Massachusetts Institute of Technology
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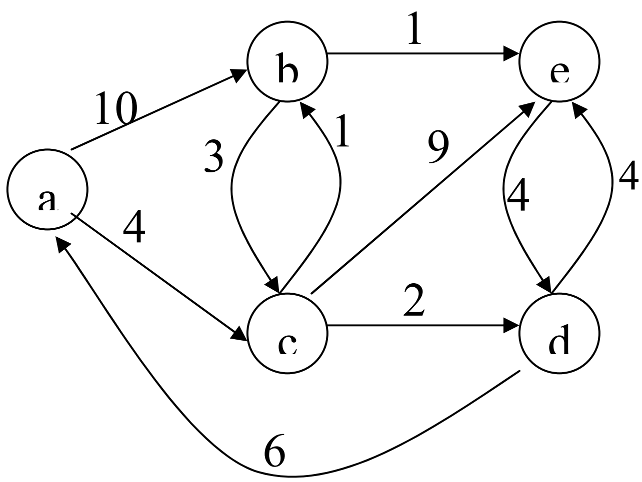
Cambridge, MA 02139

Unified Engineering
Spring 2005

Problem Set #8
Solutions

Problem C5. Graphs, Shortest Path

What is the Shortest Path through the graph shown below using Dijkstra's algorithm?



Show all the steps in the computation of the shortest path.

Initialize

V = {a, b, c, d, e}
E = {(a,b), (a,c), (b,e), (b,c), (c,b), (c,d), (c,e), (d,a), (d,e), (e.d)}
S = {∅}
Q = {a, b, c, d, e}

D = [0, ∞, ∞,∞,∞]
Previous = [0, 0, 0, 0, 0]
Start at A

Relax (a,b,10)
Relax (a,c,4)

S = {a}
Q = {b, c, d, e}

D = [0, 10, 4,∞,∞]
Previous = [0,a, a, 0, 0]

Move to C

Relax(c,b,1)
Relax (c,d,2)
Relax(c,e,9)

S = {a,c}
Q = {b, d, e}

D = [0, 5, 4, 6, 13]
Previous = [0,c, a, c, c]

Move to B

Relax(b,e,1)
Relax(b,c,3)

S = {a, c, b}
Q = { d, e}

D = [0, 5, 4, 6, 6]
Previous = [0, c, a, c, b]

Move to D (You can also move to E)

Relax(d,a,6)
Relax(d,e,4)

S = {a, c, b, d}
Q = {e}

D = [0, 5, 4, 6, 6]
Previous = [0, c, a, c, b]

Move to E

Relax(e,d,4)

S = {a, c, b, d, e}
Q = {}

D = [0, 5, 4, 6, 6]
Previous = [0, c, a, c, b]

C6 Tree Traversal

a.

i. Inorder Traversal: a/b-c*d*e/f+g

ii. Preorder Traversal: * -/ a b c / * d e + f g

iii. Postorder Traversal: ab/c-de*fg+/*

b. Algorithm for generating an expression tree from a prefix expression

1. Read the prefix expression from right to left
2. Examine the next element in the input.
3. If it is operand then
   a. create a leaf node i.e. node having no child (node- >left_child=node- >right_child=NULL)
   b. copy the operand in data part
   c. PUSH node's address on stack
4. If it is an operator, then
   a. create a node
   b. copy the operator in data part
   c. POP address of node from stack and assign it to node->left_child
   d. POP address of node from stack and assign it to node->right_child
   e. PUSH node's address on stack
5. If there is more input go to step 2
6. If there is no more input, POP the address from stack, which is the address of the ROOT node of Expression Tree.

c. Procedure for creating an expression tree from a prefix expression

```
procedure Create_Expression_Tree_From_Prefix (
      Root :    out Nodeptr) is
   Temp,
   Leftchild,
   Rightchild    : Nodeptr;
   Nodeptr_Stack : My_Stack;
   Length        : Integer;
   Expression    : String (1 .. 80);

begin
   -- create a temporary stack for
   My_Pointer_Stack.Create(Nodeptr_Stack);
   Put("Please enter an expression in prefix form: ");
   Get_Line(Expression,Length);
   for I in reverse 1 .. Length loop
      if Isdigit(Expression(I)) then
         Temp := new Node;
         Temp.Element := Expression(I);
         Temp.Left_Child := null;
```

```
         Temp.Right_Child := null;
         Push(Nodeptr_Stack,Temp);
      end if;

      if Isoperator(Expression(I)) then
         Temp := new Node;
         Temp.Element := Expression(I);
         Pop(Nodeptr_Stack, Leftchild);
         Pop(Nodeptr_Stack, Rightchild);
         Temp.Left_Child := Leftchild;
         Temp.Right_Child := Rightchild;
         Push(Nodeptr_Stack, Temp);
      end if;

   end loop;

   Pop(Nodeptr_Stack, Temp);
   if Isempty(Nodeptr_Stack) then
      Root := Temp;
   else
      Put_Line("Cannot create the expression tree");
      Root := null;
   end if;
end Create_Expression_Tree_From_Prefix;
```

C7 Spanning Trees/ Depth First and Breadth First Traversal

a. The modification to `expression_tree.ads` is shown below:

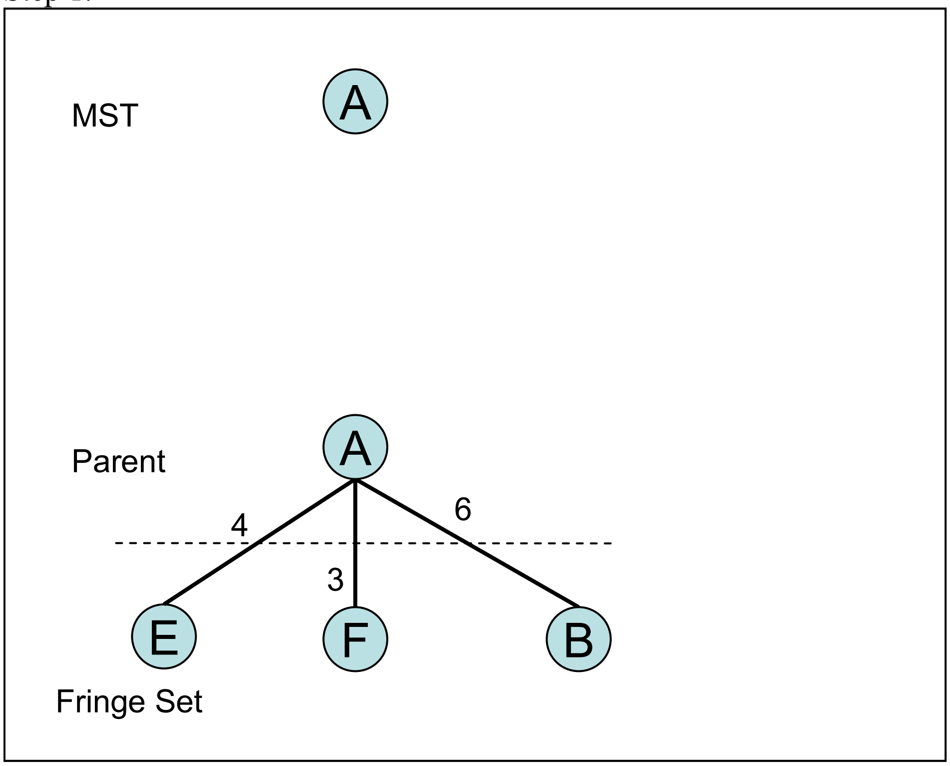
```
procedure Traverse_Dfs( Root : in Nodeptr);

procedure Traverse_Bfs( Root : in Nodeptr);
```

