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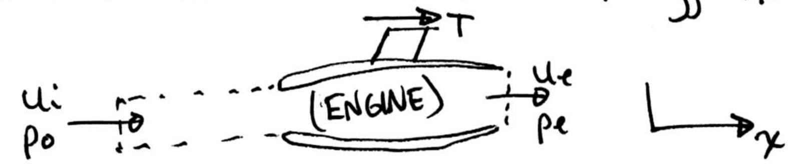
Unified Engineering
Spring 2005

Problem Set #4
Solutions

PROPULSION P1 SOLUTIONS WAITZ

INTEGRAL MOMENTUM EQN. : $\sum F_x = \int \rho u_x (\vec{u}\cdot\vec{n})\,dA$



$T + (p_o - p_e)A_e = \dot{m}u_e - \dot{m}u_o$

a) MOM. FLUX INTO ENGINE

= massflow rate * momentum per unit mass (i.e. velocity)

= $\dot{m}$ (velocity)

OR FROM THE INTEGRAL

$= \int \rho u_x (\vec{u}\cdot\vec{n})\,dA = \dot{m}(-u_{in}) = 50\frac{kg}{s}\cdot(-400\,m/s)$

$\boxed{=-20\,kN}$ (negative because u_{in} is (+) an outward normal is (−))

b) MOM. FLUX OUT – SAME AS ABOVE EXCEPT $\vec{n}$ & $\vec{u}_e$ ARE IN SAME DIRECTION SO $\vec{u}\cdot\vec{n}$ IS (+)

$= 50\frac{kg}{s}\cdot 700\,m/s = \boxed{35\,kN}$

c) NET MOM FLUX $= \boxed{15\,kN}$

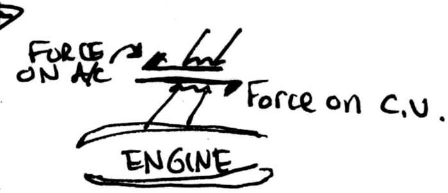
d) NET PRESSURE FORCE

$= (p_o - p_e)A_e = (30\,kPa - 25\,kPa)(1\,m^2) = \boxed{5\,kN}$

e) NET FORCE ON MASS IN C.V.

$T = \dot{m}u_e - \dot{m}u_o - A_e(p_o - p_e) = 35\,kN - 20\,kN - 5\,kN = \boxed{10\,kN}$

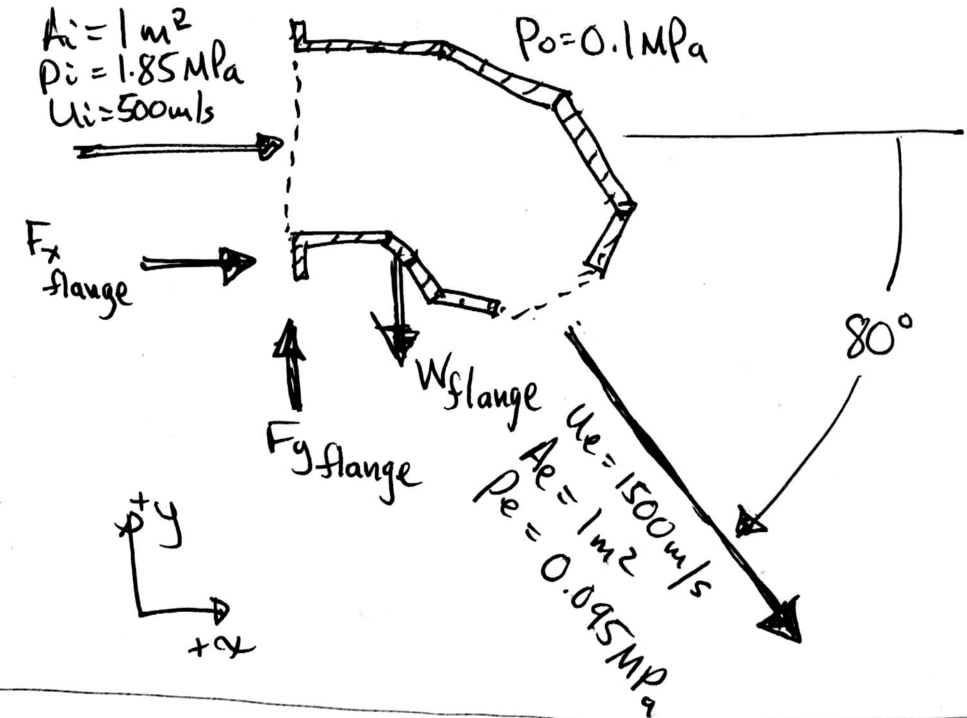
f) THIS FORCE ACTS IN THE POSITIVE X-DIRECTION ON THE MASS IN THE C.V. ⟶



g) SINCE THE AIRCRAFT IS IN STEADY LEVEL FLIGHT, THE FORCES MUST BALANCE. THEREFORE THE VECTOR SUM OF ALL OTHER FORCES (LIFT, DRAG, WEIGHT) MUST = 10 kN + x DIR

