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Unified Engineering
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Problem Set #5
Solutions

PS4 solution

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a) According to Breguet Range Equation,

$$R = \underbrace{\frac{h}{g}}_{\text{fuel type}} \underbrace{\eta_{ov}}_{\text{efficiency}} \underbrace{\left(\frac{L}{D}\right)}_{\text{aerodynamics}} \underbrace{\ln\left(\frac{W_{initial}}{W_{final}}\right)}_{\text{structure.}}$$

From the problem sentence, η_{ov} = constant

We can also assume $\frac{h}{g}$, $\ln\left(\frac{W_{initial}}{W_{final}}\right)$ constant and independent of flight condition. Thus, max range is achieven when $\frac{L}{D} = \frac{C_L}{C_D}$ is maximum.

From SPL 2 data, C_L is known, and $C_D = 0.013 + C_{D\text{-}lab2}$.

By inspecting the slope, $\left(\frac{C_L}{C_D}\right)_{max} = 8.41$ at $C_L = 0.7$.

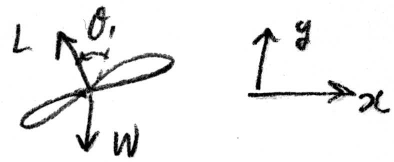
$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \Leftrightarrow V = \sqrt{\frac{W/S}{\frac{1}{2}\rho C_L}} = \sqrt{\frac{5\ (Pa)}{\frac{1}{2}\cdot \underset{(kg/m^3)}{1.22} \cdot 0.7}} = \boxed{3.42\ (m/s)}$$

b) The maximum endurance condition corresponds with the minimum energy expenditure. The minimum power required occurs when $\frac{C_L^{3/2}}{C_D}$ is maximized.

From the dragonfly $C_L^{\frac{3}{2}}$ vs C_D curves, this condition is achieved for $C_L = 1.2$

$$V = \sqrt{\frac{2(W/S)}{\frac{1}{2}\rho C_L}} = \sqrt{\frac{5\ (Pa)}{\frac{1}{2}\cdot 1.22 \cdot 1.2}} = \boxed{2.61\ (m/s)}$$

c) For steady turning flight, the equilibrium equation in vertical direction is



$$\Sigma F_y = L\cos\theta - W = 0$$

$$\Leftrightarrow \quad L = \frac{W}{\cos\theta}.$$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{W/\cos\theta}{\frac{1}{2}\rho V^2 S} \Leftrightarrow V = \sqrt{\frac{2\,(W/S)}{\frac{1}{2}\rho\cos\theta\, C_L}}$$

Banking does not change the C_L vs C_D curves.

Thus, for maximum range, $C_L = 0.7$.

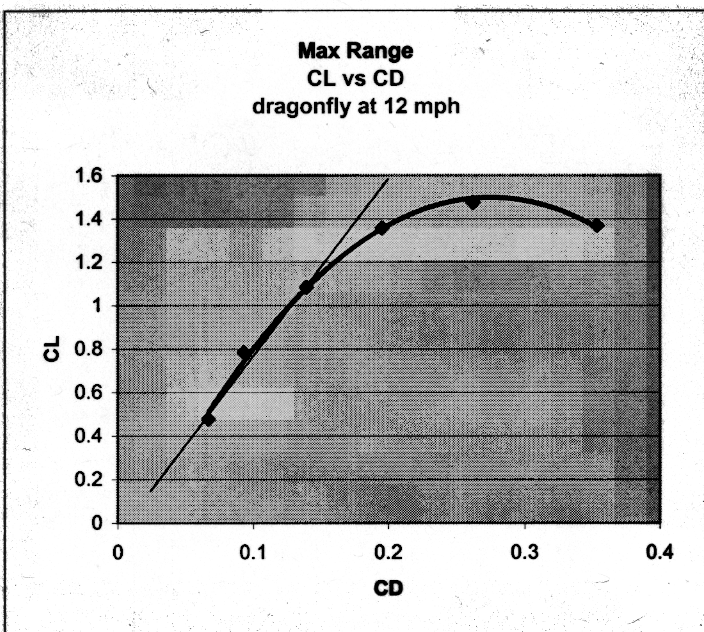
$$V = \sqrt{\frac{5\ (\mathrm{Pa})}{\frac{1}{2}\cdot 1.22\cdot 0.7\cdot \cos 20}} = \boxed{3.53\ (\mathrm{m/s})}$$