b. Implementation of depth-first and depth first search

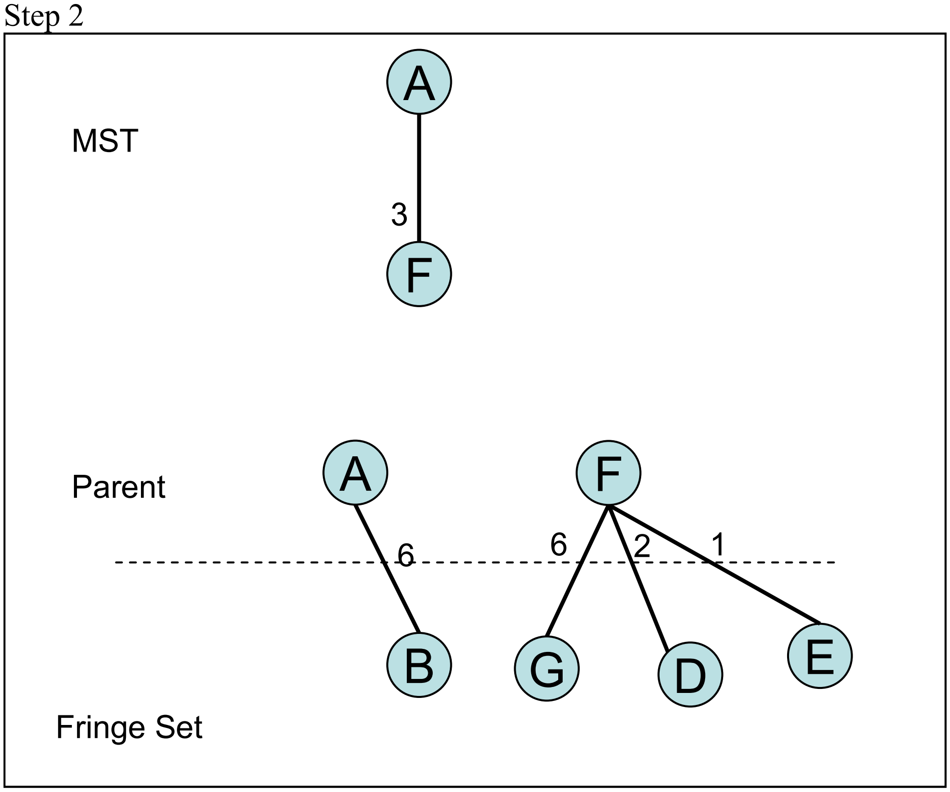
```
procedure Traverse_Dfs (
         Root : in     Nodeptr) is
   begin
      Traverse_Preorder(Root);
   end Traverse_Dfs;

   procedure Traverse_Bfs (
         Root : in     Nodeptr) is
      -- create a queue to hold node pointers
      My_Bfs_Queue : My_Queue;
      Temp         : Nodeptr;
   begin
      -- initialize the queue
      Init_Queue(My_Bfs_Queue);

      --queue in the root node
      Enqueue(My_Bfs_Queue, Root);

      -- loop until there are no nodes in the queue
      loop
         exit when Empty_Queue(My_Bfs_Queue);
         --get the first node from the queue
         Dequeue(My_Bfs_Queue, Temp);
         -- display the element
         Ada.Text_Io.Put(Temp.Element);
         Ada.Text_Io.New_Line;
         --if the left child is not null, enqueue it
         if Temp.Left_Child /= null then
            Enqueue(My_Bfs_Queue, Temp.Left_Child);
         end if;
         --if the right child is not null, enqueue it
         if Temp.Right_Child /= null then
            Enqueue(My_Bfs_Queue, Temp.Right_Child);
         end if;

      end loop;
      null;
   end Traverse_Bfs;
```

c. Minimum Weight Spanning Tree Using Prim's Algorithm

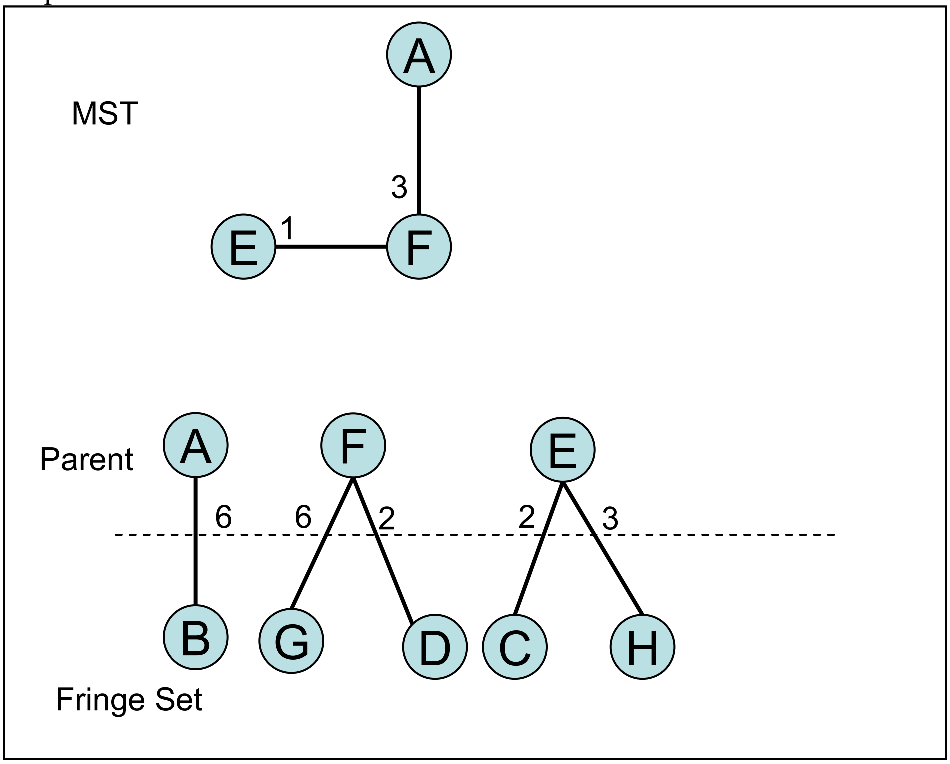
Step 1.



