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Unified Engineering
Spring 2005

Problem Set #6
Solutions

WAITZ

PROPULSION P7 SOLUTIONS

EQUATIONS:
$$\frac{T}{\dot{m}a_0} = M_0\left[\left\{\left(\frac{\theta_0}{\theta_0-1}\right)\left(\frac{\theta_t}{\theta_0\tau_c}-1\right)(\tau_c-1)+\frac{\theta_t}{\theta_0\tau_c}\right\}^{1/2}-1\right]$$

$$\eta_0 = \frac{M_0(\gamma-1)\left(\frac{T}{\dot{m}a_0}\right)}{(\theta_t-\tau_c\theta_0)}$$

$$\eta_{TH} = 1-\frac{1}{\theta_0\tau_c}$$

$$\eta_p = \eta_0/\eta_{TH}$$

$$\pi_c = \tau_c^{\gamma/\gamma-1}$$

$$\frac{T_{T_t}}{T_0} = \theta_t, \quad \frac{T_{T_t}}{T_0} = \left(1+\frac{\gamma-1}{2}M_0^2\right) = \theta_0$$

FOR THE FOLLOWING CONDITIONS:

$T_0 = 217K$, $a_0 = 295 m/s$, $\pi_c = 15$, $T_{T_t} = 1900K$, $\gamma = 1.4$

a)

	η_0	η_{TH}	η_p
M=2	0.444	0.744	0.597
M=0.8	0.182	0.591	0.308

$\eta_{TH}\uparrow$ WITH $M_0\uparrow$ BECAUSE
$$\eta_{TH} = 1-\frac{T_0}{T_{T_3}} = 1-\underbrace{\frac{1}{\theta_0\tau_c}}$$

COMPOSED OF 2 PARTS RAM TEMP RISE & COMPRESSOR TEMP. RISE. RAM TEMP RISE GOES UP W/FLIGHT SPEED

$\eta_p\uparrow$ WITH $M_0\uparrow$ BECAUSE

$u_e \rightarrow u_0$ &

$$\eta_p = \frac{2}{1+\frac{u_e}{u_0}}$$

b) $\text{RANGE} = \frac{h}{g}\eta_o \frac{L}{D} \ln \frac{W_i}{W_f}$

$$\frac{\text{RANGE}_{M=2}}{\text{RANGE}_{M=0.8}} = \frac{\eta_{o\,M=2}}{\eta_{o\,M=0.8}} \cdot \frac{D_{M=0.8}}{D_{M=2.0}}$$

$\eta_{o\,M=2} = 0.444$
$\eta_{o\,M=0.8} = 0.182$

MUST FIND DRAG DIFFERENCE FROM PROBLEM P5

$$D = \tfrac{1}{2}\rho V^2 S C_D = \tfrac{1}{2}\rho V^2 S\left[k C_L^2 + C_{D_o}\right], \quad C_L^2 = \frac{L^2}{(\frac{1}{2}\rho V^2 S)^2} = \frac{W^2}{(\frac{1}{2}\rho V^2 S)^2}$$

$$D = \tfrac{1}{2}\rho V^2 S\, k\left(\frac{W}{S}\right)^2 \frac{1}{(\frac{1}{2}\rho V^2)^2} + \tfrac{1}{2}\rho V^2 S C_{D_o}$$

$$D = \tfrac{1}{2}\rho V^2 S C_{D_o} + \frac{\left(\frac{W}{S}\right)^2 k S}{\frac{1}{2}\rho V^2}$$

$\frac{W}{S} = 3200\ N/m^2$. $\rho = 0.34\ \frac{kg}{m^3}$

	k	C_{D_o}	V	D/S
M=0.8	0.2	0.01	236 m/s	311 N/m²
M=2.0	0.4	0.026	590 m/s	1607.8 N/m²

$$\boxed{\frac{\text{RANGE}_{M=2}}{\text{RANGE}_{M=0.8}} = \frac{0.444}{0.182} \cdot \frac{311}{1608} = 0.47}$$

ALTHOUGH THE EFFICIENCY IS BETTER, YOU PAY A VERY LARGE DRAG PENALTY FOR GOING FAST ($\sim V^2$)

c)
- COULD REDUCE ENGINE WEIGHT.
- COULD ALSO INCREASE T_{T4} TO GET MORE THRUST PER UNIT MASS FLOW → BUT EFFICIENCY CAN DROP
  (smaller engine) → LESS DRAG
- EASIEST THING TO TRY MAY BE INCREASING COMPRESSOR PRESSURE RATIO — RAISING π_c TO 27 WOULD INCREASE η_o TO 0.48 @ M=2