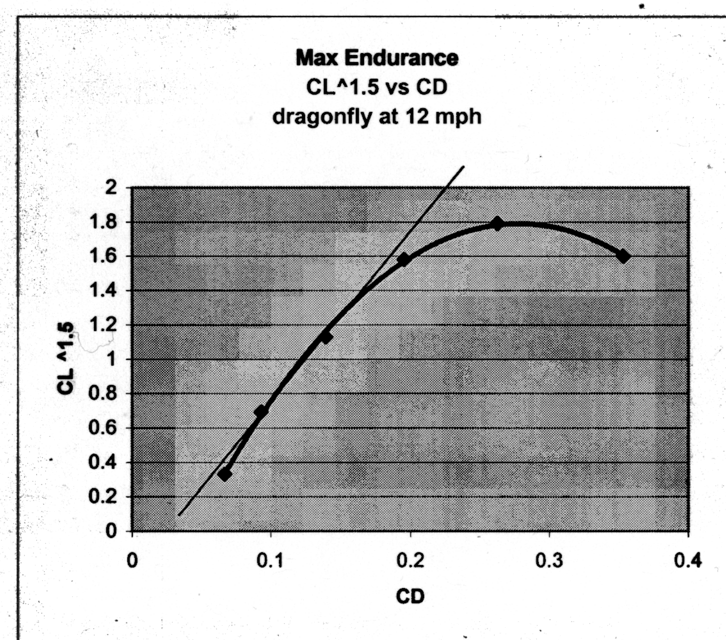
For maximum endurance. $C_L = 1.2$

$$V = \sqrt{\frac{5}{\frac{1}{2}\cdot 1.22\cdot 1.2\cos 20^\circ}} = \boxed{2.70\ (\mathrm{m/s})}$$

SUMMARY

alpha	Cx	Cy	Cmo	C_D	C_L	C_D	C_L^1.5
0	0.053747	0.4792209	0.025275	0.066747075	0.479220931	0.066747	0.331744
4	0.025348	0.787663	-0.01676	0.093230741	0.783976137	0.093231	0.694151
8	-0.0257	1.0909157	-0.0545	0.139375967	1.083875815	0.139376	1.128416
12	-0.10353	1.3649608	-0.08272	0.195518875	1.35665926	0.195519	1.580179
16	-0.16713	1.487127	-0.08334	0.262252327	1.475585379	0.262252	1.792448
20	-0.14842	1.4024648	-0.05356	0.353199335	1.368649416	0.353199	1.601173





P5 SOLUTIONS WAITZ

TO FIND MAX THRUST, USE $P_{avail} - P_{reqd} = \frac{d}{dt} P.E. + \frac{d}{dt} K.E.$
AND SOLVE FOR T FOR EACH OF
THE THREE MANEUVERS — THEN PICK THE BIGGEST ONE.

$$TV - DV = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

CASE 1) 6g COMBAT TURN $\therefore \frac{dh}{dt} = 0, \frac{d}{dt}V^2 = 0$

$T = D, \quad L = 9W \qquad D = \frac{1}{2}\rho V^2 S k C_L^2 + \frac{1}{2}\rho V^2 S C_{D_0}$

$$\frac{T}{W} = \frac{1}{2}\rho V^2\left(\frac{1}{W/S}\right) k \frac{L^2}{(\frac{1}{2}\rho V^2 S)^2} + \frac{1}{2}\rho V^2\left(\frac{1}{W/S}\right) C_{D_0}$$

$$\frac{T}{W} = \underbrace{\frac{kn^2}{\frac{1}{2}\rho V^2}\left(\frac{W}{S}\right)}_{3.18} + \underbrace{\frac{C_{D_0}}{\left(\frac{W}{S}\right)}\frac{1}{2}\rho V^2}_{0.0227}$$

$V = 0.7(295\,m/s) = 206.5$
$\rho = 0.34\ kg/m^3$
$k = 0.2$
$C_{D_0} = 0.01$
$n = 6$
$W/S = 3200\ N/m^2$

