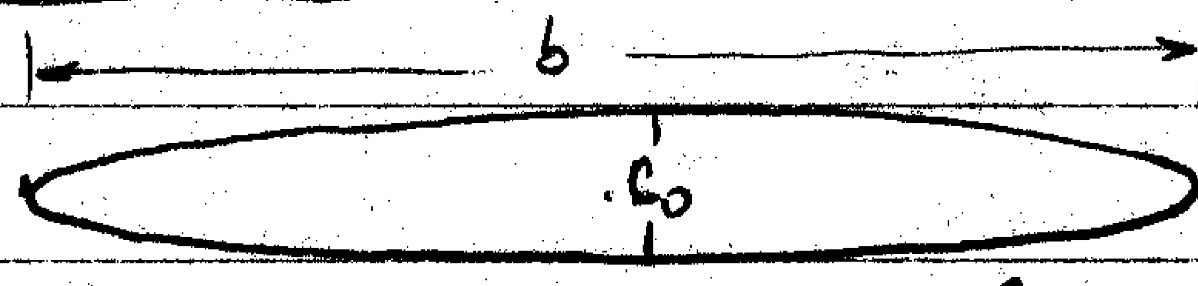
**Massachusetts Institute of Technology**
**Department of Aeronautics and Astronautics**
**Cambridge, MA 02139**

# Unified Engineering
# Spring 2005

## Problem Set #3 Solutions

1a) Constant $c_\ell$, so $c_\ell = C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = 1.0 \rightarrow S = \frac{L}{\frac{1}{2}\rho V^2 1.0} = 0.333\ m^2$

$\boxed{c(y) = c_0\sqrt{1-(2y/b)^2}} \rightarrow S = c_0 b \frac{\pi}{4} = 0.333\ m^2 \rightarrow \boxed{c_0 = \frac{0.333\ m^2}{(\pi/4)\cdot 1.8\ m} = 0.236\ m}$




Can also get $c_0$ via $L \rightarrow \Gamma_0 \rightarrow c_0$

1b) $AR = b^2/S = 9.73$, $\alpha_i = \frac{C_L}{\pi AR} = 0.0327\ rad = 1.87°$

$\alpha + \alpha_{aero} = \frac{c_\ell}{2\pi} + \alpha_i$, but $\alpha = 0$ as given

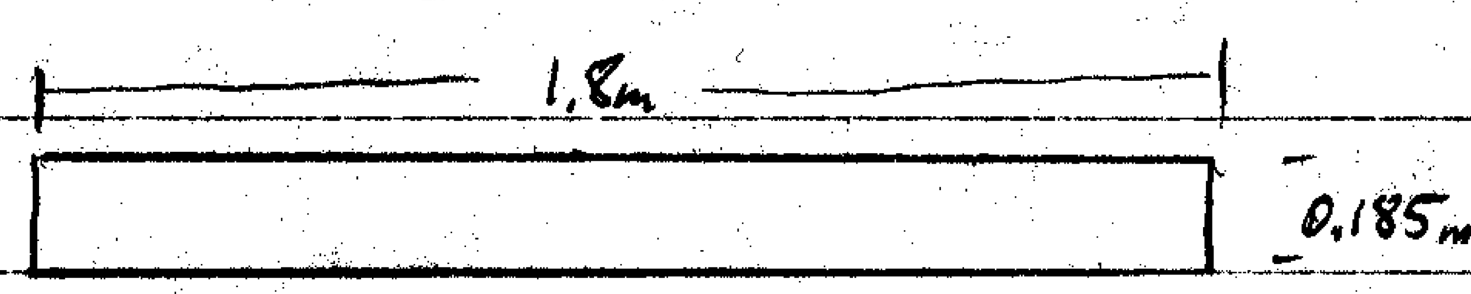
$\rightarrow \boxed{\alpha_{aero} = \frac{1.0}{2\pi} + 0.0327 = 0.1918 = 11.0°}$ constant in $y$.

1c) With the same airfoil across span, $\alpha_{L=0}$ is constant, and $\alpha_{L=0} < 0$ for +cam

$\rightarrow \boxed{\alpha_{geom} = \alpha_{aero} + \alpha_{L=0} \quad \text{constant}}$ shifted by $\alpha_{L=0}$

2a) Same $C_L = 1.0$ as in 1a), so $S = \frac{L}{\frac{1}{2}\rho V^2 C_L} = 0.333\ m^2 = bc$