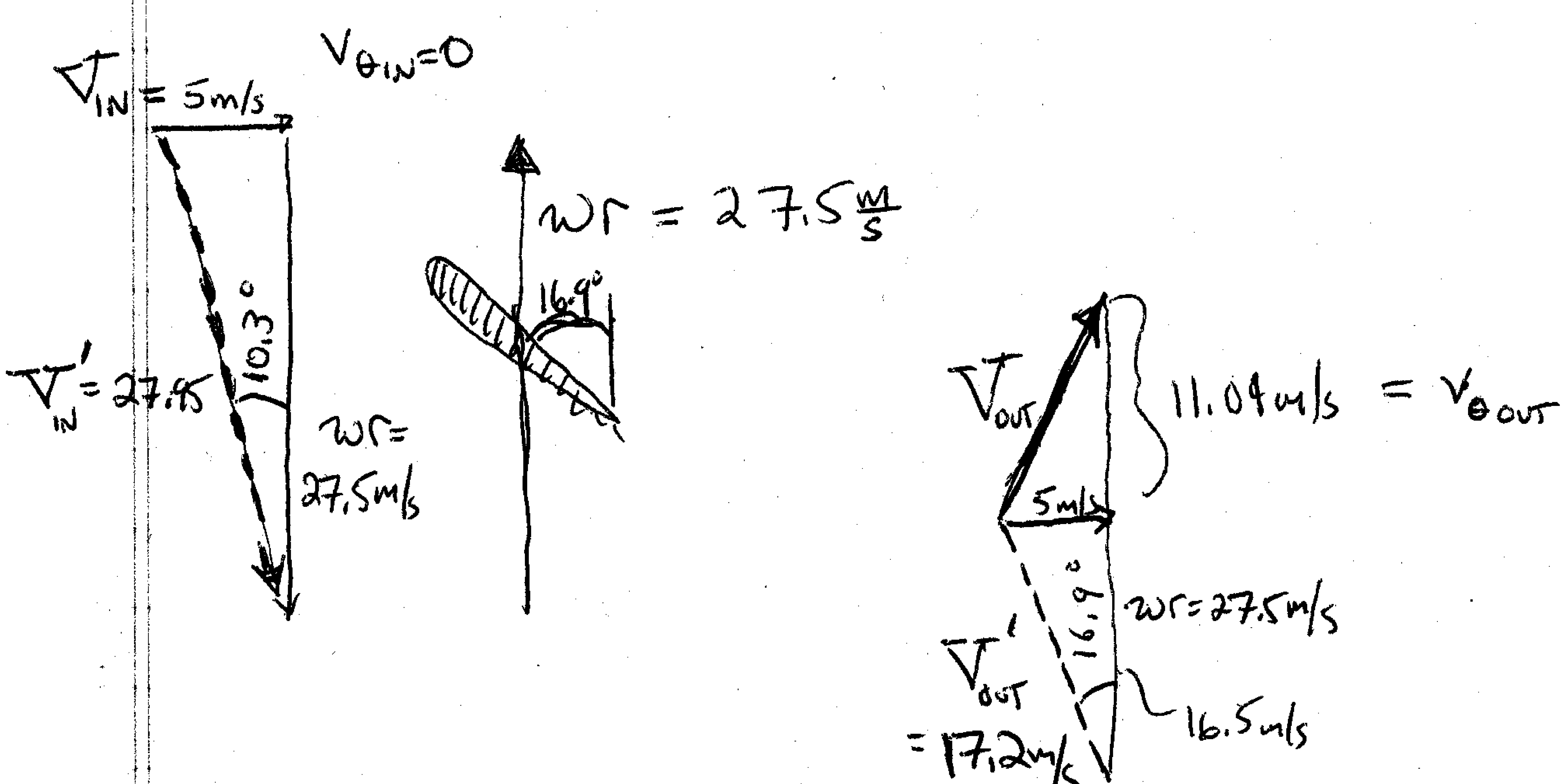
P8 PROPULSION SOLUTIONS WAITZ

a) $R_{TIP} = 0.075\,m \therefore r = 0.0525\,m$

ZERO LIFT LINE ANGLE AT THIS RADIUS, $\beta_{z\ell} = \tan^{-1}(P/2\pi r)$

WHERE $P \equiv$ PITCH $= 0.10\,m$

SO $\beta_{z\ell} = 16.7°$

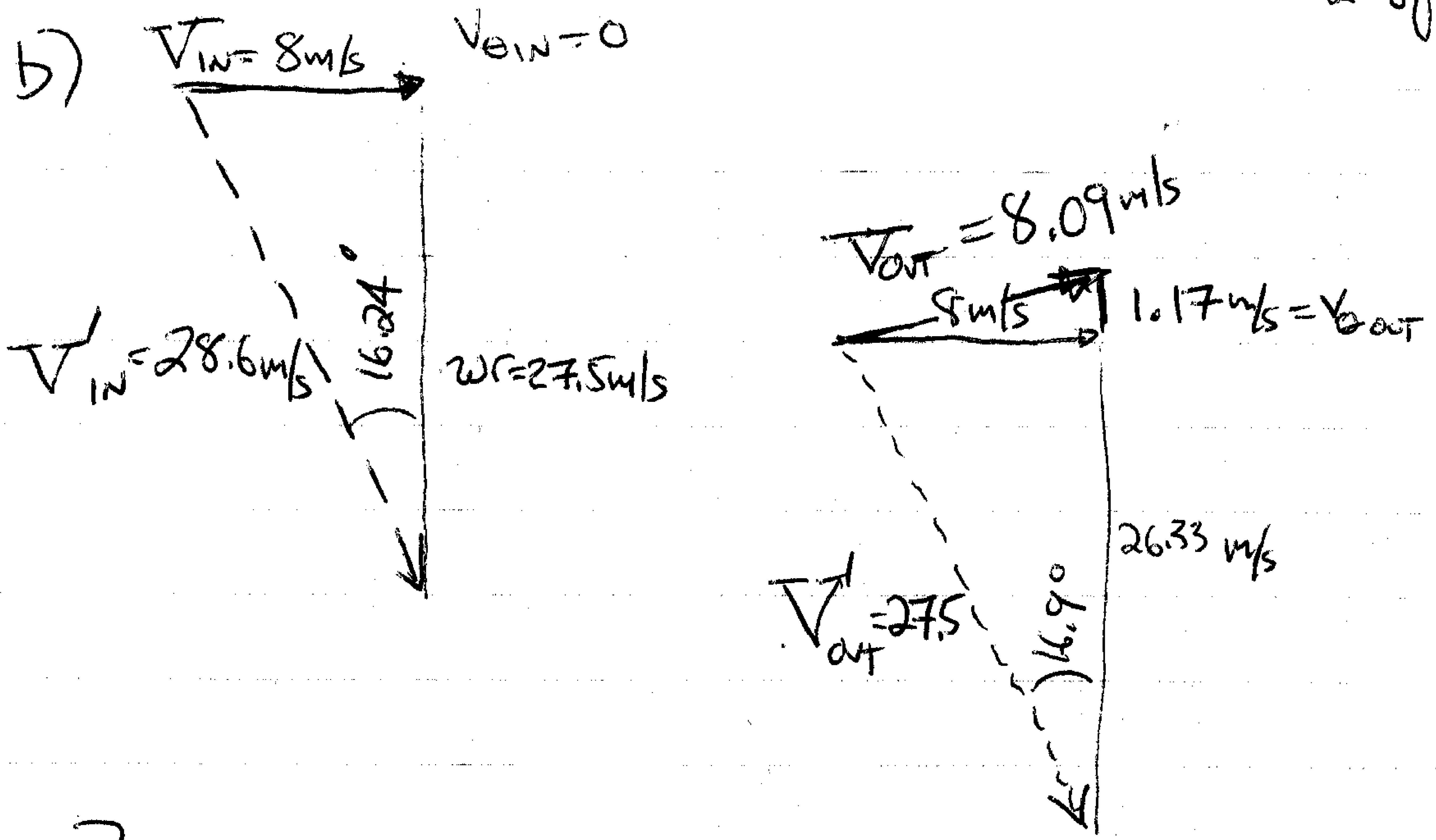