$3.2 = \frac{T}{W}$

$\therefore T = (3.2)\, 20{,}000 \cdot 9.8$

$$\boxed{T = 627.2\ kN}$$

CASE 2) M=0.8 TO M=2.0 IN 15 SECONDS

$V_{FINAL} = 2(295\,m/s) = 590\,m/s$
$V_{initial} = 0.8(295\,m/s) = 236\,m/s$

$$\frac{\Delta V}{\Delta t} = \frac{590\,m/s - 236\,m/s}{15\,s} = 23.6\,\frac{m}{s^2} = \frac{dV}{dt}$$

$$\frac{T}{W} = \frac{kn^2}{\frac{1}{2}\rho V^2}\left(\frac{W}{S}\right) + \frac{C_{D_0}}{\left(\frac{W}{S}\right)}\frac{1}{2}\rho V^2 + \frac{1}{V}\frac{d}{dt}\left(\frac{1}{2}\frac{V^2}{g}\right)$$

$n = 1$

TRY WORST CASE FOR V & K & C_{D_0} @ $M=2$

$$\frac{T}{W} = \underbrace{\frac{k}{\frac{1}{2}\rho V^2}\left(\frac{W}{S}\right)}_{0.022} + \underbrace{\frac{C_{D_0}}{\left(\frac{W}{S}\right)}\frac{1}{2}\rho V^2}_{0.481} + \underbrace{\frac{1}{g}\frac{dV}{dt}}_{2.41}$$

$k = 0.4$
$C_{D_0} = 0.026$
$V = 590\ m/s$
$\frac{W}{S} = 3200\ N/m^2$
$\rho = 0.34\ kg/m^3$

$\Rightarrow \frac{T}{W} = 2.91 \quad \therefore T = (2.91)(9.8)(20{,}000)$

$$\boxed{T = 570.3\ kN}$$

CASE 3) STRAIGHTUP AT $M = 0.5$

$$\frac{T}{W} = \frac{k}{\frac{1}{2}\rho V^2}\left(\frac{W}{S}\right) + \frac{C_{D_0}}{\left(\frac{W}{S}\right)}\frac{1}{2}\rho V^2 + \frac{1}{\cancel{W}V}\left(\cancel{W}\frac{dh}{dt}\right)$$

& $\frac{dh}{dt} = V$ SO $\frac{T}{W} = \underbrace{\frac{k}{\frac{1}{2}\rho V^2}\left(\frac{W}{S}\right)}_{0.182} + \underbrace{\frac{C_{D_0}}{\left(\frac{W}{S}\right)}\frac{1}{2}\rho V^2}_{0.016} + \underbrace{1}_{1}$

$V = 0.5(295\ m/s) = 147.5\ m/s$
$W/S = 3200\ N/m^2$, $\rho = 0.34$, $k = 0.21$, $C_{D_0} = 0.01$

$\frac{T}{W} = 1.19 \quad \therefore T = 9.8 \cdot 20{,}000\,(1.19) = \boxed{233.9\ KN}$

$$\boxed{\therefore \text{ SIZE ENGINE FOR MAX} = 627\ kN}$$

P6 SOLUTIONS WAITZ

a) $T = \dot{m} u_e + A_e(p_e - p_o)$

$$\dot{m} = \frac{P_c\sqrt{\gamma}}{\sqrt{RT_c}} \frac{M}{\left[1 - \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} A = \frac{\frac{P_c\sqrt{\gamma}}{\sqrt{RT_c}}}{\left[1 - \frac{\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \cdot A^*$$

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{1 + \frac{\gamma-1}{2}M_e^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$p_e = \frac{P_c}{\left[1 + \frac{\gamma-1}{2}M_e^2\right]^{\gamma/\gamma-1}} \qquad T_e = \frac{T_c}{1 + \frac{\gamma-1}{2}M_e^2} \qquad u_e = M_e\sqrt{\gamma R T_e}$$

I PUT THESE INTO A SPREADSHEET AND FOUND

$\frac{A_e}{A^*} = 13.333 \Rightarrow M_e = 4.04 \Rightarrow u_e = 2448.9$ m/s

$\Rightarrow p_e = 27{,}545$ Pa

$\dot{m} = 1.98$ kg/s

$\therefore T = 4257$ N

$I_{sp} = \frac{T}{\dot{m} g} = 219.5$ s — TAKE-OFF

b) FOR IDEALLY EXPANDED $T = \dot{m} u_e \Rightarrow$

$I_{sp} = \frac{u_e}{g} = 249.9$ s — ideal exp.