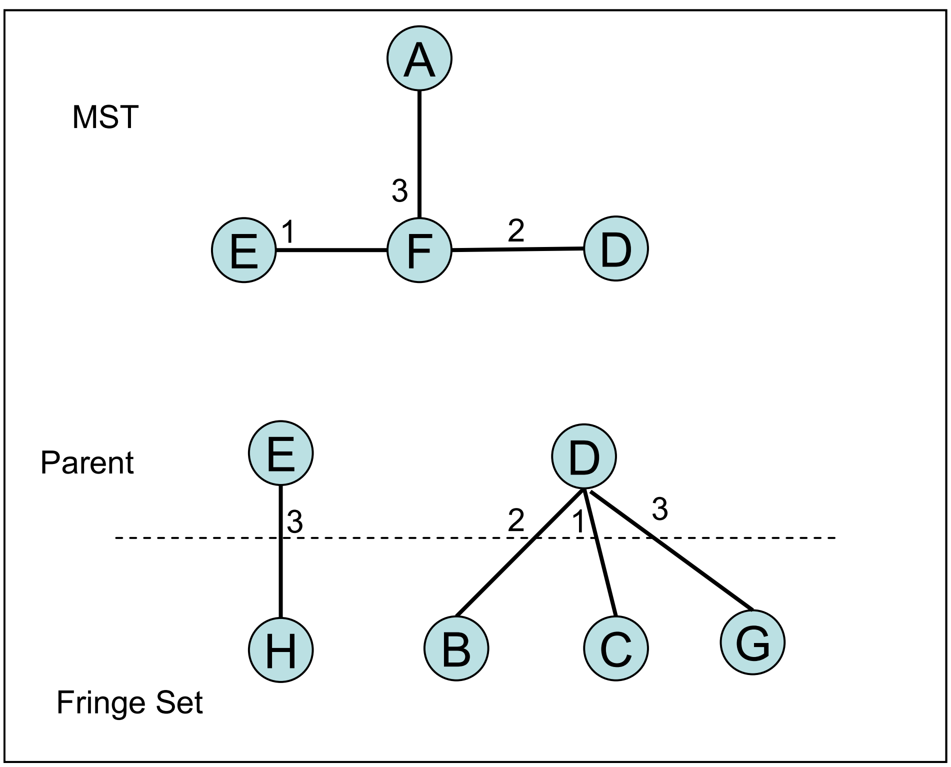
Step 2



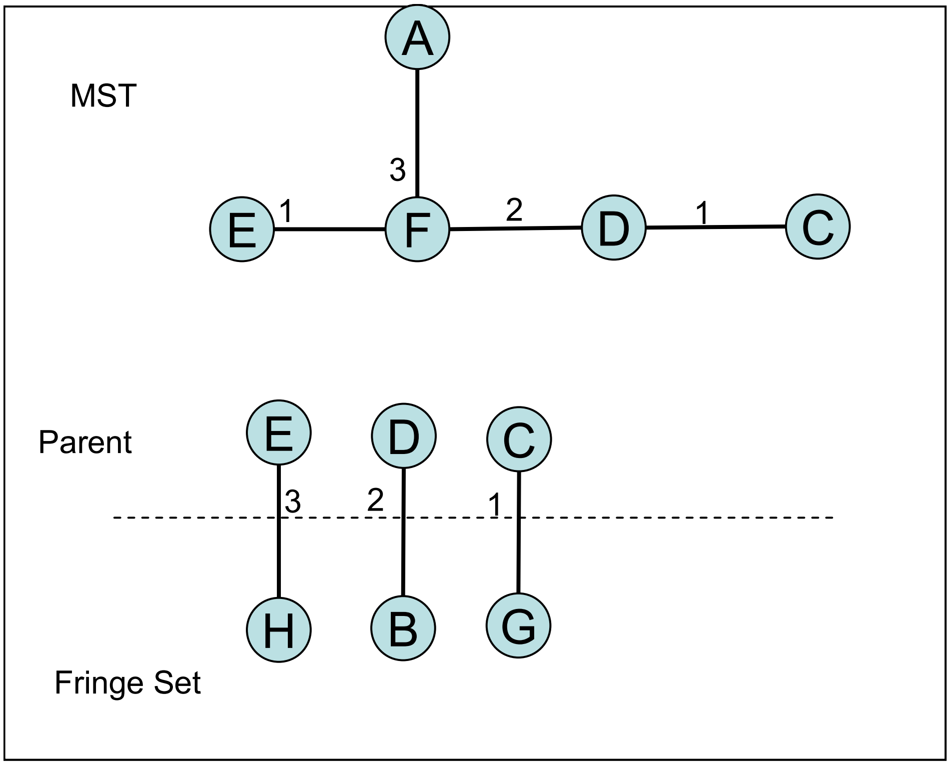
Step 3.