$$P = \omega r \dot{m} (V_{\theta OUT} - V_{\theta IN})$$

$$\frac{P}{A} = \omega r \rho V_{IN} (V_{\theta OUT} - V_{\theta IN}) = (27.5)(1.2 \tfrac{kg}{m^3})(5)(11.04 - 0)$$

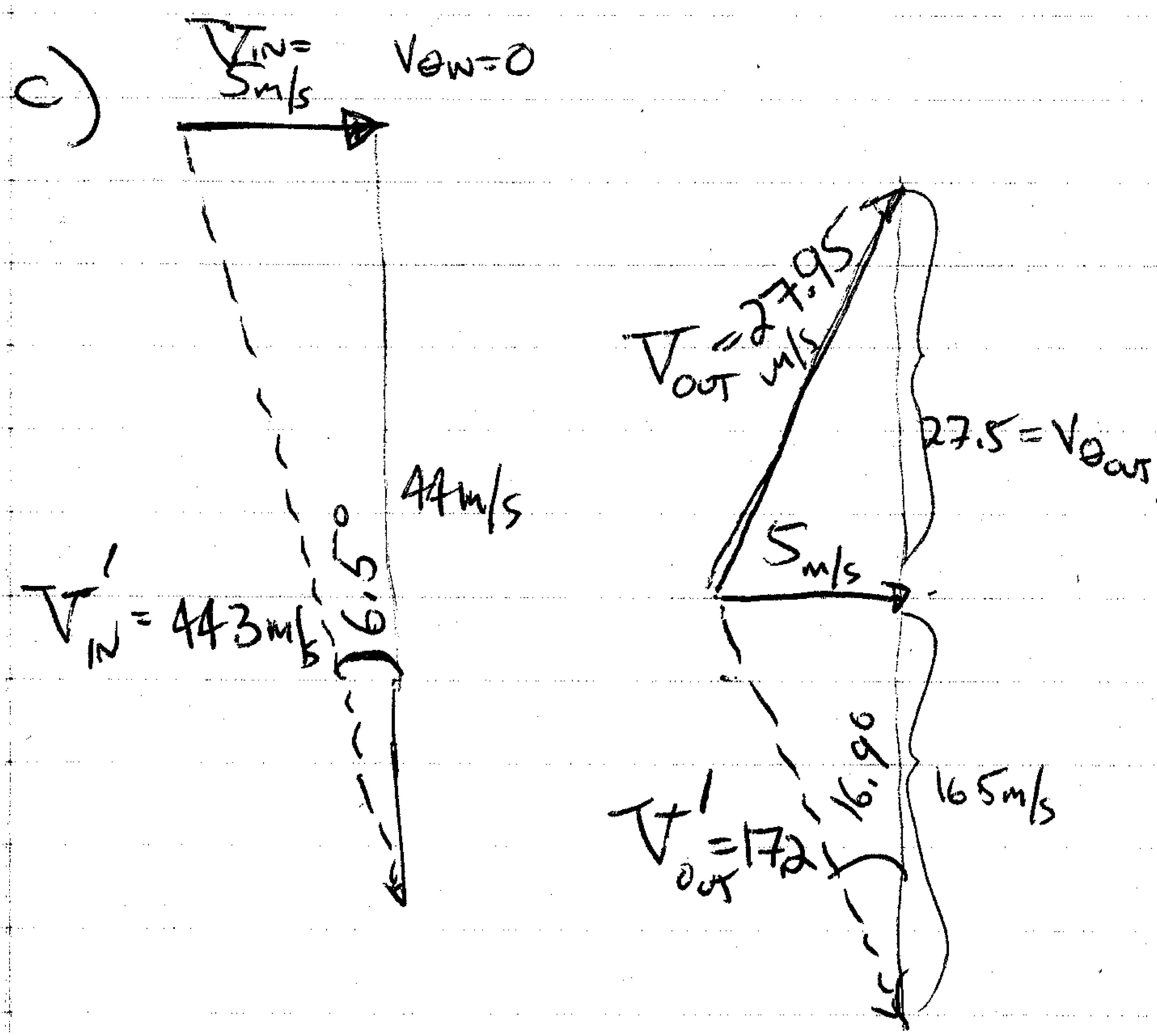
$$= \underline{1.8\ \tfrac{kJ}{m^2}}$$

b)



$$\frac{P}{A} = (27.5)(1.2)(8)(1.17 - 0) = 0.31\ \frac{kJ}{m^2}$$

c)