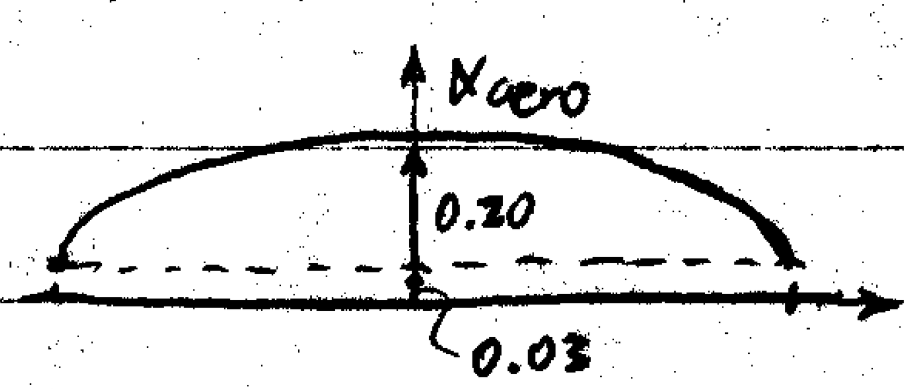
$c = 0.333\ m^2/b = 0.185\ m$




2b) $\frac{1}{2} c V c_\ell = \Gamma = \Gamma_0\sqrt{1-(2y/b)^2}$, $\Gamma_0 = \frac{4}{\pi} L/\rho V b = 0.589\ m^2/s$

$c_\ell(y) = \frac{2\Gamma_0}{cV}\sqrt{1-(2y/b)^2} = 1.27\sqrt{1-(2y/b)^2}$

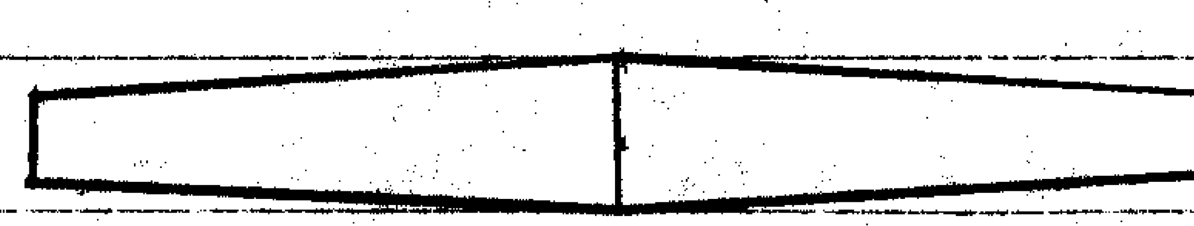
$\boxed{\alpha_{aero}(y) = \frac{c_\ell(y)}{2\pi} + \alpha_i = 0.20\sqrt{1-(2y/b)^2} + 0.0327}$




2c) $\boxed{\alpha_{aero}(b/2) - \alpha_{aero}(0) = -0.20\ rad = -11.6°\ \text{washout}}$ (a lot!)

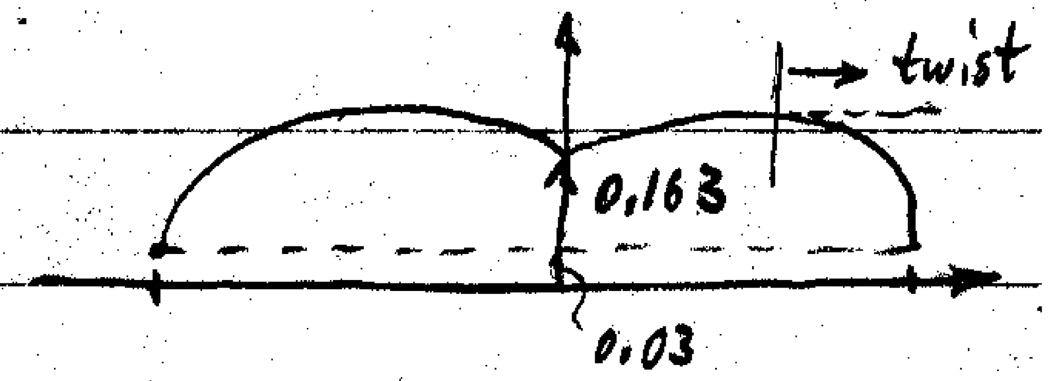
3a) Still have the same $S = b\frac{c_r + c_t}{2} = bc_r\frac{1+r}{2} = bc_r\, 0.8 = 0.333\ m^2$

$c_r = 0.230\ m$ $\quad c_t = 0.138\ m$



3b) $c_\ell(y) = \frac{2\Gamma(y)}{c(y)V} = \frac{2\Gamma_0}{c_r V}\frac{\sqrt{1-(2y/b)^2}}{1-0.4|2y/b|}$

$\boxed{\alpha_{aero}(y) = \frac{c_\ell(y)}{2\pi} + \alpha_i = 0.163\frac{\sqrt{1-(2y/b)^2}}{1-0.4|2y/b|} + 0.0327}$




3c) $\alpha_{aero}(b/2) - \alpha_{aero}(0) = -0.163 = -9.34°$ washout

most of the twist is on the outer half of the wing.

