Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

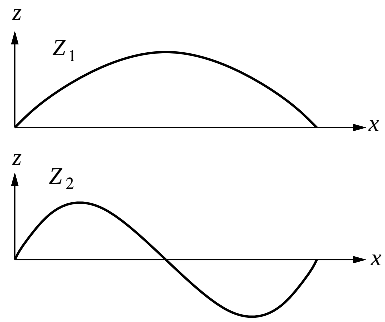
Unified Engineering
Spring 2005
Problem Set #2

Due Date: Tuesday, February 15, 2005 at 5pm

	Time Spent (minutes)
F4	
F5/F6	
M2	
M3	
M4	
M5	
Study Time	

Name: ________________

A camberline shape consists of a parabolic $Z_1(x)$ part plus a cubic $Z_2(x)$ part:



$$Z(x) = 4\varepsilon \left[Z_1(x) + r Z_2(x) \right]$$

$$Z_1(x) = x - \frac{x^2}{c}$$

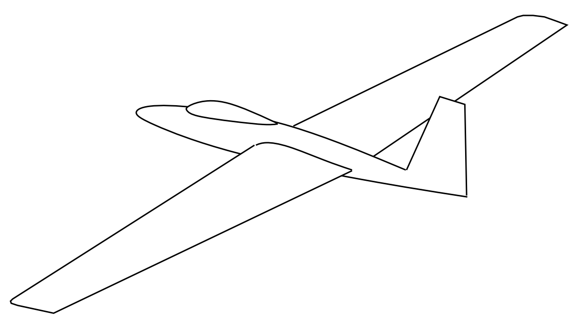
$$Z_2(x) = \frac{4}{3}x - 4\frac{x^2}{c} + \frac{8}{3}\frac{x^3}{c^2}$$

Note that the $Z_1(x)$ part is exactly what's used in the example in the Lecture 4 Notes. The constant "r" is a parameter controlling the amount of *reflex*, or upturning of the trailing edge, which is added by the $Z_2(x)$ part.

a) Determine dZ_2/dx, and verify that it can be written as

$$\frac{dZ_2}{dx} = \cos 2\theta + \frac{1}{3}$$

b) Perform a standard Thin Airfoil Theory Analysis: For some given angle of attack α and camber scaling factor ε, determine the Fourier coefficients $A_0, A_1, \ldots$ corresponding to the overall $Z(x)$ camber shape. Then determine the airfoil's c_ℓ and $c_{m,c/4}$.

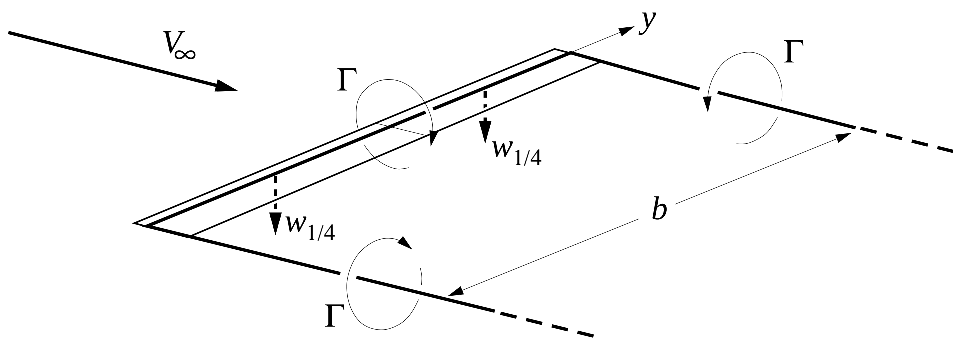


c) In order for a tailless aircraft to be both stable and trimmed in pitch, its wing airfoil must have a near-zero pitching moment, or $c_{m,c/4} \simeq 0$. Determine the value of r which achieves this objective. Will this zero pitching moment requirement hold for any reasonable flight condition?

d) Plot the resulting camberline shape $Z(x)$. Also overlay the shape corresponding to $r = 0$ for comparison. Pick an exaggerated $\varepsilon = 0.2$ so the shapes are easier to see.

Unified Engineering Spring 2005
Fluids Problem 5+6

Although the constant-circulation wing model is too crude for accurate induced drag predictions, it is still useful to get physical insight into the lifting-wing problem. Consider such a wing model, shown in the figure.