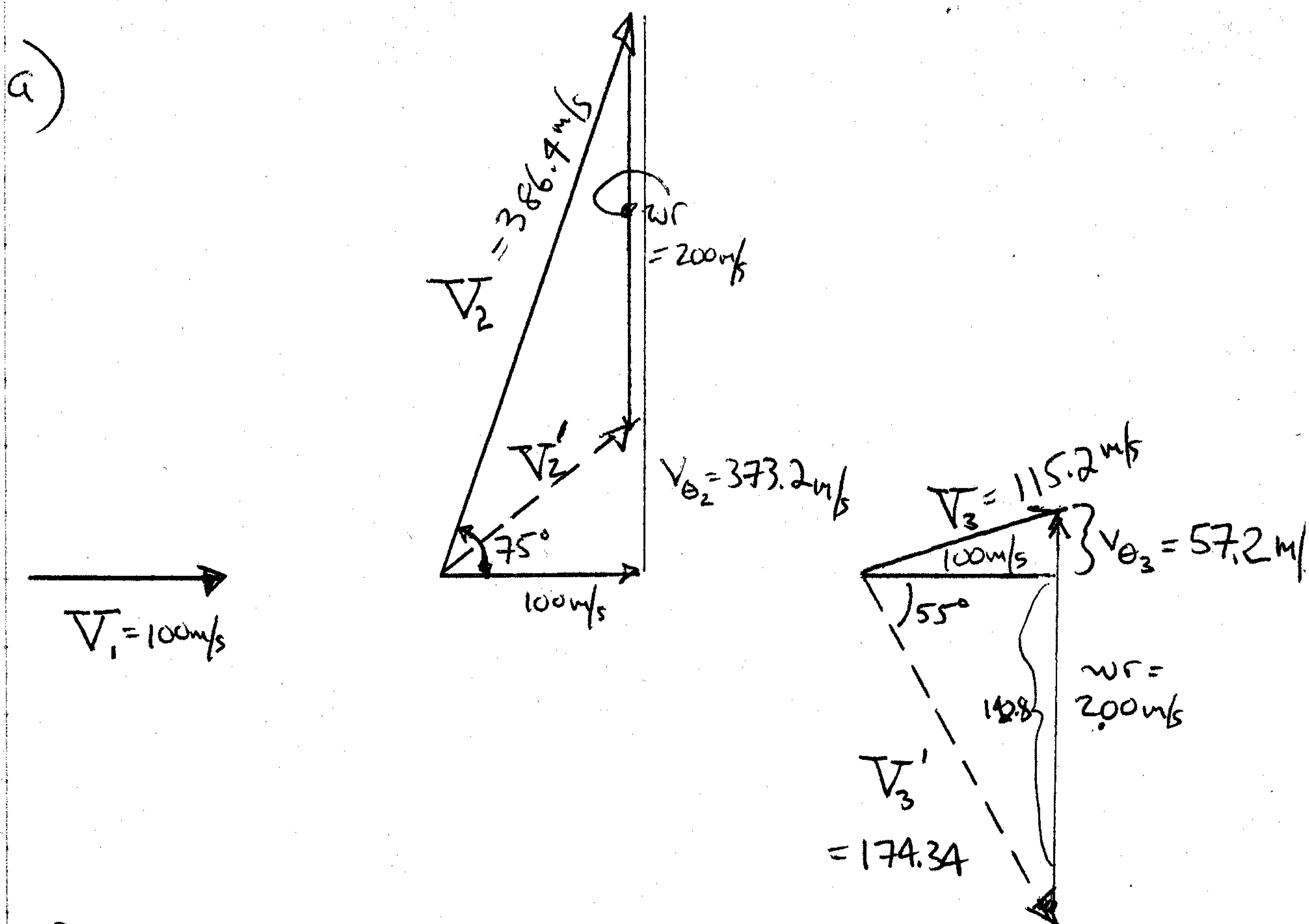
$$\frac{P}{A} = (44\text{ m/s})(1.2)(5)(27.5 - 0) = 7.3\ \frac{kJ}{m^2}$$

d) AT LOW $\lambda = \frac{V_{IN}}{\omega R}$ THE ANGLE OF ATTACK ON THE BLADE CAN BE LARGE LEADING TO STALL.

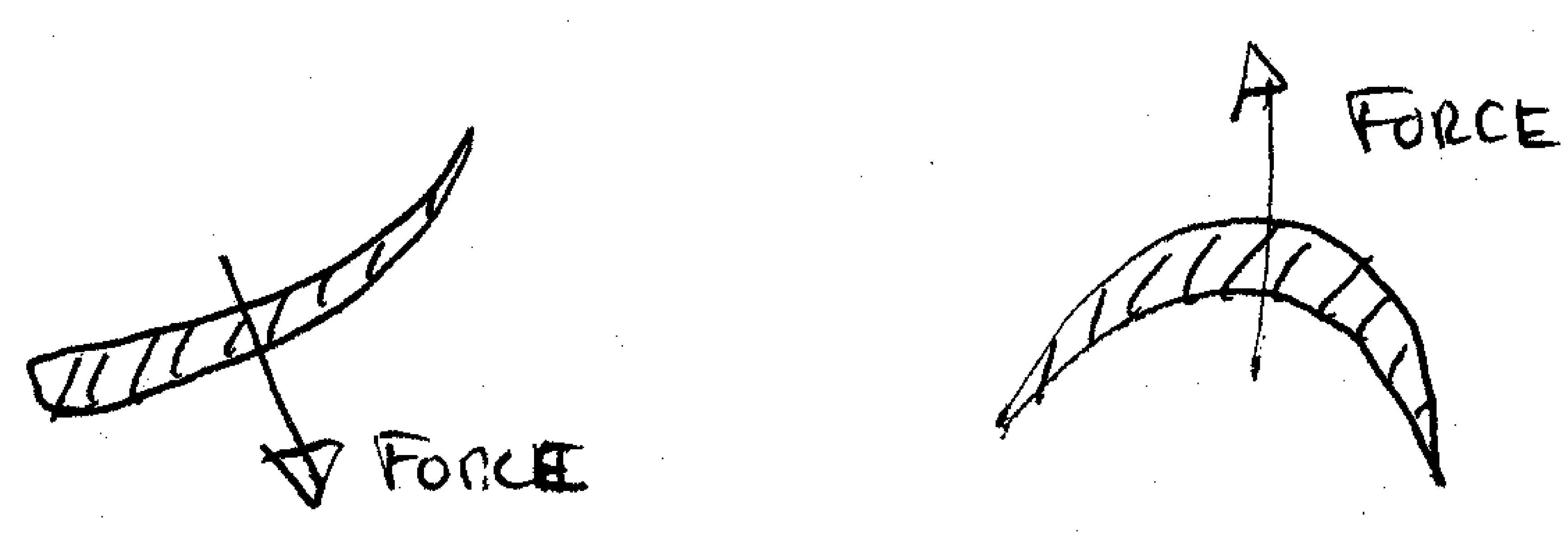
AT HIGH $\lambda = \frac{V_{IN}}{\omega R}$, THE ANGLE OF ATTACK CAN BE NEGATIVE SUCH THAT $V_{\theta OUT}$ IS < 0