MST
A
3
E
1
F
Parent
A
6
B
F
6
G
2
D
E
2
C
3
H
Fringe Set

Step4

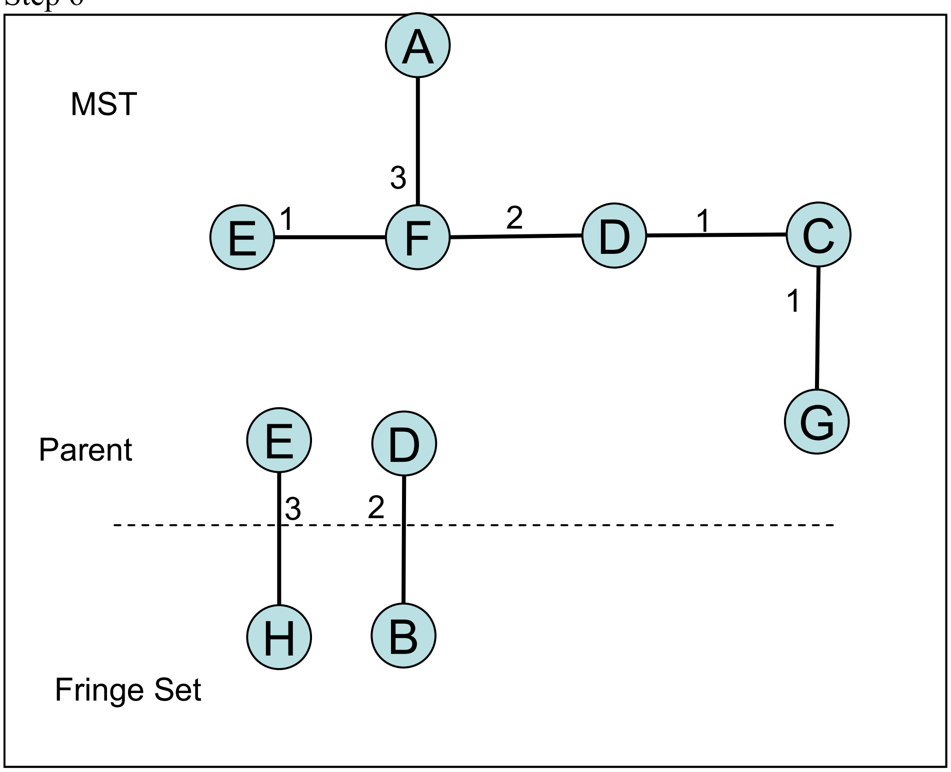

MST
A
3
E
1
F
2
D
Parent
E
3
H
D
2
B
1
C
3
G
Fringe Set

Step 5



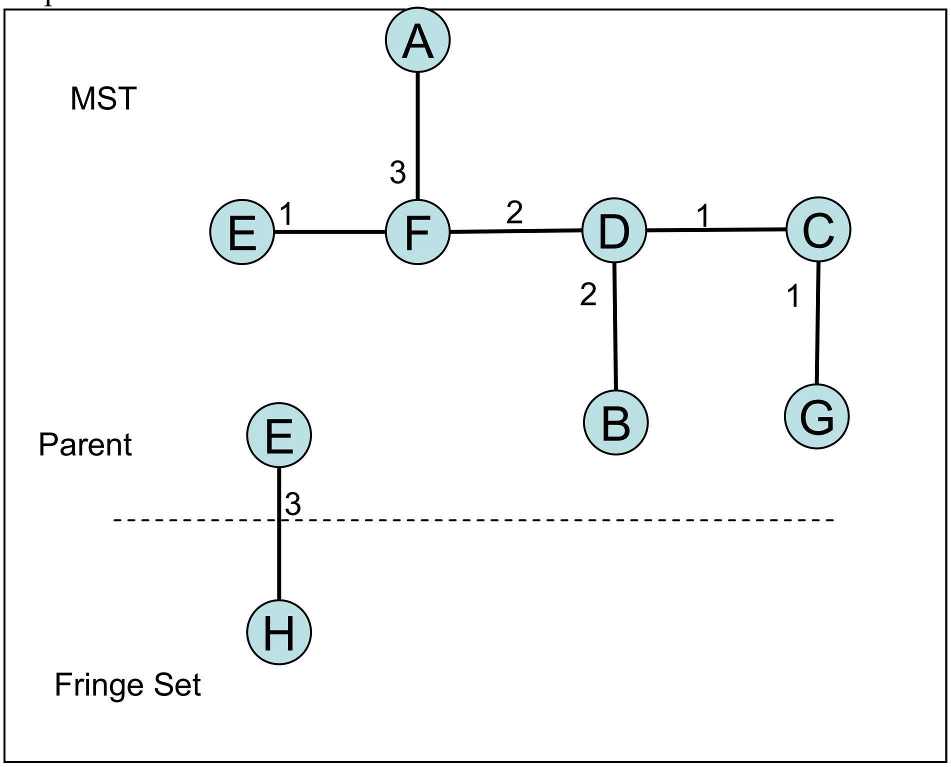
MST
A
3
E
1
F
2
D
1
C
Parent
E
D
C
3
2
1
H
B
G
Fringe Set

Step 6

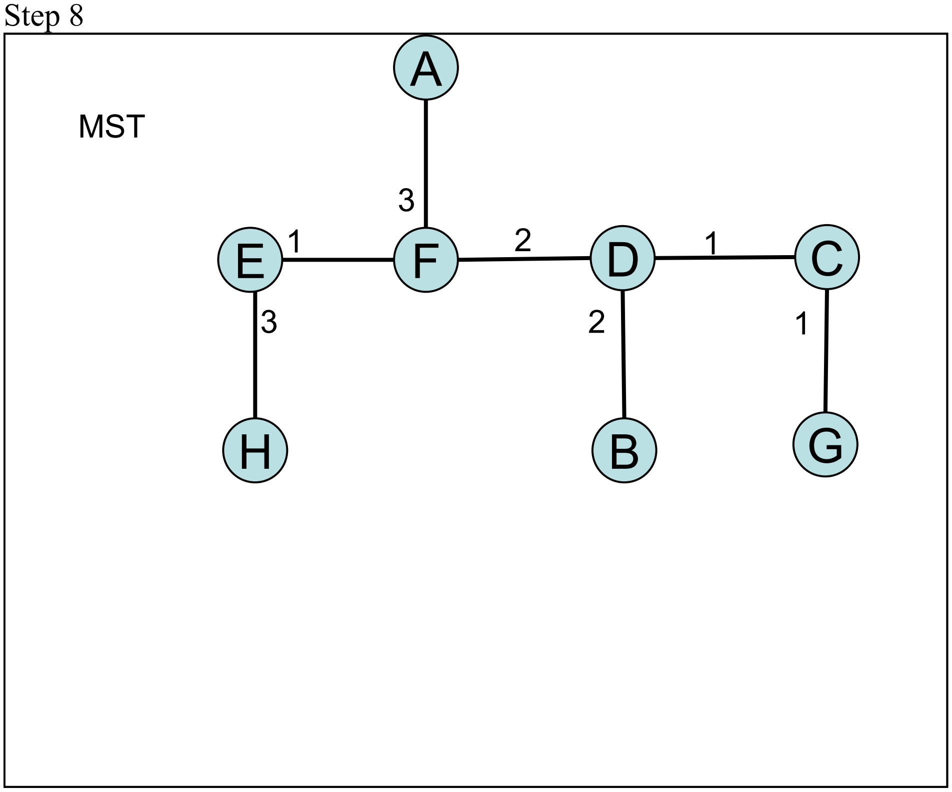


MST
A
3
E
1
F
2
D
1
C
1
G
Parent
E
D
3
2
H
B
Fringe Set

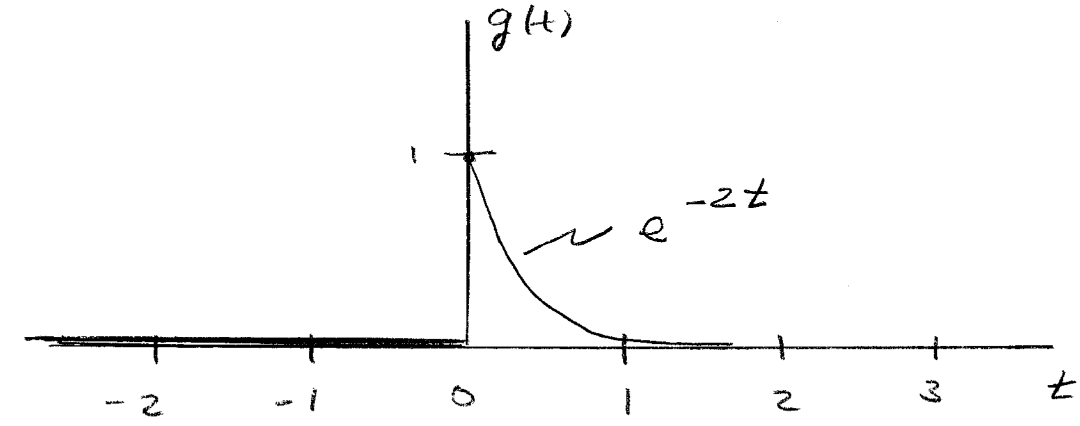
Step 7

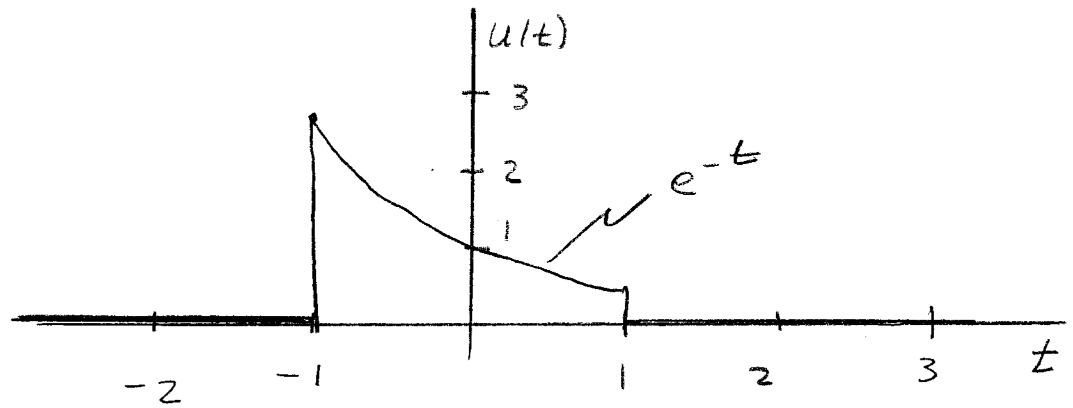

MST
A
3
E
1
F
2
D
1
C
2
1
B
G
Parent
E
3
H
Fringe Set

Step 8


MST
A
3
E
1
F
2
D
1
C
3
2
1
H
B
G

1.