UE Fluids    Problem 9+10 Solution    Spring '05

a) $\int_0^{\pi/2} \left(A_1 \sin\theta + A_3 \sin 3\theta\right) \underbrace{\cos\theta \sin\theta}_{\frac{1}{2}\sin 2\theta} \, d\theta$

$\frac{1}{4}\left(A_1 \sin\theta + A_3 \sin 3\theta\right) 2 \sin 2\theta$

$\frac{1}{4}\left(A_1\, 2\sin\theta \sin 2\theta + A_3\, 2 \sin 3\theta \sin 2\theta\right)$

$\int_0^{\pi/2} \frac{1}{4}\left[A_1\left(\cos\theta - \cos 3\theta\right) + A_3\left(\cos\theta - \cos 5\theta\right)\right] d\theta$

$\frac{1}{4}\left[A_1\left(\sin\theta - \frac{1}{3}\sin 3\theta\right) + A_3\left(\sin\theta - \frac{1}{5}\sin 5\theta\right)\right]_0^{\pi/2} = \frac{1}{4}\left[A_1\left(1+\frac{1}{3}\right) + A_3\left(1 - \frac{1}{5}\right)\right]$

$= \frac{1}{4}\left[A_1 \frac{4}{3} + A_3 \frac{4}{5}\right]$

$M_r = \frac{1}{2}\rho V_\infty^2 b^3 \left[\frac{1}{3}A_1 + \frac{1}{5}A_3\right]$

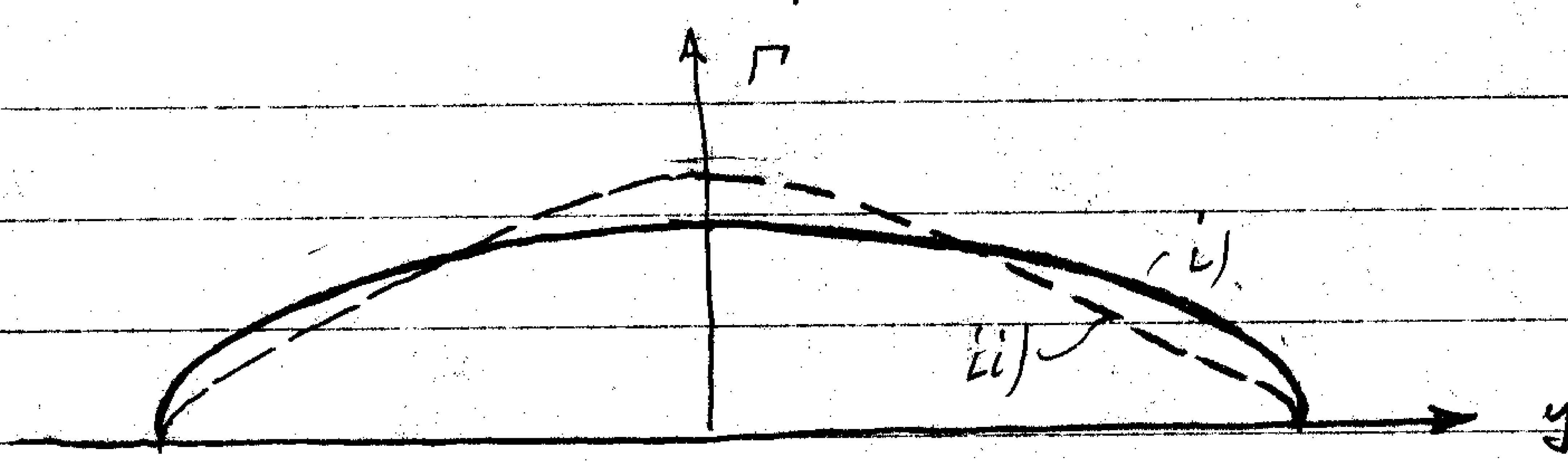
$L = \frac{\pi}{2}\rho V_\infty^2 b^2 A_1$

$1+\delta = 1 + 3\left(\frac{A_3}{A_1}\right)^2 = \frac{1}{e}$

Compare:

| | $A_1$ | $A_3$ | $M_r / \frac{1}{2}\rho V_\infty^2 b^3$ <br> $\left[\frac{1}{3}A_1 + \frac{1}{5}A_3\right]$ | $L / \frac{\pi}{2}\rho V_\infty^2 b^2$ <br> $A_1$ | $1/e$ <br> $1+3\left(\frac{A_3}{A_1}\right)^2$ |
|---|---|---|---|---|---|
| i) | 0.03 | 0 | 0.010 | 0.03 | 1.0 |
| ii) | 0.03 | -0.005 | 0.009 | 0.03 | 1.0833 |
| | | | 10% decrease in $M_r$ | | 8.3% increase in $D_i$ |

b)




Case ii) wing can be lighter, since it needs less structure for the 10% smaller $M_r$.

But case ii) also has an 8.3% larger $D_i$

tradeoff between weight and drag

Not clear which wing is better without more detailed analysis