Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Spring 2005

Problem Set #3

Due Date: Tuesday, February 22, 2005 at 5pm

	Time Spent (minutes)
F7/F8	
F9/F10	
M6	
M7	
M8	
Study Time	

Name: ________________

You are to design candidate wings for a modified Dragonfly, to the following requirements:

density:	$\rho = 1.2\,\mathrm{kg/m^3}$
speed:	$V = 5\,\mathrm{m/s} = 11.2\,\mathrm{mph}$
lift:	$L = 5.0\,\mathrm{N} = 18\,\mathrm{oz}$
span:	$b = 1.8\,\mathrm{m} = 71\,\mathrm{in}$

Assume an elliptic load distribution $\Gamma(y) = \Gamma_0\sqrt{1-(2y/b)^2}$ for each of the design cases below. Assume $dc_\ell/d\alpha = 2\pi$ for the wing airfoils. Assume the wing reference line (e.g. fuselage axis) is aligned with the flight direction, so that $\alpha = 0°$.

Baseline elliptical wing: Assume a spanwise-constant $c_\ell = 1.0$.

1a) Determine the chord distribution $c(y)$ and sketch the planform.
1b) Determine $\alpha_{\mathrm{aero}}(y)$.
1c) What will $\alpha_{\mathrm{geom}}(y)$ look like if the same cambered airfoil is used all across the span?

Rectangular wing: Assume a spanwise constant chord c.

2a) Determine this c so that the overall lift coeffcient is $C_L = 1.0$.
2b) Determine and plot $\alpha_{\mathrm{aero}}(y)$.
2c) Note the amount of washin or washout twist required, defined as $\alpha_{\mathrm{aero}}(b/2) - \alpha_{\mathrm{aero}}(0)$.

Tapered wing: Assume a tapered planform

$$c(y) = c_r + (c_t - c_r)2y/b$$

with taper ratio $c_t/c_r \equiv r = 0.6$.

3a) Determine c_r and c_t so that the overall lift coefficient is $C_L = 1.0$.
3b) Determine and plot $\alpha_{\mathrm{aero}}(y)$.
3c) Note the amount of washin or washout twist required, defined as $\alpha_{\mathrm{aero}}(b/2) - \alpha_{\mathrm{aero}}(0)$.

Wing structure is designed primarily to withstand the airload-imposed bending moment, which typically has the largest value M_r at the wing root. For some given lift distribution $L'(y)$ or $L'(\theta)$, the root bending moment is

$$M_{\rm r} \;=\; \int_0^{b/2} L'\, y\, dy \;=\; \frac{b^2}{4}\int_0^{\pi/2} L'\, \cos\theta\, \sin\theta\, d\theta$$

a) Determine M_r for the following lift distribution, for some arbitrary A_1 and A_3.

$$\begin{aligned} \Gamma(\theta) &= 2bV_\infty\,(A_1 \sin\theta \,+\, A_3 \sin 3\theta) \\ L' &= \rho V_\infty \Gamma \\ \rightarrow\quad M_r &= \frac{1}{2}\rho V_\infty^2\, b^3 \int_0^{\pi/2} (A_1 \sin\theta \,+\, A_3 \sin 3\theta)\, \cos\theta\, \sin\theta\, d\theta \end{aligned}$$

b) Consider the following two load cases:
i) $A_1 = 0.03$, $A_3 = 0$
ii) $A_1 = 0.03$, $A_3 = -0.005$

Assuming the same flight speed V_∞ and span b, determine and compare the relative lift L, root bending moment M_r, and induced drag factor $1+\delta = 1/e$ between the two cases.

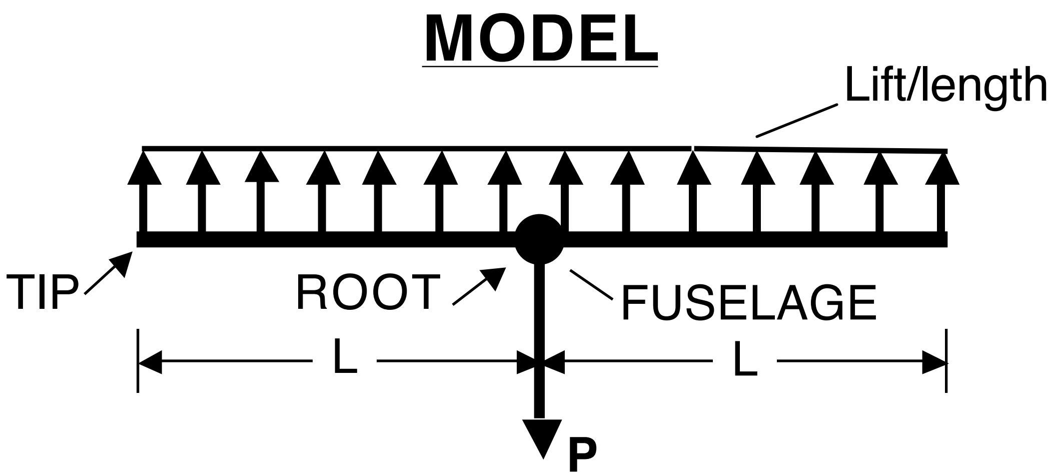
c) Sketch the loading shapes i) and ii) versus y on one plot to help visualize the two caess. For an aircraft which has to fly at minimum power, what are the relative merits between wings i) and ii) ?

Note: The following trigonometric identity should be useful for part a) ...

$$2\, \sin m\theta\, \sin m\theta \;=\; \cos(m{-}n)\theta \,-\, \cos(m{+}n)\theta$$

Unified Engineering Problem Set #3 **Units M4.4,M4.5**
Spring, 2005

M6 Let's continue exploring our simple model of how wings carry load in level flights. We can expand this to consider stress distributions and deflections. The model of the load configuration is again shown below. Use the results from the solution for problem set #2 for the axial force, shear force, and bending moment as appropriate. Assume that the wing has constant cross-sectional properties of I and A and is made of an isotropic material with a modulus of E.