P9 SOLUTIONS — WAITZ

a)

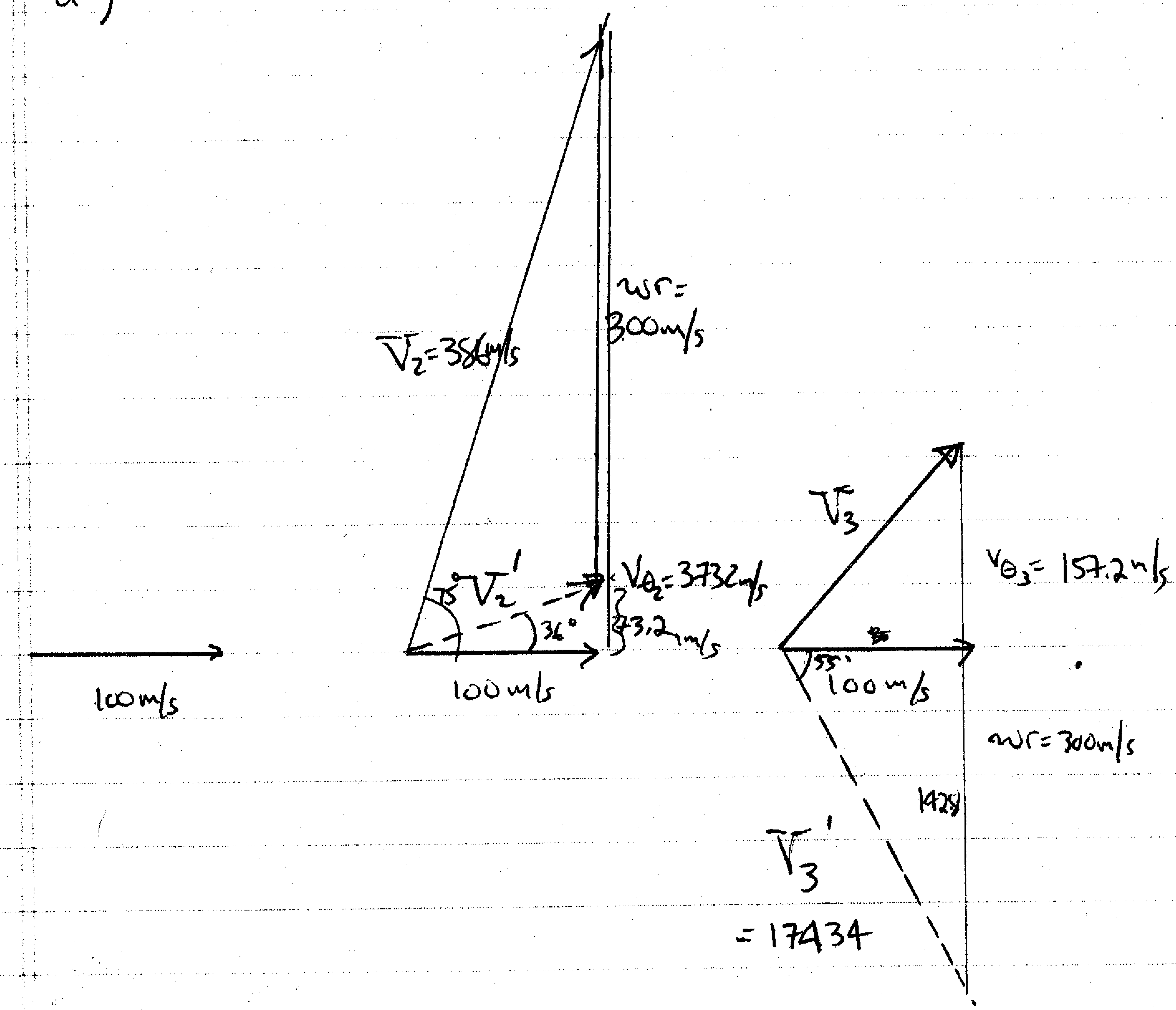


b) THERE IS A TORQUE APPLIED TO BOTH BLADE ROWS (THERE IS A CHANGE IN DIRECTION OF THE FLOW)



c) $$P = \omega \dot{m} \left[r V_{\theta_3} - r V_{\theta_2} \right]$$

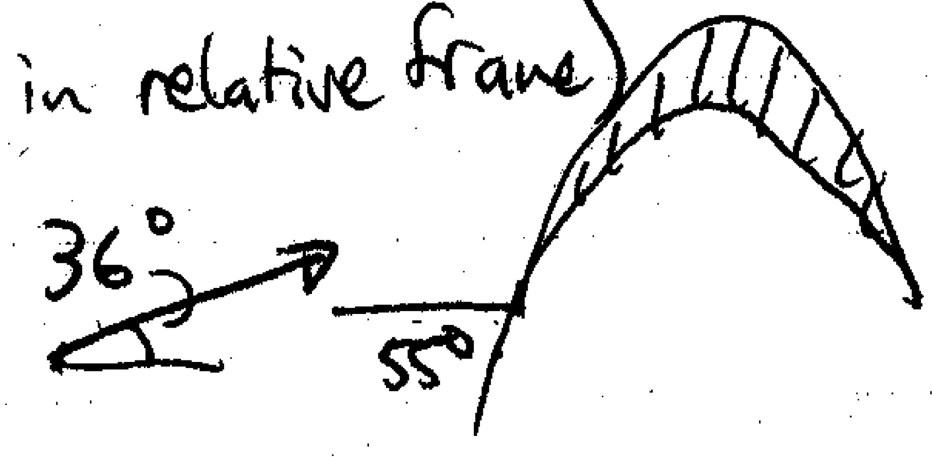
THIS IS A TURBINE THE TANGENTIAL COMPONENT OF VELOCITY IS REDUCED ACROSS THE MOVING (in abs. frame) BLADE ROW (WHERE + IS DEFINED IN THE DIRECTION OF ROTOR MOTION) d)