2. To convolve, flip g(t), slide left and right, multiply by u(t), and find the area

There will be 3 important ranges:

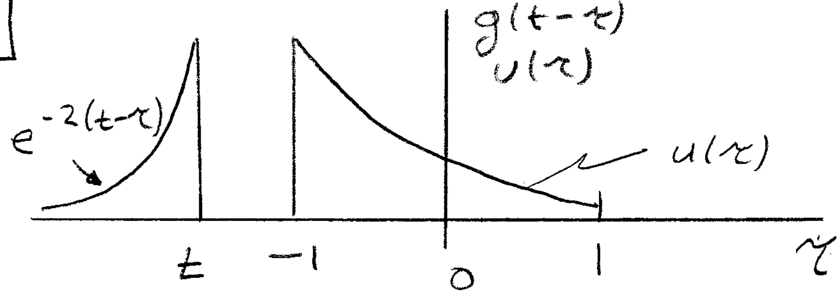
$t < -1$ no overlap

$-1 < t < 1$ overlap from -1 to t

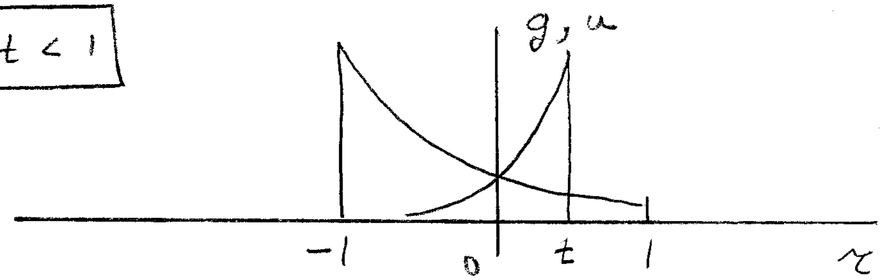
$t > 1$ overlap from -1 to 1

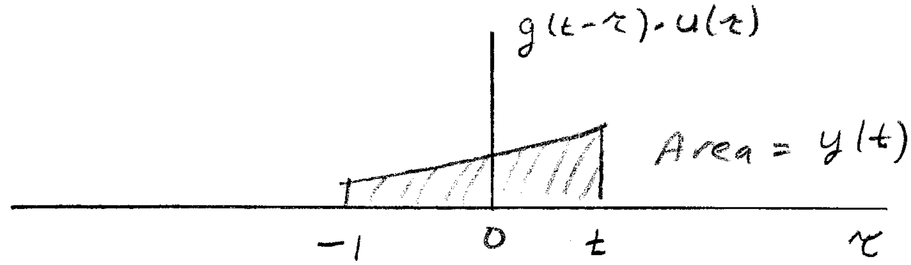
These are illustrated below:

$\boxed{t < -1}$

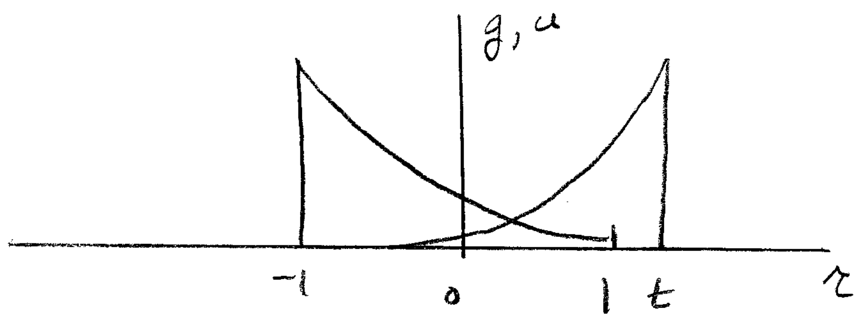


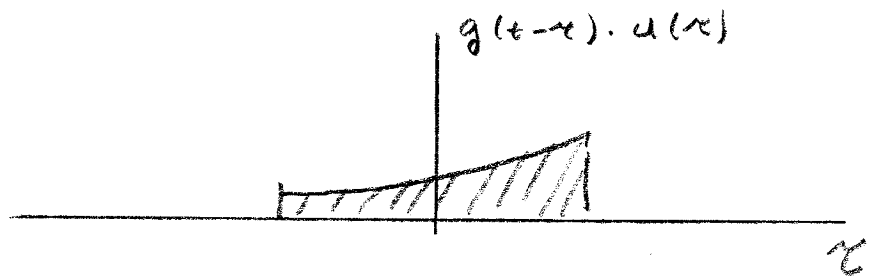
$\boxed{-1 < t < 1}$



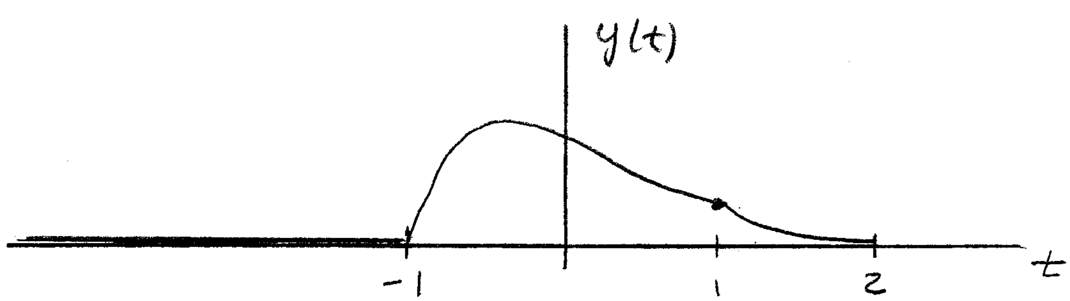


$\boxed{t > 1}$





So the convolution will look like:



3. The convolution is:

$\underline{t < -1}$ $\quad y(t) = 0$

$\underline{-1 < t < 1}$ $\quad y(t) = \int_{-1}^{t} e^{-2(t-\tau)} e^{-\tau} \, d\tau$