$$\Sigma F_x = \int_S \rho u_x (\vec{u}\cdot\vec{n})\, dA$$

$$\Sigma F_y = \int_S \rho u_y (\vec{u}\cdot\vec{n})\, dA$$



$$\underbrace{F_{x\,flange}}_{\text{x-force on flange}} + \underbrace{(p_i - p_o)A_i}_{\substack{\text{x-pressure force on inlet}\\ \text{(relative to } p_o)}} - \underbrace{(p_e - p_o)A_e\cos\theta}_{\substack{\text{x-pressure force on exit}\\ \text{(relative to } p_o)}} = \underbrace{\rho u_i(-u_i)A_i}_{\text{x-mom. flux inlet}} + \underbrace{\rho u_e\cos\theta\,(u_e)A_e}_{\text{x-mom. flux exit.}}$$

$$\underbrace{F_{y\,flange}}_{\text{y-force on flange}} + \underbrace{0}_{\substack{\text{y comp. of press. force}\\ \text{on inlet}}} + \underbrace{(p_e - p_o)A_e\sin\theta}_{\substack{\cancel{\text{x}}\text{-comp of press. force}\\ \text{on exit}}} - \underbrace{m_{flange}\cdot g}_{\substack{\text{y comp. of nozzle}\\ \text{weight}}} = \underbrace{0}_{\text{y-mom flux inlet}} + \underbrace{\rho u_e\sin\theta\,(u_e)A_e}_{\text{y-mom flux exit}}$$

a) NET PRESSURE FORCE IN x-DIR = $(p_i - p_o)A_i - (p_e - p_o)A_e\cos\theta$

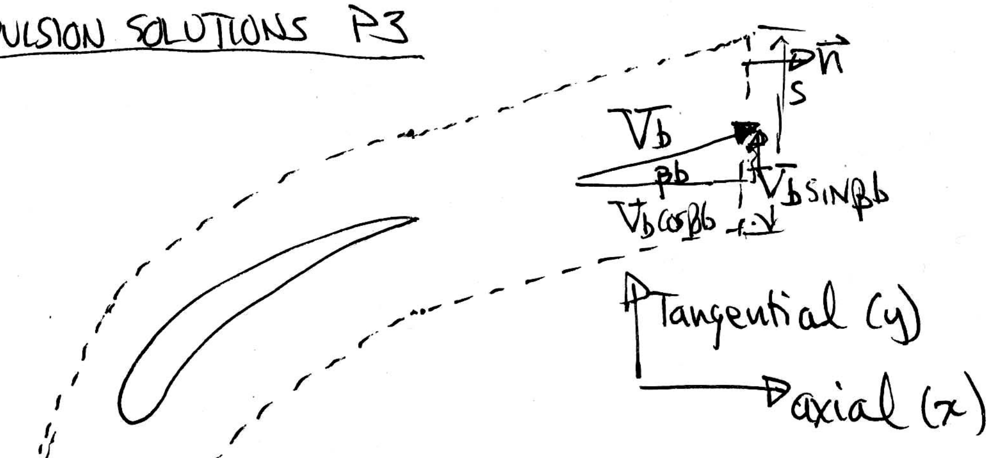
$$= (1.85\times10^6 - 0.1\times10^6)\,1\,m^2 - (0.095\times10^6 - 0.1\times10^6)\,1\,m^2 \cos 80^\circ$$

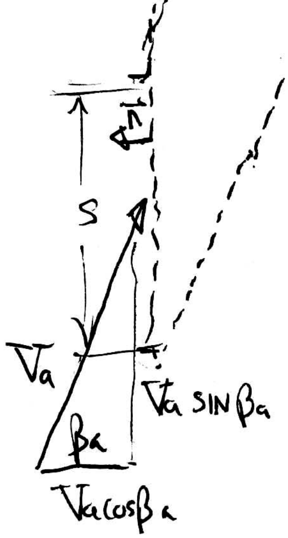
$$\boxed{= 1.7509\times10^6\ N}$$

b) THIS ACTS TO PUT THE FLANGE IN TENSION (WOULD BE BALANCED BY NEGATIVE FORCE ON FLANGE)