THIS NOZZLE IS <u>OVEREXPANDED</u> AT LAUNCH $p_e < p_o$

c) FOR IDEALLY EXPANDED, $D=0$, vertical launch
$g = \text{const.}$

$$u_{burnout} = g\left[Isp_{ideal}\ln\left(\frac{m_{initial}}{m_{final}}\right) - t_{burnout}\right]$$

$$h_{burnout} = g\left[-\frac{t_{bo}\,Isp_{ideal}\ln\left(\frac{m_i}{m_f}\right)}{\left(\frac{m_i}{m_f}-1\right)} + t_{bo}Isp_{ideal} - \frac{1}{2}t_{bo}^2\right]$$

$$t_{bo} = \frac{m_{propellant}}{\dot{m}} = \cancel{} = 15.15\,s$$

$$\boxed{u_{burnout} = 3246\ m/s \qquad h_{burnout} = 18837\ m}$$

GLIDE PHASE: $\frac{1}{2}m_{final}u_{final}^2 = m_{final}\,g\,\Delta h$

$h_{glide} = \Delta h = 537692\,m$

$$\boxed{h_{TOTAL} = 556530\,m}$$

d) $m_i = 41\ kg$, $m_{final} = 11\ kg$ (still 30 kg fuel)

$AR = 14.667 \Rightarrow M_e = 4.13$

TAKE OFF THRUST HIGHER, Isp_{ideal} also higher BUT WEIGHT PENALTY.

$T = \dot{m}u_e + (p_e - p_a)A_e$

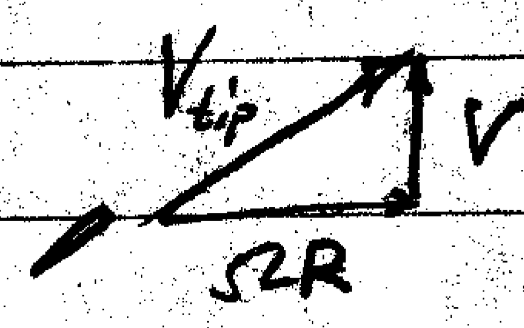
NET EFFECT IS BAD $\boxed{h_{TOTAL} = 506305\,m}$

UE Fluids F13 Solution Spring 05

$\Omega = RPM \cdot \frac{\pi}{30} = 6000 \cdot \frac{\pi}{30} = 628.32$ rad/s

$\Omega R = 628.3$ rad/s $\cdot 3.5$ in $\cdot$ 0.0254 m/in $= 55.8$ m/s

Velocity triangle:



$V_{tip}^2 = V^2 + (\Omega R)^2 = 3156$ m²/s²

$h_0 = c_p T + \frac{1}{2} V_{tip}^2 = 1000 \cdot 290 + \frac{1}{2} \cdot 3156 = 291580$ m²/s²

$\boxed{T_0 = \frac{h_0}{c_p} = 291.6\ K}$ 1.6°K warmer than ambient.

$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left(\frac{291.6}{290}\right)^{2.5} = 1.0137$, $\boxed{\rho_0 = 1.0137\,\rho = 1.216\ kg/m^3}$

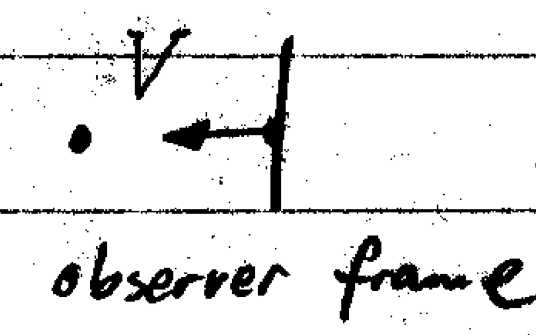
$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{291.6}{290}\right)^{3.5} = 1.0192$, $\boxed{p_0 = 1.0192\,p = 1.0192 \times 10^5\ Pa}$