$$= e^{-2t} \int_{-1}^{t} e^{\tau} \, d\tau$$

$$= e^{-2t} \, e^{\tau} \Big|_{\tau=-1}^{t}$$

$$= e^{-2t} \left(e^{t} - e^{-1} \right)$$

$$= e^{-t} - e^{-2t-1}$$

$\underline{t > 1}$ $\quad y(t) = \int_{-1}^{1} e^{-2(t-\tau)} e^{-\tau}$

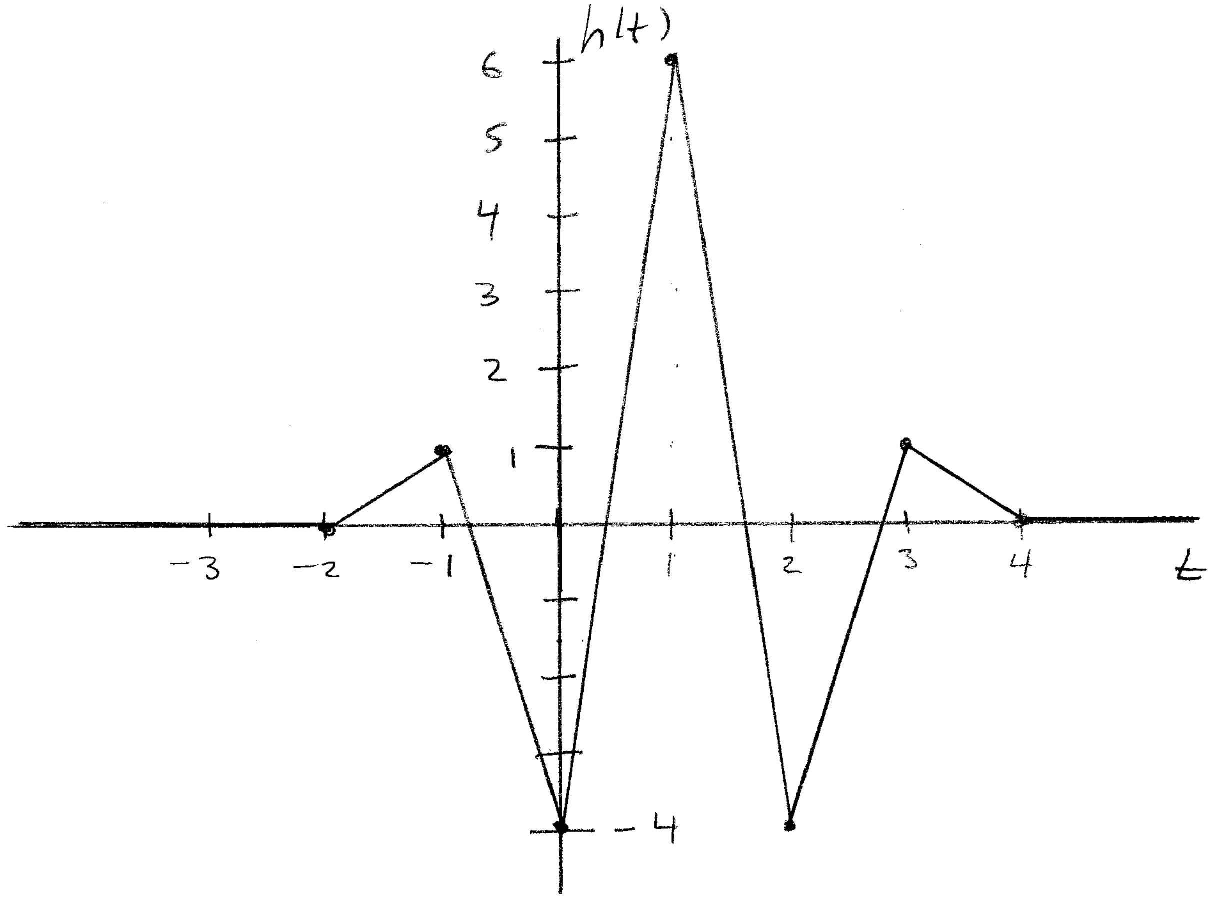
$$= e^{-2t} \, e^{\tau} \Big|_{\tau=-1}^{1}$$

$$= e^{-2t} \left(e^{1} - e^{-1} \right)$$

So,

$$y(t) = \begin{cases} 0, & t > -1 \\ e^{-t} - e^{-2t-1}, & -1 < t < 1 \\ e^{-2t}\left(e^{1} - e^{-1}\right), & t > 1 \end{cases}$$

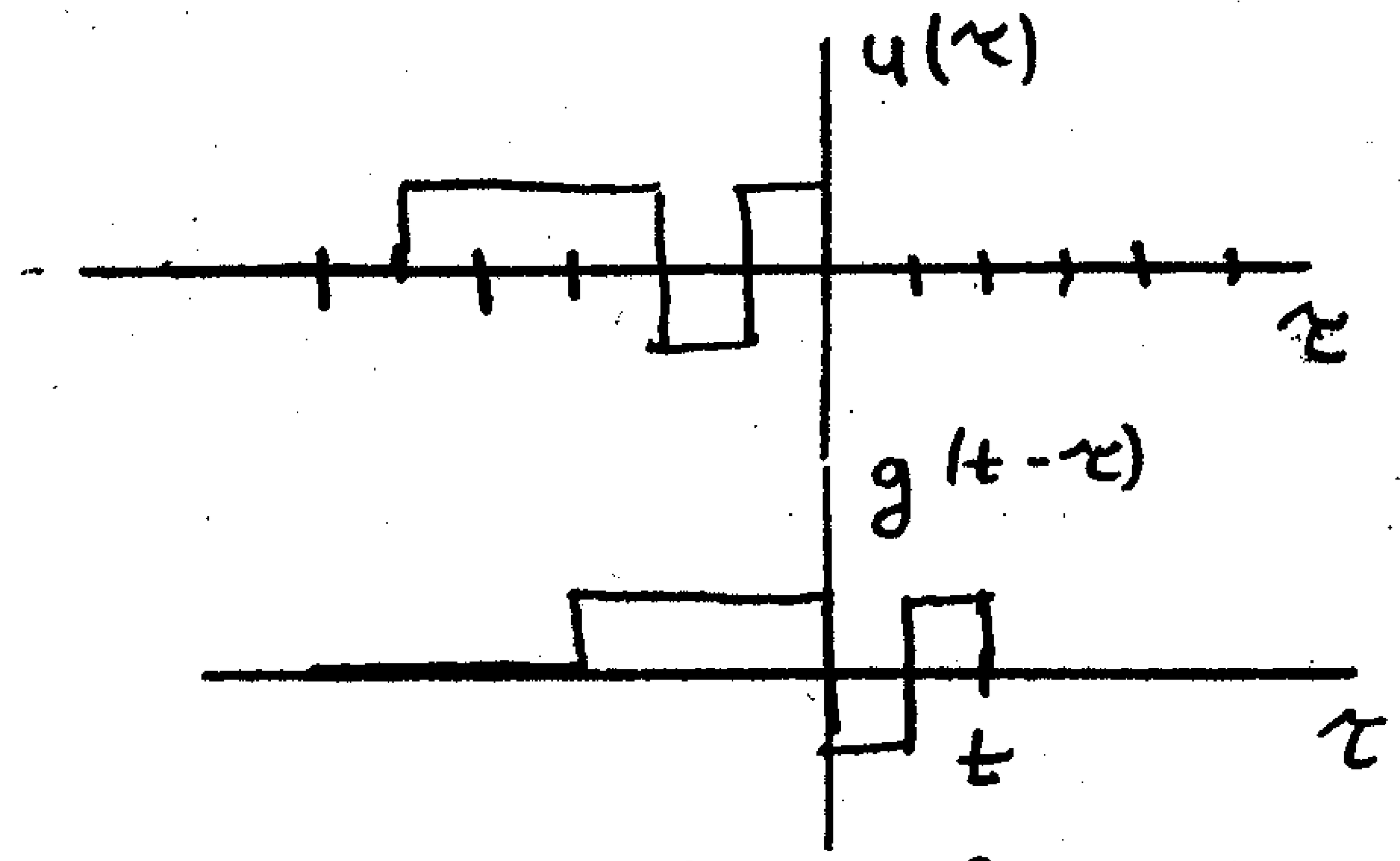
4. This part is very much like S6. See S6 solution for general approach. The result is



