.756 | −0.066 |
| 2502 | 6.222 | −0.039 |
| T.A.T. | 6.283 | const. |




Plotting vs. thickness:




T.A.T. assumes small t/c (thickness/chord).

XFOIL results approach T.A.T.'s results as $t/c \to 0$

consistent

$$c_{m_x} = c_m + \left(\frac{x}{c} - \frac{1}{4}\right) c_l \quad , \quad \text{find } x \text{ where } \frac{dc_{m_x}}{d\alpha} = 0$$

$$\rightarrow \frac{dc_{m_x}}{d\alpha} = \frac{dc_m}{d\alpha} + \left(\frac{x}{c} - \frac{1}{4}\right)\frac{dc_l}{d\alpha} = 0 \quad \rightarrow \quad \frac{x_{AC}}{c} = \frac{1}{4} - \frac{dc_m/d\alpha}{dc_l/d\alpha}$$

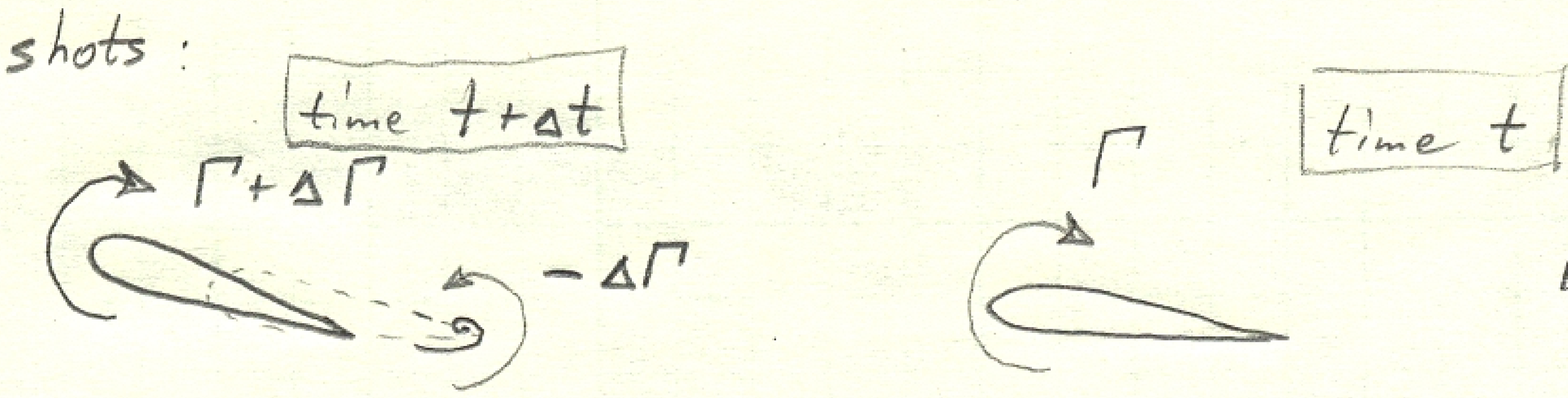
evaluate for the 3 airfoils.

| | $x_{AC}/c$ |
|---|---|
| 2520 | 0.272 |
| 2510 | 0.260 |
| 2502 | 0.256 |
| T.A.T. | 0.250 |

XFOIL's $\frac{x_{AC}}{c}$ approches 0.25, predicted by T.A.T

a) Two snapshots:

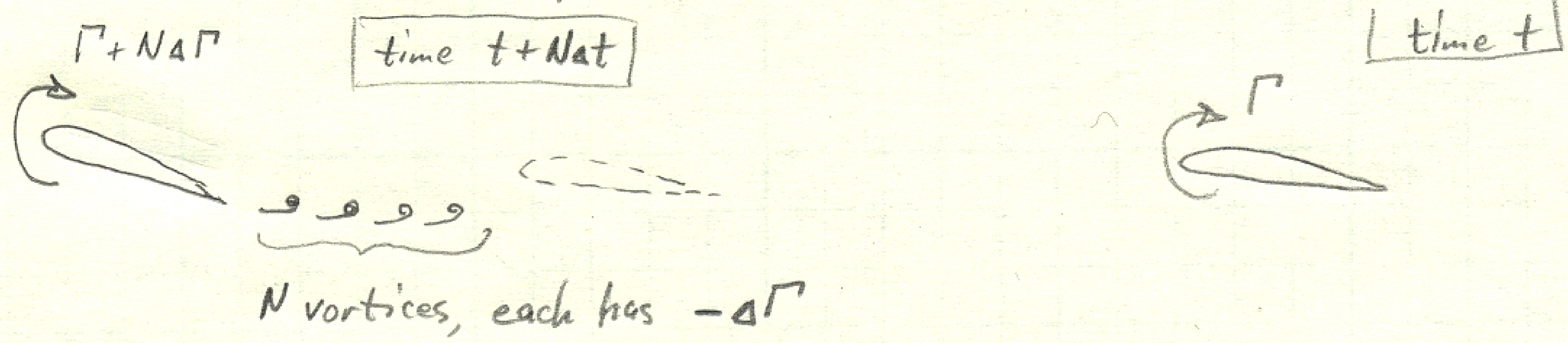
$\Delta\Gamma = \dot{\Gamma}\,\Delta t$



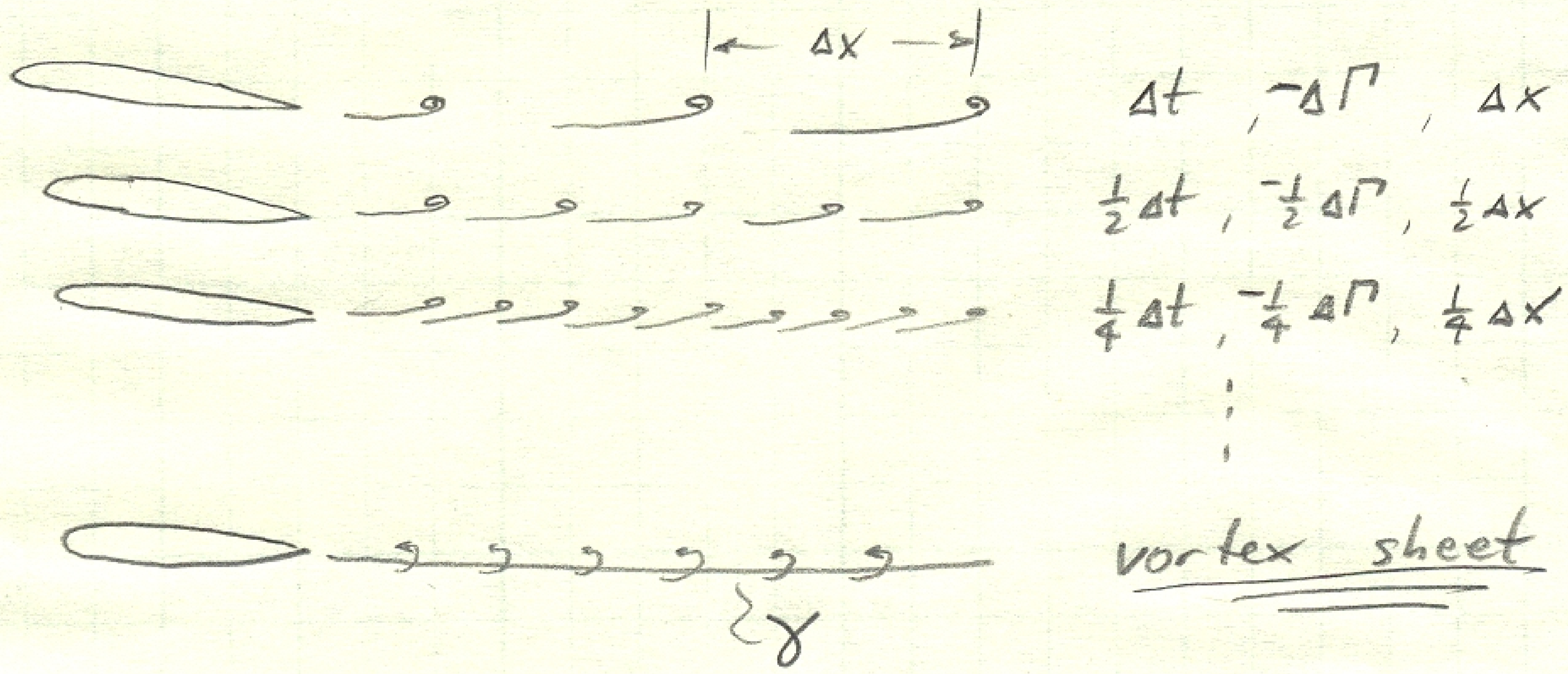

$\Gamma = \Gamma_0 + \dot{\Gamma}t$

Shed vortex of strength $-\Delta\Gamma = -(\dot{\Gamma}\Delta t)$ cancels $\Delta\Gamma$ of airfoil so that overall $\Gamma$ does not change (Kelvin's Theorem)