$$\dot{P} = \omega \dot{m} \left[r V_{\theta_3} - r V_{\theta_2} \right]$$

$$\frac{\left(\frac{P}{\dot{m}}\right)_{\omega r = 300 m/s}}{\left(\frac{P}{\dot{m}}\right)_{\omega r = 200 m/s}} = \frac{\left[157.2 - 373.2\right]}{\left[57.2 - 373.2\right]} = 0.68$$

AT $\omega r = 300 m/s$ THERE IS A NEGATIVE ANGLE OF ATTACK ON THE TURBINE (19° in relative frame) MAY CAUSE SEPARATION OF FLOW.



Solution

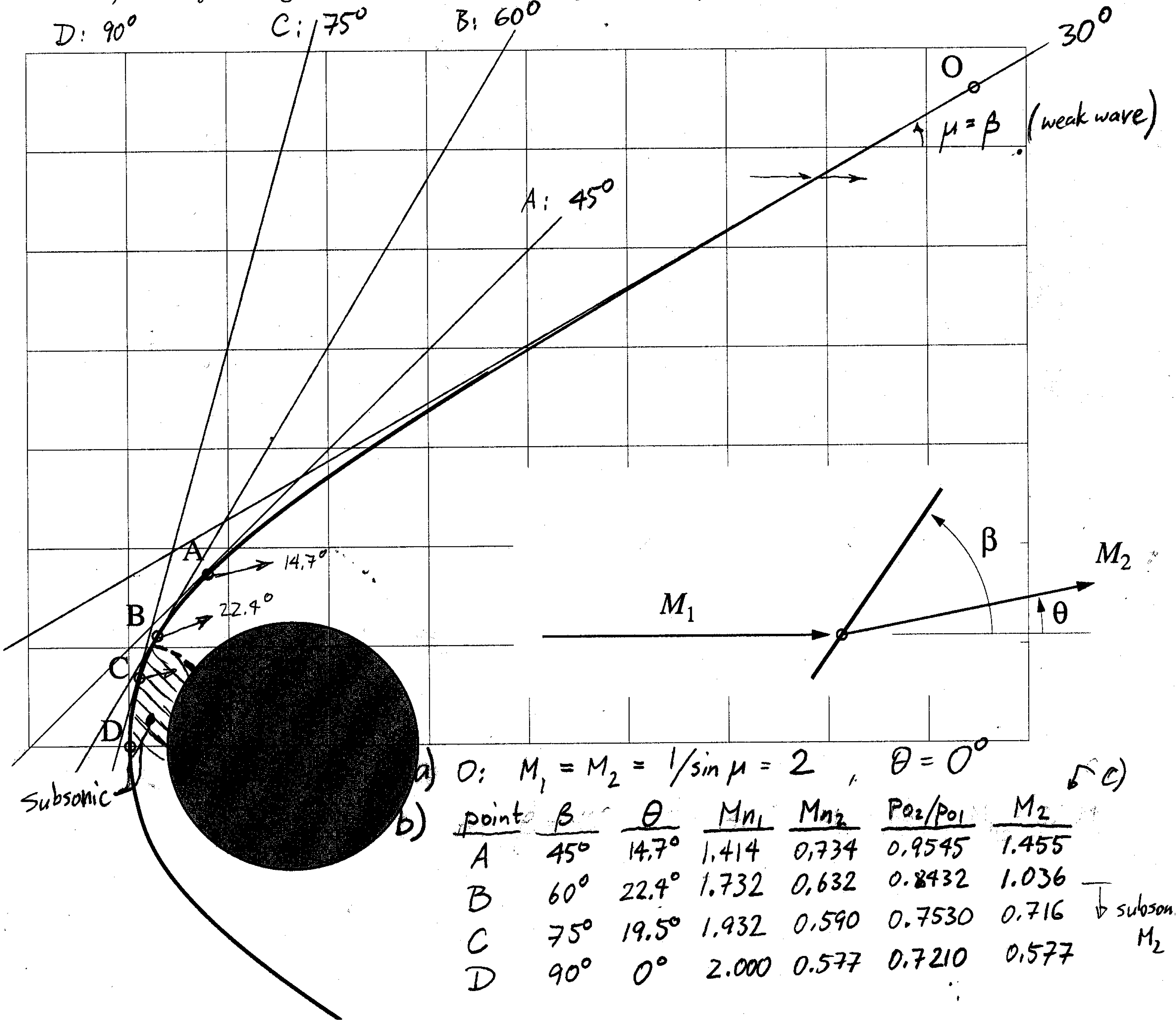
Unified Engineering Spring 2005
Fluids Problem F16+17

The figure shows the bow shock around a supersonic sphere, taken from a photo so that it's fairly accurate. The grid has been added to allow graphical estimation of the local shock wave angles via tangent lines and trigonometry.

a) Using the drawing at point O, where the shock is extremely weak, estimate the freestream Mach number $M_\infty = M_1$. Locate this point on the β vs θ vs M_1 oblique shock chart (Anderson p 513). Submit a xerox copy of the chart.