No. 5505
Engineer's Computation Pad

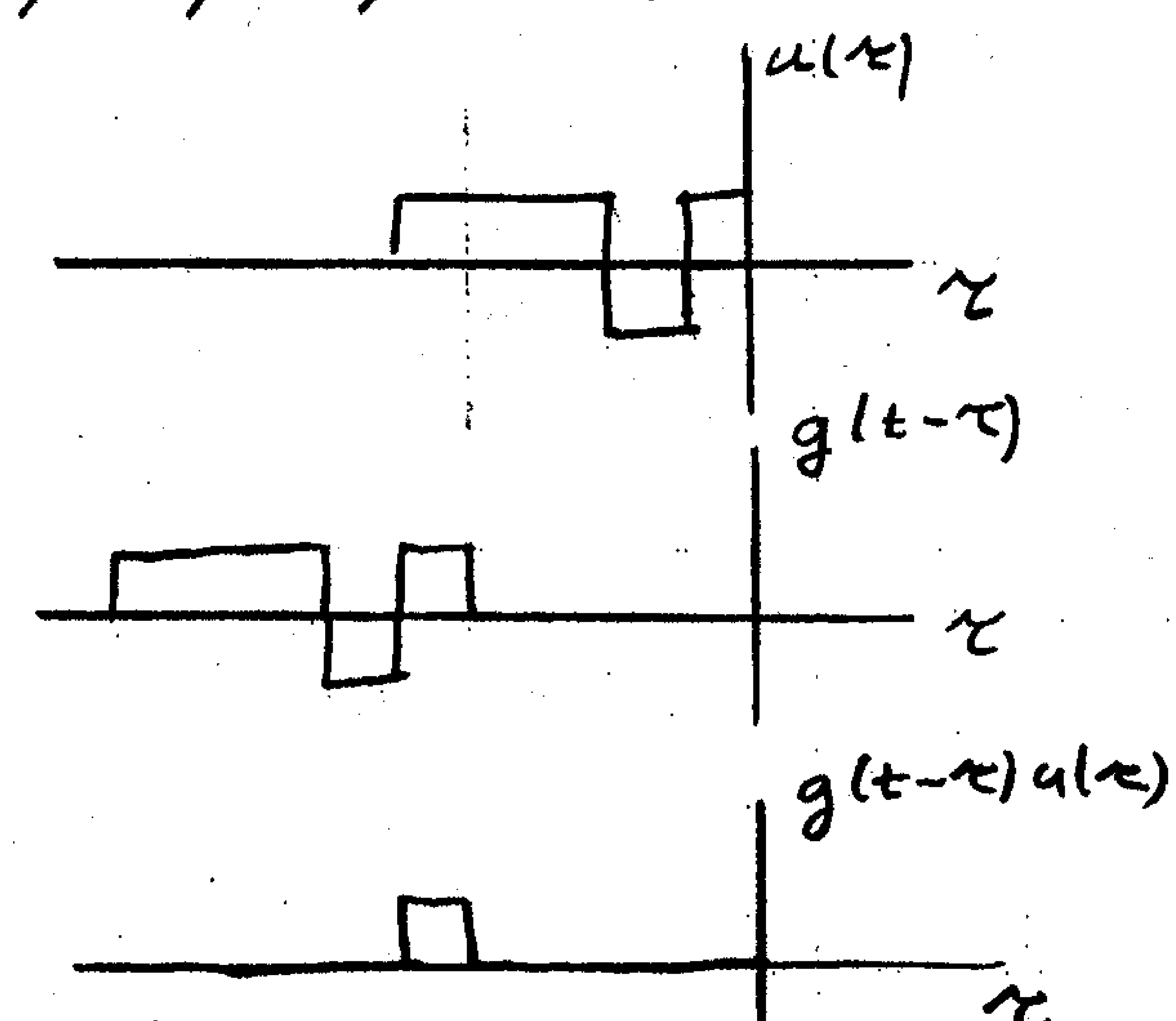
1. Because $g(t)$ & $u(t)$ are piecewise constant, $y(t)$ will be continuous and piecewise linear. We can find $y(t)$ at the "corners" by evaluating at $t =$ integer, since the corners of $g(t)$ & $u(t)$ occur at the integers.

So do flip & slide:



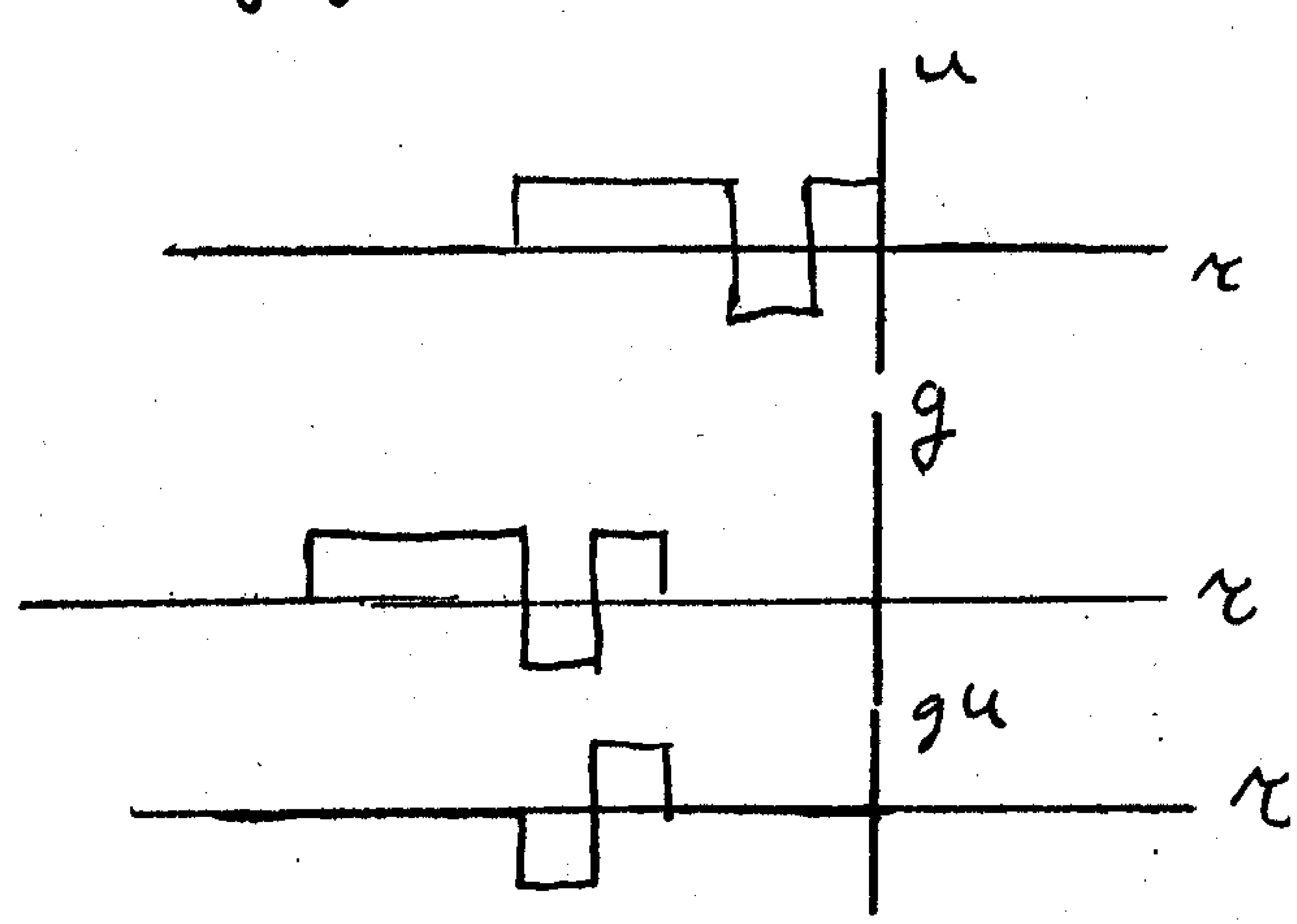
There is no overlap for $t < -5$ or $t > 5$. So do $t = -4, -3, -2, \ldots 4$