If we consider $N$ snapshots, we see:




b) If we make $\Delta t$ smaller and smaller, we get the continuous limit:




For given $\Delta t$, vortex spacing is $\Delta x = V\Delta t$ (distance traveled by airfoil)

$$\boxed{\gamma \equiv \lim_{\substack{\Delta x \to 0 \\ (\Delta t \to 0)}} \frac{\Gamma_{vortex}}{\Delta x} = \frac{-\Delta\Gamma}{\Delta x} = \frac{-\dot{\Gamma}\Delta t}{V\Delta t} = -\frac{\dot{\Gamma}}{V}}$$

sheet strength

FAL
2/5/05
①

# UNIFIED ENGINEERING

## Problem Set #1 -- SOLUTIONS

M1. (a) There are three key sets of equations:

**Equilibrium Equations** $\left( \frac{\partial \sigma_{mn}}{\partial x_n} + f_m = 0 \right)$
gives 3 equations

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + f_1 = 0$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + f_2 = 0$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + f_3 = 0$$

These are based on the fundamental of $\boxed{\text{equilibrium}}$

**Strain-displacement** $\epsilon_{mn} = \frac{1}{2} \left( \frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m} \right)$

gives 6 equations

$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

$$\epsilon_{33} = \frac{\partial u_3}{\partial x_3}$$

$$\epsilon_{21} = \epsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\epsilon_{31} = \epsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$$

$$\epsilon_{32} = \epsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$$

These are based on <u>geometrical</u> <u>relationships</u> and have the key assumption that strains are small such that angular changes are small. (This can be measured as $\cos\theta \approx 1$; $\sin\theta \approx \theta$)

<u>Stress-Strain</u> $\sigma_{mn} = E_{mnpq}\,\epsilon_{pq}$

gives 6 equations

$$\sigma_{11} = E_{1111}\epsilon_{11} + E_{1122}\epsilon_{22} + E_{1133}\epsilon_{33} + 2E_{1123}\epsilon_{23} + 2E_{1113}\epsilon_{13} + 2E_{1112}\epsilon_{12}$$

$$\sigma_{22} = E_{1122}\epsilon_{11} + E_{2222}\epsilon_{22} + E_{2233}\epsilon_{33} + 2E_{2223}\epsilon_{23} + 2E_{2213}\epsilon_{13} + 2E_{2212}\epsilon_{12}$$

$$\sigma_{33} = E_{1133}\epsilon_{11} + E_{2233}\epsilon_{22} + E_{3333}\epsilon_{33} + 2E_{3323}\epsilon_{23} + 2E_{3313}\epsilon_{13} + 2E_{3312}\epsilon_{12}$$

$$\sigma_{23} = E_{1123}\epsilon_{11} + E_{2223}\epsilon_{22} + E_{3323}\epsilon_{33} + 2E_{2323}\epsilon_{23} + 2E_{1323}\epsilon_{13} + 2E_{1223}\epsilon_{12}$$

$$\sigma_{13} = E_{1113}\epsilon_{11} + E_{2213}\epsilon_{22} + E_{3313}\epsilon_{33} + 2E_{2313}\epsilon_{23} + 2E_{1313}\epsilon_{13} + 2E_{1213}\epsilon_{12}$$

$$\sigma_{12} = E_{1112}\epsilon_{11} + E_{2212}\epsilon_{22} + E_{3312}\epsilon_{33} + 2E_{2312}\epsilon_{23} + 2E_{1312}\epsilon_{13} + 2E_{1212}\epsilon_{12}$$

This is based only on linear relationships between stress and strain (constitutive)

(b) These come from geometrical restrictions as manifested in the strain - displacement equations. Displacements must be continuous functions of $x_1$, $x_2$, and $x_3$, thus with three such functions, the six strains cannot be independent.

They relate the strain fields to be compatible with the continuity of the displacement.

They are derived by using the strain - displacement equations, taking "cross" derivatives and equating these.

They express geometrical restrictions

(c) In using engineering equations, the form of the equations change (e.g. $\sigma_x$ rather than $\sigma_{11}$), but the underlying fundamental and associated assumptions stay the same and the equations represent the same thing. Only the notation changes.

One key change due to definition is that engineering shear strain is 2x tensorial shear strain, so this factor of 2 must be incorporated in all equations with engineering shear strain.