b) For points A, B, C, D on the drawing, determine or compute the following: Table, App B
— Shock wave angle β and turning angle θ from chart
— Normal Mach numbers M_{n_1} and M_{n_2} $\quad M_{n_1} = M_1 \sin\beta$, $M_{n_2} = f(M_{n_1})$, $M_2 = \dfrac{M_{n_2}}{\sin(\beta-\theta)}$
— Total pressure ratio p_{o_2}/p_{o_1} from Table, App B
— Location on the oblique shock chart

c) Identify the region where the flow is locally subsonic (Hint: determine M_2 at the points).



a) O: $M_1 = M_2 = 1/\sin\mu = 2$, $\theta = 0^\circ$ c)

b)

point	β	θ	M_{n_1}	M_{n_2}	p_{o_2}/p_{o_1}	M_2
A	45°	14.7°	1.414	0.734	0.9545	1.455
B	60°	22.4°	1.732	0.632	0.8432	1.036
C	75°	19.5°	1.932	0.590	0.7530	0.716
D	90°	0°	2.000	0.577	0.7210	0.577

subson. M_2

F16 + 17 Solution Spring '05

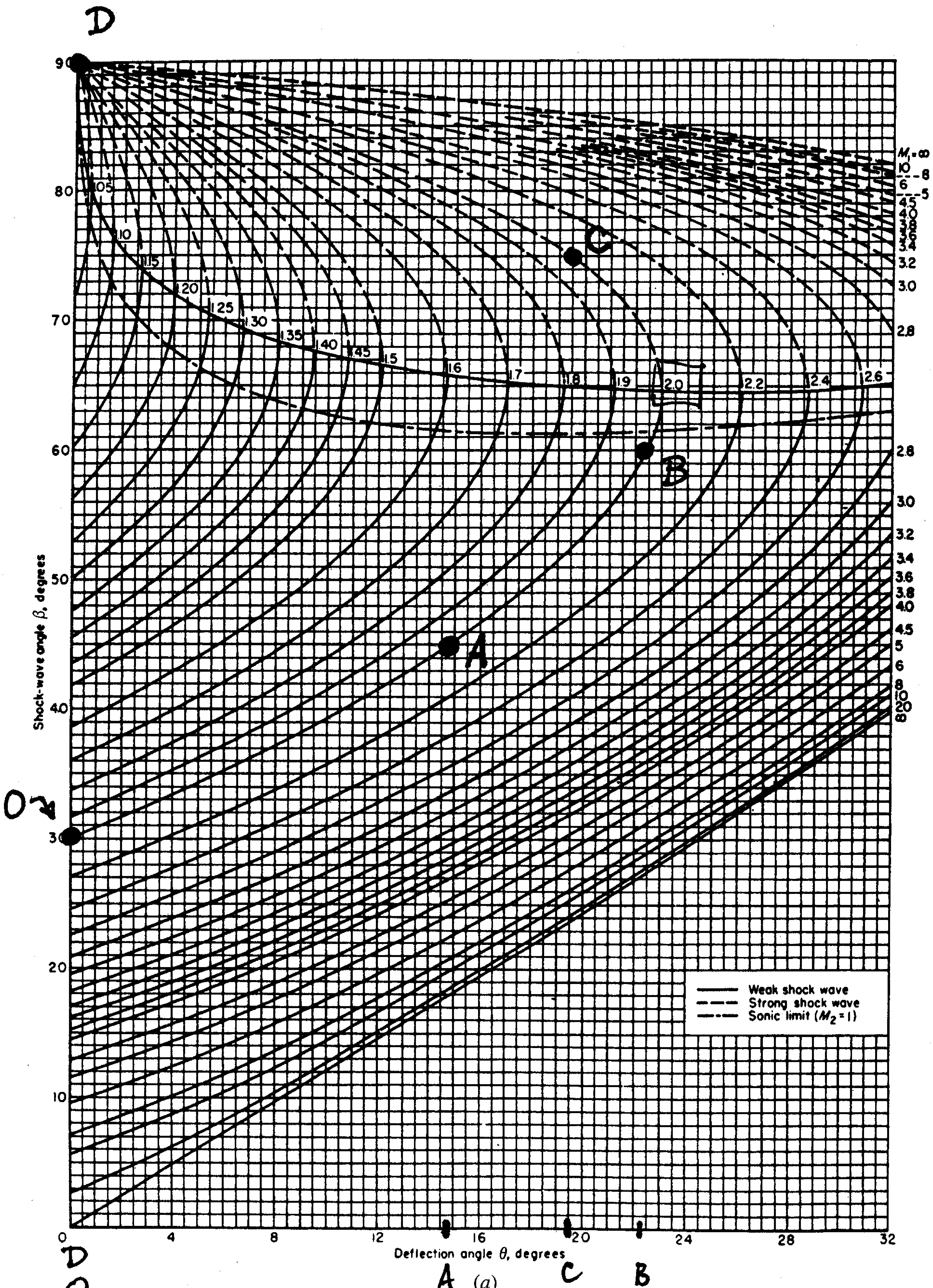


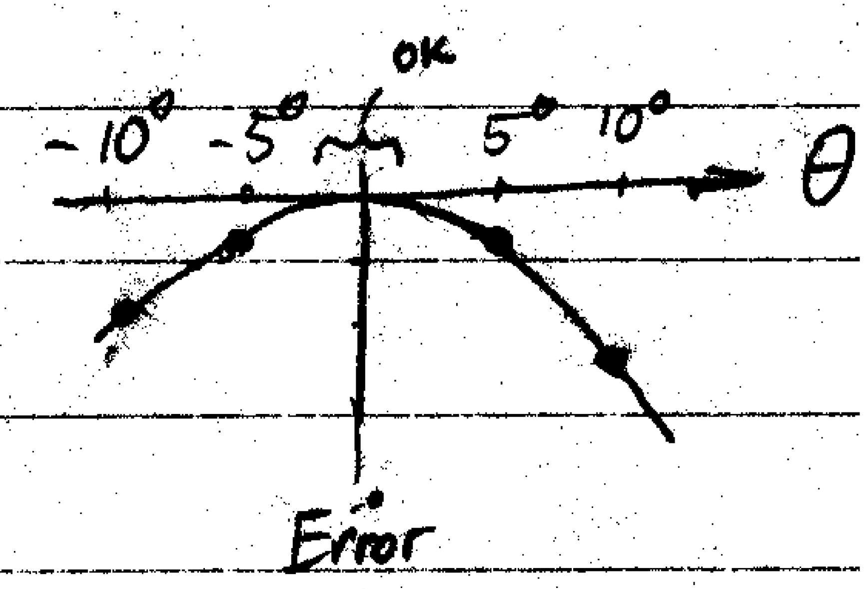
Figure 9.7 Oblique shock properties: $\gamma = 1.4$. The θ-β-M diagram. *(Source: NACA Report 1135, Ames Research Staff, "Equations, Tables and Charts for Compressible Flow," 1953.)*

UE Fluids F18 Solution Spring '05

	M_1	θ	β	M_{n_1}	p_2/p_1	$\frac{2}{\gamma M_1^2}\left(\frac{p_2}{p_1}-1\right)$	$\frac{2\theta}{\sqrt{M_1^2-1}}$	% Error
A	1.5	5°	47.8°	1.111	1.271	0.172	0.156	−9.3%
B	1.5	10°	56.5°	1.251	1.656	0.417	0.312	−25.0%

	M_1	$\nu(M_1)+\theta$ = $\nu(M_2)$ →	M_2	p_2/p_1	$\frac{2}{\gamma M_1^2}\left(\frac{p_2}{p_1}-1\right)$	$\frac{-2\theta}{\sqrt{M_1^2-1}}$	% Error
C	1.5	$11.91^\circ + 5^\circ = 16.91^\circ$	1.67	0.778	−0.141	−0.156	−9.3%
D	1.5	$11.91^\circ + 10^\circ = 21.91^\circ$	1.84	0.601	−0.253	−0.312	−18.8%

Approximate ΔC_p formula OK only for very small θ.



Relations used:

For A, B (oblique shocks): $\beta(M_1, \theta)$ from Chart, p513

$M_{n_1} = M_1 \sin\beta$

$p_2/p_1 = f(M_{n_1})$ from normal-shock Table, App B

For C, D (expansion fans): $\nu(M_1)$ from P-M table, App. C

$\nu(M_2) = \nu(M_1) + \theta$

M_2 = inverse of $\nu(M_2)$ from P-M table, App C.

