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# Unified Engineering Spring 2005 

Problem Set \#6
Due Date: Tuesday, March 15, 2005 at 5pm

|  | Time <br> Spent <br> (minutes) |
| :--- | :--- |
| P7 |  |
| P8 |  |
| P9 |  |
| F16/17 |  |
| F18 |  |
| F20 |  |
| Study <br> Time |  |

## Problem P7. (Propulsion) L.O. F

Use ideal cycle analysis to estimate how engine design and performance are related to aircraft performance. Assume the supersonic aircraft in Problem 5 is powered by an ideal turbojet engine. The engine uses a conventional hydrocarbon fuel ( $\mathrm{h}=42.8 \mathrm{MJ} / \mathrm{kg}$ ), $\gamma=$ $1.4, \mathrm{c}_{\mathrm{p}}=1000 \mathrm{~J} /(\mathrm{kg}-\mathrm{K})$, the turbine inlet temperature is 1900 K , and the compressor pressure ratio is 15 . Assume an altitude of 11 km where the pressure is 22.6 kPa , the temperature is 217 K , the density is $0.34 \mathrm{~kg} / \mathrm{m}^{3}$, and the speed of sound is $295 \mathrm{~m} / \mathrm{s}$.
a) What are the overall efficiency, thermal efficiency and propulsive efficiency of the engine at $\mathrm{M}=2.0$ and $\mathrm{M}=0.8$ ? Explain why each of the efficiencies changes for the two flight speeds.
b) Assuming the weight fraction is the same (but drag and overall efficiency change), by what percent would the range change if the airplane was flown at these two speeds for each of these ideal cycles?
c) If you were asked to change the engine design to improve the range at $\mathrm{M}=2$ at fixed altitude by $10 \%$ what things might you do?

Problem P8. (Propulsion) L.O. G

Use the nominal Dragonfly propeller for the conditions (a-c) shown below. First draw velocity triangles in the blade-relative and absolute reference frames at inlet and exit to the blade row, then estimate the power imparted to the flow per unit area.

For the purposes of this assignment, assume that the flow exits the blade parallel to the zero-lift line. [Although this is in general a poor assumption for propellers. For the "sparse" blading of a propeller (compared to a compressor), assuming the exit flow lines up with the blade will over predict the turning. At typical conditions for your props, Professor Drela estimates that the turning is over predicted by about 70\%.]
a) Radial station $r=0.7 \mathrm{R}_{\text {tip }}, \mathrm{V}=5 \mathrm{~m} / \mathrm{s}, 5000 \mathrm{rpm}$
b) Radial station $\mathrm{r}=0.7 \mathrm{R}_{\text {tip }}, \mathrm{V}=8 \mathrm{~m} / \mathrm{s}$, 5000 rpm
c) Radial station $r=0.7 \mathrm{R}_{\text {tip }}, \mathrm{V}=5 \mathrm{~m} / \mathrm{s}, 8000 \mathrm{rpm}$
d) A typical propeller efficiency curve is shown below. Why is the efficiency reduced at low values of the advance ratio $(\lambda=\mathrm{V} / \omega \mathrm{R})$ and again at high values of the advance ratio?

APC 7*4 at 6000 rpm


Problem P9. (Propulsion) L.O. G

Two rows of turbomachine blades are shown below. At inlet to the first blade row, the flow is purely axial. Assume that the axial component of velocity (w) then remains constant through the stage.

a) Neatly draw and label the velocity triangles assuming the blade speed $\omega \mathrm{r}=2 \mathrm{w}$, where $\mathrm{w}=100 \mathrm{~m} / \mathrm{s}$ is the axial velocity.
b) On which blade row(s) is there a torque applied? Why? In what direction does it act?
c) Write an expression for the power extracted by each blade row. Is this a compressor or a turbine? How do you know?
d) Draw the velocity triangles for a blade speed of 3w. Assuming the flow is well behaved, does the power per unit mass flow increase or decrease and by how much (percent change)? What aerodynamic problems might be encountered at this condition?

The figure shows the bow shock around a supersonic sphere, taken from a photo so that it's fairly accurate. The grid has been added to allow graphical estimation of the local shock wave angles via tangent lines and trigonometry.
a) Using the drawing at point O , where the shock is extremely weak, estimate the freestream Mach number $M_{\infty}=M_{1}$. Locate this point on the $\beta$ vs $\theta$ vs $M_{1}$ oblique shock chart (Anderson p 513). Submit a xerox copy of the chart.
b) For points A, B, C, D on the drawing, determine or compute the following:

- Location on the oblique shock chart
- Shock wave angle $\beta$ and turning angle $\theta$
- Normal Mach numbers $M_{n_{1}}$ and $M_{n_{2}}$
- Total pressure ratio $p_{o_{2}} / p_{o_{1}}$
c) Identify the region where the flow is locally subsonic (Hint: determine $M_{2}$ at the points).


After computing the pressure ratio $p_{2} / p_{1}$ across a shock or expansion wave, in aerodynamic problems it is useful to then compute and work with the equivalent pressure coefficient jump.

$$
\begin{equation*}
\Delta C_{p} \equiv \frac{p_{2}-p_{1}}{\frac{1}{2} \rho_{1} V_{1}^{2}}=\frac{p_{2}-p_{1}}{\frac{\gamma}{2} p_{1} M_{1}^{2}}=\frac{2}{\gamma M_{1}^{2}}\left(\frac{p_{2}}{p_{1}}-1\right) \tag{1}
\end{equation*}
$$

Rather than go through the full shock or expansion calculation procedure to get $p_{2} / p_{1}$ and then $\Delta C_{p}$, a much simpler and commonly used approximation for a weak shock or expansion is

$$
\begin{equation*}
\Delta C_{p} \simeq \frac{2 \theta}{\sqrt{M_{1}^{2}-1}} \tag{2}
\end{equation*}
$$

in which $\theta$ is in radians, and is set positive for a shock, and negative for an expansion.
a) Consider four simple corner flows $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, all with $M_{1}=1.5$, and with $\theta= \pm 5^{\circ}$ and $\pm 10^{\circ}$ (two shock cases and two expansion cases). For each of these four cases, determine $p_{2} / p_{1}$ and then the "exact" $\Delta C_{p}$ using equation (1).
b) Evaluate the accuracy of the $\Delta C_{p}$ obtained from the approximate formula (2).


Anderson problem 10.9 (p 586).