UE Fluids F14 + F15 Solution Spring '05

Still air: $T = 300\,K$, $p = 10^5\,Pa$

$$\rightarrow a = \sqrt{\gamma R T} = \sqrt{1.4 \cdot 287 \cdot 300} = 347.2\ m/s$$

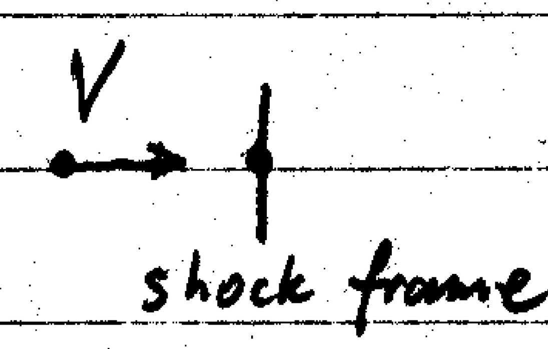
a) $\boxed{M_1 = \frac{V}{a} = \frac{450\ m/s}{347.2\ m/s} = 1.296 \approx 1.30}$



b) From shock table (p 871)

for $M_1 = 1.3$; $\boxed{\frac{T_2}{T_1} = 1.191,\ \frac{p_2}{p_1} = 1.805}$

$\boxed{T_2 = 1.191\, T_1 = 357\ K}$ (183 F toasty)

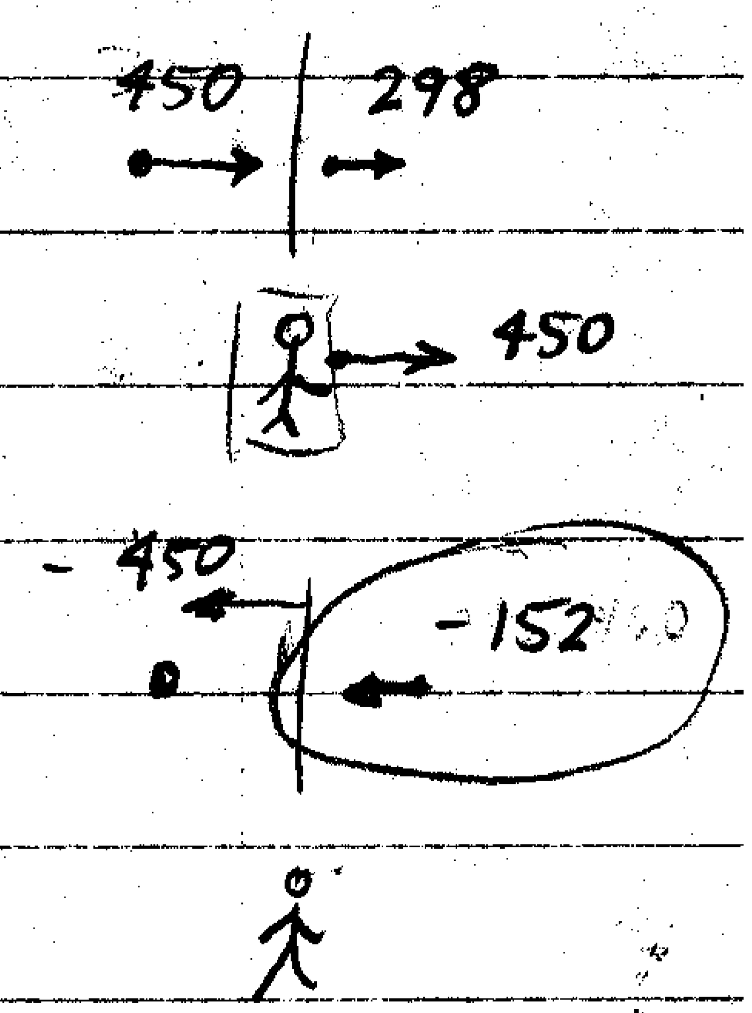


c) $\boxed{p_2 = 1.805\, p_1 = 1.805 \times 10^5\ Pa}$

d) $M_2 = 0.786$ (in shock frame)