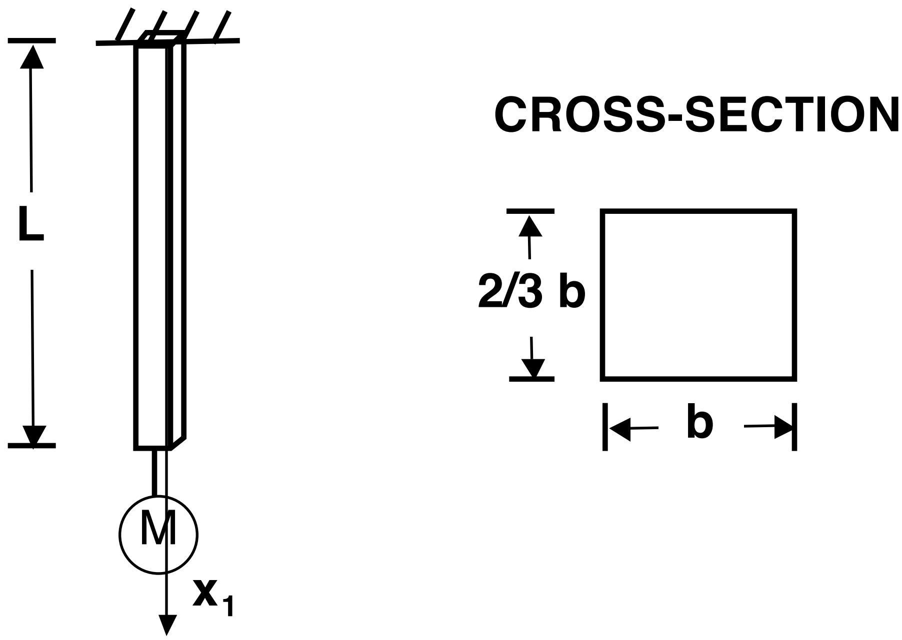
a) Determine the vertical downwash velocities $w_{1/4}$ of the two trailing semi-infinite vortices, at the $y = \pm b/4$ quarter-span locations shown on the figure.

b) Assume that the downwash is constant and equal to $w_{1/4}$ all across the span, i.e. $w(y) = w_{1/4}$. Construct the velocity diagram seen by a location on the wing, and determine the local force/span components L' and D_i'. Assume that $|w| \ll V_\infty$.

c) Determine the overall forces L and D_i on the wing.

d) Using your L result, eliminate Γ from your D_i result. Compare this D_i result to that predicted for an elliptically-loaded wing. How far off is it? Is your result at least qualitatively correct (i.e. can it predict trends)?

M2 A rod with a rectangular cross-section is suspended from an overhead support as indicated in the accompanying figure. The rod is of length L, width b, and the cross-section has an aspect ratio of 3/2. The rod is made of an isotropic material with a longitudinal modulus of E, Poisson's ratio of ν, shear modulus of G, and density of ρ. A large package of mass M hangs at the end of the rod. The entire arrangement is subjected to a gravity field of value g.



For the first part of this problem, ignore the effects of the mass of the rod and only consider the effects of the package hanging at the end.

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

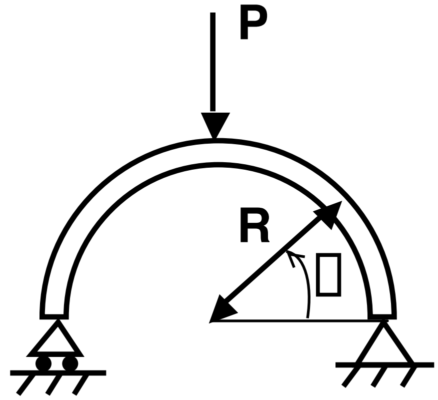
(e) If the cross-section of the rod varies with the aspect ratio changing such that the width has a value of b at the top of the rod and a value of 2/3 b (a square cross-section) at the bottom of the rod, can the rod model still be used? Why or why not? Be sure to explain clearly using equations if/as needed.

Now give consideration to the effects of the mass of the rod.

(f) Repeat parts (a) through (d) including the effects of the mass of the rod.

(g) Compare the solutions for the two cases (including and ignoring the mass of the rod) and make comments. When does the mass of the rod become important?

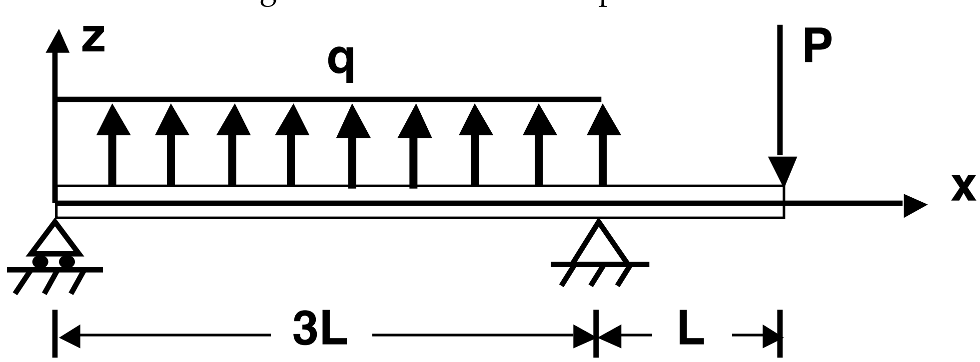
M3 A semi-circular arch is simply-supported and is loaded at its peak by a downward load of magnitude P. The arch has a radius of R.