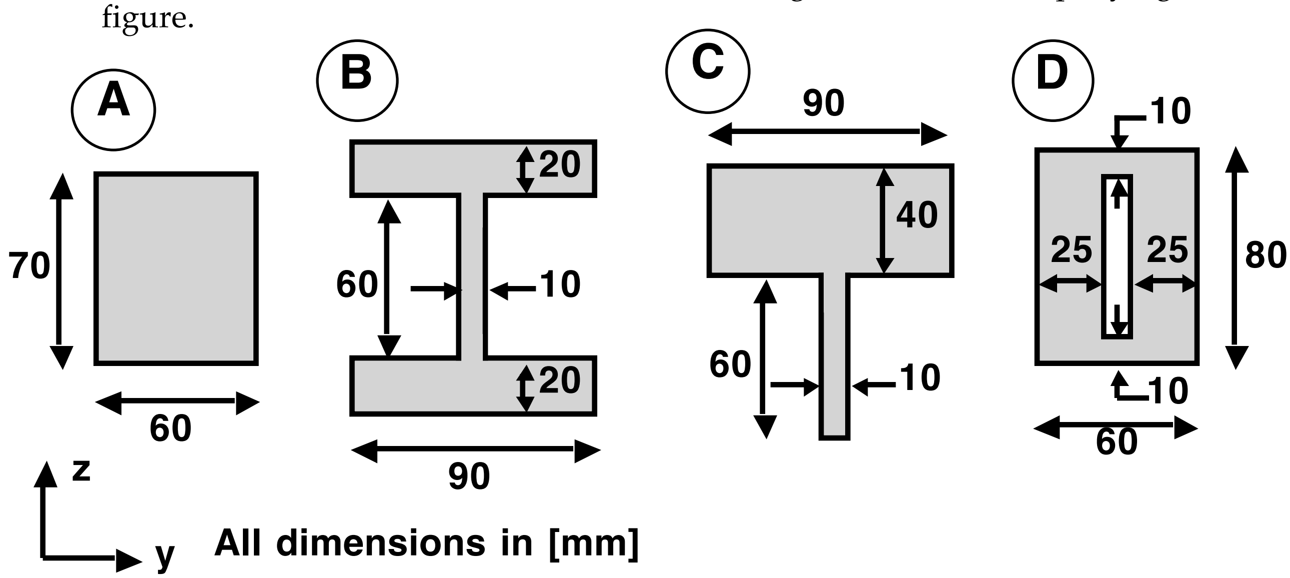
(a) Determine and sketch the distribution of the axial stress, σ_{xx}, along the wing; and find the location of the maximum value along the wing.

(b) Determine and sketch the distribution of the shear stress, τ_{xz}, along the wing; and find the location of the maximum value along the wing.

(c) Determine and sketch the deflection of the wing, w; and find the location of the maximum value along the wing.

(for thought) As in problem set #2, we now increase the wingspan by 10% (i.e. by a factor of 1.1) while keeping the total lift and the center load P the same. What is the effect on the maximum values of the axial and shear stresses and their location? What is the effect on the maximum value of the deflection w and its location? Compare the slope of the wing, dw/dx, at this location (the slope is known as the "*dihedral angle*").

M7 You are asked to evaluate different designs of a 5-meter long statically determinate beam to be made out of titanium. Four different cross-sections are under consideration: a solid rectangle, an I-beam, a T, and a rectangular tube. In each case, the cross-sectional area of the beam is the same: 4200 mm^2. The dimensions of each of the cross-sections are given in the accompanying figure.



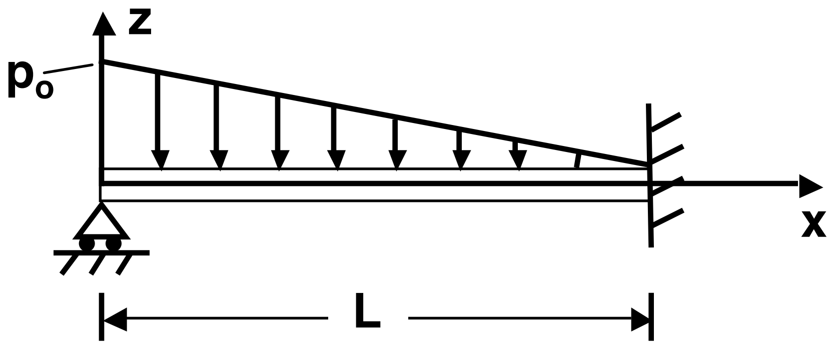
(a) Determine the cross-section which will give the smallest deflection of the beam.

(b) Determine the cross-section which will have the smallest value of the maximum magnitude of the axial stress σ_{xx} and find the location in the cross-sectional plane.

(c) Determine the cross-section which will have the smallest value of the maximum magnitude of the shear stress τ_{xz} and find the location in the cross-sectional plane.

(d) Comment on the possible beam selections.

M8 A beam of length L is clamped at one end and supported by a roller at the other. The beam has a constant cross-section with area A and moment of inertia I, and is made of a material with modulus E and Poisson's ratio ν. The beam is loaded by a linearly-varying downward load of intensity equal to zero at the clamped end and p_o at the other end.



(a) Determine the maximum deflection of this beam and its location.

(b) Determine the maximum axial stress magnitude, σ_{xx}, and its location in the x-direction.

(c) Determine the maximum shear stress magnitude, τ_{xz}, and its location in the x-direction.