$a_2 = \sqrt{\gamma R T_2} = 378.7\ m/s$

$V_2 = M_2 a_2 = 297.7\ m/s$ (in shock frame)



$$(V_2)_{\substack{\text{observer}\\ \text{frame}}} = V_2 - V_{\substack{\text{observer}\\ \text{rel to shock}}} = V_2 - 450\ m/s$$

$$(V_2)_{\substack{\text{observer}\\ \text{frame}}} = 297.7 - 450 = -152\ m/s$$

e) Sound waves in fluid behind shock travel at $a_2 = 378.7\ m/s$

Fluid itself is traveling at 152 m/s to left

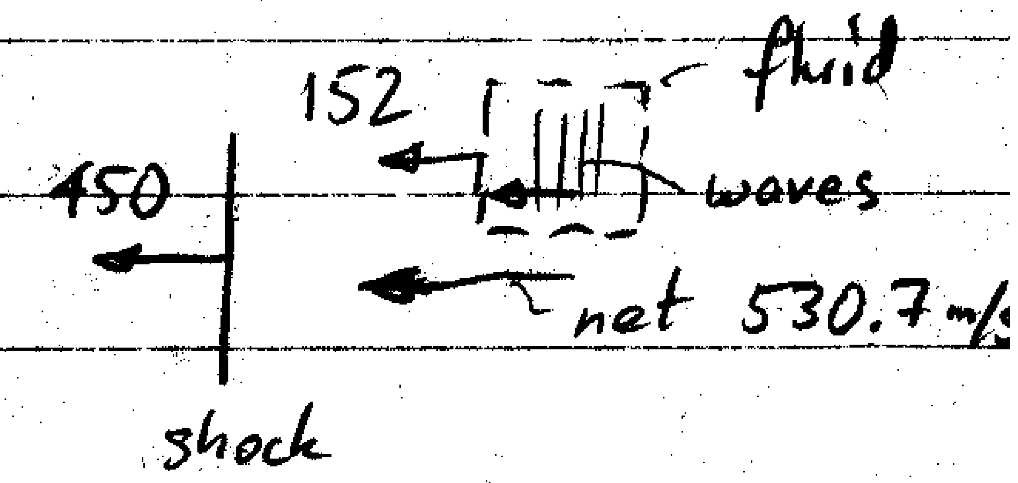
So the waves move at $378.7 + 152 = 530.7\ m/s$ faster than shock

The sound waves <u>will</u> catch up with shock.

One can also look in the shock frame:

downstream $M_2 < 1$, so waves can

make their way upstream.



UE Fluids F19 Solution Spring '05

a) Module is a gas reservoir with $T_0 = 300K$, $p_0 = 10^5 Pa$, $\rho_0 = \frac{\gamma R T_0}{p_0} = 1.205$

$R = 287\ J/kg\,K$

Hole is a sonic throat: $A^* = \frac{\pi}{4}(0.01m)^2 = 7.85 \times 10^{-5}\ m^2$

$$\dot{m} = \rho^* a^* A^* = \rho_0 \left[1 + \frac{\gamma-1}{2}\right]^{\frac{-1}{\gamma-1}} a_0 \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{1}{2}} A^*, \quad a_0 = \sqrt{\gamma R T_0} = 347\ m/s$$

$$\boxed{\dot{m} = 0.019\ kg/s}$$

b) Volume of module: $V = \frac{\pi}{4} D^2 l = \frac{\pi}{4}(3m)^2 \, 8m = 56.55\ m^3$

Mass of air in module: $m = \rho V = 68.1\ kg$

State eq'n: $p = \rho R T = \frac{m}{V} R T = m \left(\frac{RT}{V}\right)$

since $\frac{RT}{V}$ is constant: $\dot{p} = \dot{m} \frac{RT}{V} = 0.019\ kg/s \cdot \frac{287 \cdot 300}{56.55\ m^3}$

$$\boxed{\dot{p} = 28.9\ Pa/s}$$

c) $\rho = \frac{m}{V}$, so $\dot{\rho} = \frac{\dot{m}}{V}$ since $V = $ constant

$\dot{m} = \rho_0 a_0 A^* \cdot$ const,

ρ_0 is unaffected by temperature, since volume is fixed.

$\rightarrow a_0 = \sqrt{\gamma R T}$

$$\boxed{\text{Reducing } T \text{ will reduce } a_0 \text{ and hence reduce } \dot{m}}$$