(a) Determine the reactions for this structural configuration.

(b) Determine the axial force, shear force, and bending moment as functions of the arch angle □. Draw these as functions of □.

M4 A beam of total length 4L has a roller support at one end and is pinned at the three-quarters point (3L). The beam is loaded by a constant uniform upward loading of magnitude q between the two support points and by a concentrated downward load of magnitude P at the beam tip.

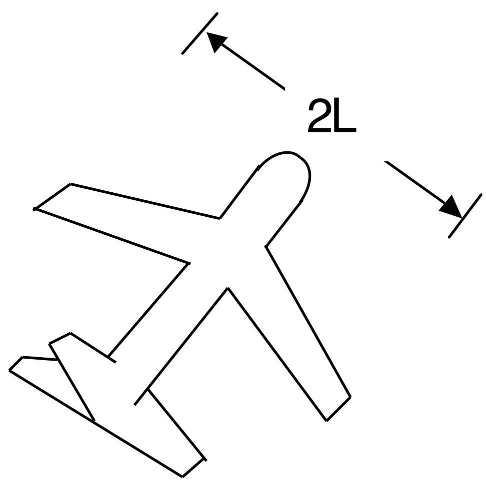


(a) Determine the reactions for this structural configurations.

(b) Using the relationships between loading, shear, and moment, determine the loading, shear, and moment diagrams.

(c) Check the obtained values for these parameters at the beam mid-span.

M5 Now that we have begun to learn about beams, we can start out on simple models of airplanes. Let's first explore how wings carry load in level flight. The wing of the airplane shown below can be modeled as a beam of total span 2L which has no supports. The beam has a concentrated load (the weight of the *fuselage*[1], its contents, and the *empennage*[2]) P at its center and a distributed load (the lift of the wing) along its span. You will learn from Fluids that this distribution is often modeled as varying, with different possible variations, along the span. For simplicity in an initial model, let us assume that we can model the distribution as being a distributed load of constant magnitude. We also ignore the weight of the wing at this point. The model is shown subsequently.



(a) Determine the reactions for this structural configuration.

(b) Determine the axial force, shear force, and bending moment as functions of the distance from the *root*[3] of the wing.

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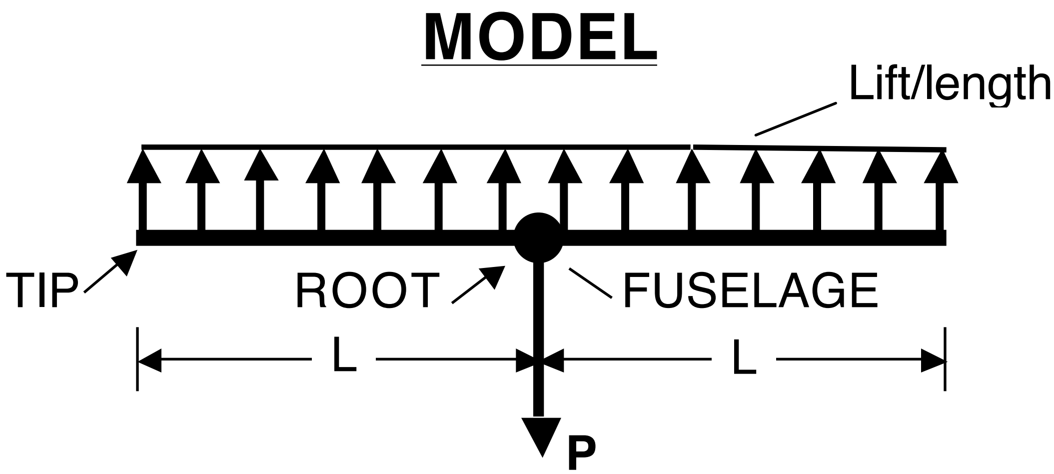
[1] The *fuselage* is that part of the airplane between the wings where the passengers and/or freight are carried.

[2] The *empennage* is more commonly known as the tail.

[3] The *root* of the wing is the location where the wing is joined to the fuselage.



(c) Using common sense arguments and the results from part (b), describe where it is likely that the wing is most highly loaded.

(for thought) We now increase the wingspan by 10% (i.e. by a factor of 1.1) while keeping the total lift and the center load P the same. What is the effect on the shear and bending moment at the root? (**NOTE:** You should be able to answer this by inspection.)