$t = -4$:



so $y(-4) = \int g(t-\tau)\,u(\tau)\,d\tau = 1$

$t = -3$:

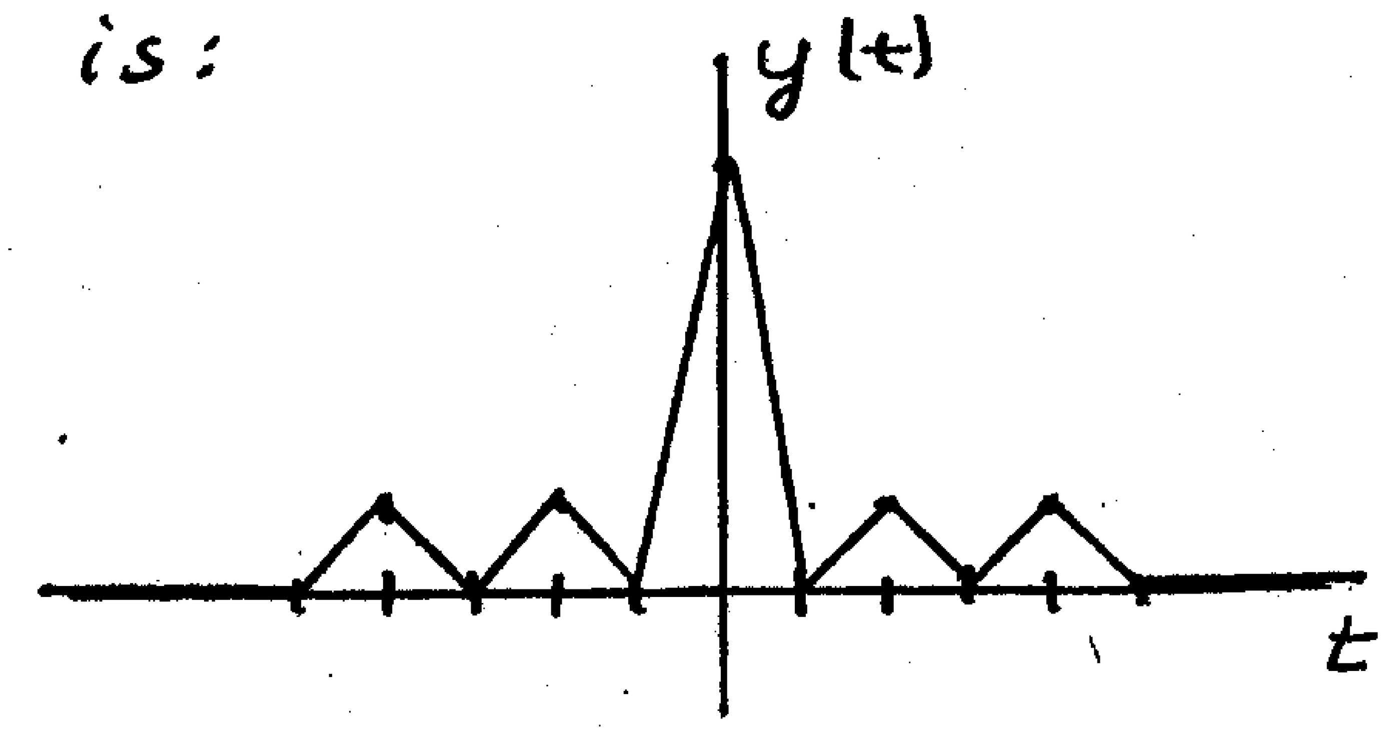


So $y(-3) = \int g(t-\tau)\, u(\tau)\, d\tau = 0$

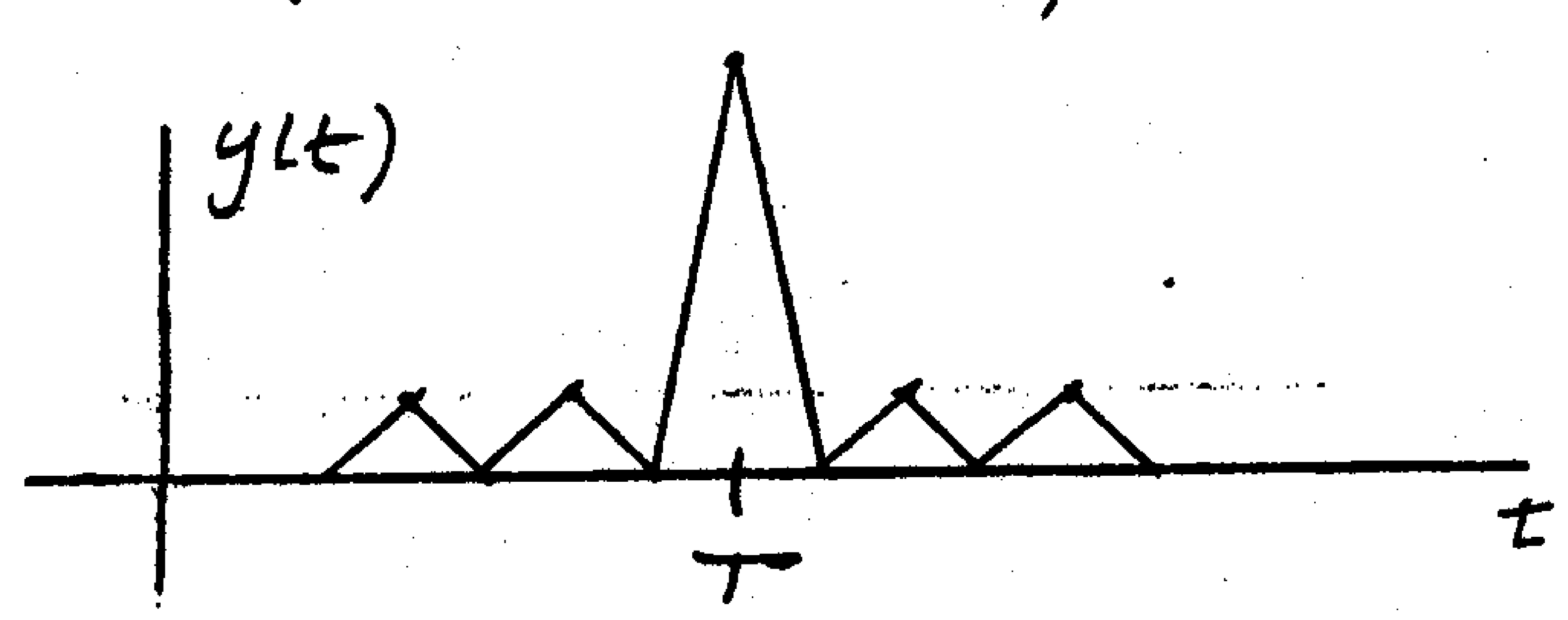
Continuing in this fashion, we have

t	$y(t)$
-4	1
-3	0
-2	1
-1	0
0	5
1	0
2	1
3	0
4	1
5	0

So $y(t)$ is:



2. By linearity and time invariance, delaying $u(t)$ by T will simply delay $y(t)$, so the convolution $g(t) * u(t-T)$ is as above, shifted right by T.



3. T is easily identified as the time at which the max of $y(t)$ occurs.

4. Since we are modeling a notch filter we want one frequency and one time to "show up". So we want to use a "flipped" $u(t)$ for $g(t)$ because they will match at only one point and "conflict" at other points producing small values for $y(t)$.