NET PRESSURE FORCE IN y-DIR = $(p_e - p_o)A_e\sin\theta = (0.095\,MPa - 0.1\,MPa)\cdot 1(\sin 80^\circ)\,m^2$

$$\boxed{= -4.9\ kN}$$





a) SINCE THE MASS FLOW IS THE SAME AT INLET AND OUTLET, THEN THE <u>AXIAL COMPONENT OF VELOCITY MUST BE THE SAME</u> AT INLET AND AT OUTLET. HOWEVER, THE TANGENTIAL (SWIRL) COMPONENT OF VELOCITY DECREASES ACROSS THE BLADE ROW

b) THE AXIAL FLUX OF TANGENTIAL MOMENTUM IS EQUAL TO THE MASS FLOW RATE TIMES THE TANGENTIAL MOM. PER UNIT MASS (i.e. THE TANG. VEL.)

$$\text{AXIAL FLUX OF TANG. MOM.} = \dot{m} u_y$$

IT DECREASES ACROSS THE BLADE ROW. THE FLOW HAS LESS SWIRL VELOCITY LEAVING THE BLADE ROW.

c) SINCE THE FLOW IS INCOMPRESSIBLE & THE MAGNITUDE OF THE VELOCITY DECREASES, THEN THE ~~MAGN~~ PRESSURE INCREASES.

d) $\sum F_x = R_x + \underset{\text{in } x}{\text{Pressure forces}} = \int_S \rho u_x \vec{u}\cdot\vec{n}\, dS$

- By symmetry, pressure forces on top & bottom streamlines are the same and thus balance. ∴ Only need to consider pressure forces on left and right of C.V.
- Since upper & lower surfaces of C.V. are streamlines there is no flux across them. ∴ Only need to consider fluxes at inlet & outlet.

$$R_x + p_a S - p_b S = \rho V_a \cos\beta_a(-V_a\cos\beta_a)S + \rho V_b\cos\beta_b(V_b\cos\beta_b)S$$

mass flow $= \rho V_a \cos\beta_a S = \rho V_b \cos\beta_b S$

∴ $R_x = (p_b - p_a)S$ = force ON C.V.

$p_b > p_a$ so R_x is IN (+) x-direction

∴ FORCE FELT BY BLADES IS ← (−) x

$\sum F_y = R_y + \cancel{\underset{\text{in } y}{\text{pressure forces}}}^{\,0} = \int_S \rho u_y \vec{u}\cdot\vec{n}\, dS$

$$R_y = \rho V_a \sin\beta_a(-V_a\cos\beta_a)S + \rho V_b\sin\beta_b(V_b\cos\beta_b)S$$

$$R_y = \rho S V_a\cos\beta_a(V_b\sin\beta_b - V_a\sin\beta_a) \quad < 0$$

= FORCE ON C.V.

∴ FORCE FELT BY BLADES IS IN ↑ +y direction

c) X-COMP. OF NET MOM. FLUX $= \rho u_i (-u_i) A_i + \rho u_e \cos\theta (u_e) A_e$

$= 100\frac{kg}{s}\left(-500\frac{m}{s}\right) + 100\frac{kg}{s}\left(1500\frac{m}{s}\right)\cos 80°$

$\boxed{= -23.9\,kN}$

y-COMP OF NET MOM. FLUX $= \rho(-u_e \sin\theta) u_e A_e$

$= 100\frac{kg}{s} \cdot \left(-1500\frac{m}{s}\right)\sin 80°$

$\boxed{= -147.7\,kN}$

d) X-COMP OF NET MOM. FLUX WOULD BE BALANCED BY A NEGATIVE FORCE ON FLANGE ∴ FLANGE IS IN TENSION.

e) $F_{x\,flange} + 1.7509\times10^{6}\,N = -23.9\times10^{3}\,N$

$\therefore \boxed{F_{x}\ flange = -1.77\times10^{6}\,N}$

$F_{y}flange + (-4.9\times10^{3}N) - 400kg \cdot 9.8\frac{m}{s^2} = -147.7\times10^{3}N$

$\therefore \boxed{F_{y}\ flange = -0.139\times10^{6}\,N}$

f) THE FORCE IS TRANSMITTED FROM THE FLUID TO THE NOZZLE STRUCTURE BY VISCOUS AND PRESSURE FORCES ON THE WALLS OF THE NOZZLE.