$$\frac{p_2}{p_1} = \cancel{\frac{p_{o2}}{p_{o1}}}\left\{\frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\frac{\gamma-1}{2}M_1^2\right]}\right\}^{\frac{-\gamma}{\gamma-1}} , \quad \frac{p_{o2}}{p_{o1}} = 1$$

UE Fluids F20 Solution Spring '05

$\frac{A_e}{A_{throat}} = 1.53$ (given)

For each case, first assume no choking, $p_{oe} = p_r$.

a) $p_e = 0.94$ atm: $\frac{p_e}{p_o} = 0.94 = \left[1+\frac{\gamma-1}{2}M_e^2\right]^{\frac{-\gamma}{\gamma-1}} \rightarrow \boxed{M_e = 0.2986}$

$\rightarrow \boxed{\frac{A_e}{A^*} = 2.03 > 1.53 \text{ OK not choked}}$

b) $p_e = 0.886$ atm: $\frac{p_e}{p_o} = 0.886 = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}} \rightarrow \boxed{M_e = 0.4195}$

$\boxed{\frac{A_e}{A^*} = 1.53 = 1.53 \text{ OK}}$ barely choked

c) $p_e = 0.75$ atm will be choked. From b) result $\dot{m}$ is known.

$$\dot{m} = \rho u A = \rho a M A = \frac{\gamma p}{(\gamma-1)^{1/2}} \frac{M}{h_o^{1/2}}\left[1+\frac{\gamma-1}{2}M^2\right]^{1/2} A \quad \text{(general result)}$$

Equating c) and b) $\dot{m}$'s at exit: $\dot{m}_{c)} = \dot{m}_{b)}$

$$\left\{\frac{\gamma p_e}{(\gamma-1)^{1/2}} \frac{M_e}{h_o^{1/2}}\left[1+\frac{\gamma-1}{2}M_e^2\right]^{1/2} A_e\right\}_{c)} = \left\{\frac{\gamma p_e}{(\gamma-1)^{1/2}} \frac{M_e}{h_o^{1/2}}\left[1+\frac{\gamma-1}{2}M_e^2\right]^{1/2} A_e\right\}_{b)}$$

$$\left\{p_e M_e\left[1+\frac{\gamma-1}{2}M_e^2\right]^{1/2}\right\}_{c)} = \left\{p_e M_e\left[1+\frac{\gamma-1}{2}M_e^2\right]^{1/2}\right\}_{b)}$$

Plugging in for $M_{e_b}, p_{e_b}, p_{e_c}$: $M_e^2\left[1+\frac{\gamma-1}{2}M_e^2\right] = \left(\frac{0.886}{0.75}\right)^2 0.4195^2\left[1+\frac{\gamma-1}{2}0.4195^2\right]$

$= 0.2542$

$\frac{\gamma-1}{2}M_e^4 + M_e^2 - 0.2542 = 0 \rightarrow M_e^2 = \frac{1}{\gamma-1}\left[-1+\sqrt{1+2(\gamma-1)0.2542}\right] = 0.2425$

$\boxed{M_e = 0.4924}$

d) Supersonic outflow, $p_o = p_r = 1$ atm

$\frac{p_e}{p_o} = 0.154 = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}} \rightarrow \boxed{M_e = 1.88} \rightarrow \boxed{\frac{A}{A^*} = 1.53}$ match nozzle