UE Fluids — F11 Soln — Spring '05

a) Anderson (p.463)

given $\rho_\infty = 0.819 \ kg/m^3$, $P_\infty = 0.61 \ atm = 0.616 \times 10^5 \ Pa$

$$h_\infty = \frac{\gamma}{\gamma-1}\frac{P_\infty}{\rho_\infty} = 2.633 \times 10^{-5} \ m^2/s^2$$

$$V_\infty = 300 \ m/s$$

$$h_{0\infty} = h_\infty + \frac{1}{2}V_\infty^2 = 3.083 \times 10^5 \ m^2/s^2$$

(constant everywhere)

$$P_{0\infty} = P_\infty\left(\frac{h_{0\infty}}{h_\infty}\right)^{\frac{\gamma}{\gamma-1}} = 1.737 \, P_\infty = 1.07 \times 10^5 \ Pa$$

$$P = P_{0\infty}\left[1 - \frac{1}{2}\frac{V^2}{h_{0\infty}}\right]^{\frac{\gamma}{\gamma-1}} = 0.505 \times 10^5 \ Pa$$

$$1 - \frac{1}{2}\frac{V^2}{h_{0\infty}} = \left(\frac{P}{P_{0\infty}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{0.505}{1.07}\right)^{\frac{1}{3.5}} = 0.807$$

$$V = \left(2h_{0\infty}(1 - 0.807)\right)^{\frac{1}{2}} = 345 \ m/s$$

$$M_\infty^2 = \frac{V_\infty^2}{\frac{\gamma P_\infty}{\rho_\infty}} = 0.8547 \Leftarrow \left\{ \begin{array}{l} \text{Using the Mach Number} \\ \text{instead} \end{array} \right.$$

$$\frac{P_0}{P_{0\infty}} = \frac{P}{P_\infty}\frac{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}} = \frac{0.5 \ atm}{0.6 \ atm}\frac{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}(0.8547)\right)^{\frac{\gamma}{\gamma-1}}} = 1$$

$$M^2 = 1.197 \ ; \ a^2 = (\gamma-1)h_0\left[1+\frac{\gamma-1}{2}M^2\right]^{-1} = 0.995 \times 10^{5} \ m^2/s^2$$

$$V = \sqrt{M^2 a^2} = 345\ m/s$$

b) Anderson 7.10

Incompressible Bernoulli:

$$P_0 = P_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = P + \frac{1}{2}\rho_\infty V^2$$

$$V = \left[V_\infty^2 + \frac{2(P_\infty - P)}{\rho}\right]^{\frac{1}{2}} = \left[300^2 + \frac{2(0.61-0.5)\cdot 1.01\times10^5}{\cancel{0.189}\ 0.819}\right]^{1/2}$$

$$\boxed{V = 342\ m/s}$$

UE Fluids F12 Solution Spring '05

a) $\oiint \rho \vec{V} \cdot \hat{n} \, h_o \, dA = \iiint \rho \dot{q} \, d\mathcal{V}$

$$-\rho_1 V_1 h_{o_1} A_1 + \rho_2 V_2 h_{o_2} A_2 = \dot{Q}$$

By mass conservation we also have $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

Also, for low speed flow, $h_{o_1} = h_1 + \frac{1}{2} V_1^2 \simeq h = c_p T_1 = 290000 \ m^2/s^2$

$$\rho_1 V_1 A_1 (h_{o_2} - h_{o_1}) = \dot{Q} \quad , \quad \text{max } h_{o_2} \simeq h_2 = c_p T_2 = 340000 \ m^2/s^2$$

$$V_1 (340000 - 290000) = \frac{\dot{Q}}{\rho_1 A_1} = \frac{1000 \, W}{1.2 \, kg/m^3 \cdot 0.01 \, m^2} = 83333 \ m^3/s^2$$

$$\text{minimum} \quad \boxed{V_1 = \frac{83333}{340000 - 290000} = 1.67 \ m/s}$$

b) State eq'n; $\rho_2 = \frac{\gamma}{\gamma - 1} \frac{p_2}{h_2} = \frac{1.4}{0.4} \frac{10^5 \, Pa}{340000} = 1.03 \ kg/m^3$

Continuity: $\rho_2 V_2 A_2 = \rho_1 V_1 A_1$

$$\boxed{V_2 = V_1 \frac{\rho_1}{\rho_2} \frac{A_1}{A_2} = 1.67 \, m/s \cdot \frac{1.2}{1.03} = 1.947 \ m/s}$$