Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Spring 2005
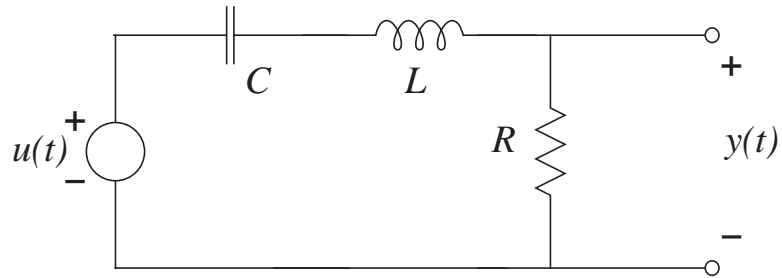## Problem Set #10

Due Date: Wednesday, April 20, 2005 at 5pm

	Time Spent (minutes)
S11	
S12	
S13	
C11	
C12	
M10	
M11	
M12	
Study Time	

Name: _____

Unified Engineering II   **Spring 2005**

Problem S11 (Signals and Systems)

Find the step response of the circuit below, using Laplace methods. The component values are $C = 0.5$ F, $L = 1$ H, and $R = 3$ Ω.

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Problem S12 (Signals and Systems)

There have been a number of aviation accidents in which the aerodynamic controls (elevator, etc.) were lost. Usually this is a result of a complete hydraulic failure, due to some other cause, such as a catastrophic engine failure. In these cases, pilots have attempted to control altitude with throttle variations, with mixed success. In the crash of a DC10 in Sioux City, Iowa, where the aircraft lost elevator control, the captian was able to land the airplane with 184 survivors of 296 on board using throttle only to control altitude. It can be done, but it is very hard. This problem examines why that is so.

Consider an aircraft flying in cruise at 250 knots, so that

$$v_0 = 129 \text{ m/s}$$

Assume that the aircraft has lift-to-drag ratio

$$\frac{L_0}{D_0} = 15$$

Then the transfer function from changes in thrust to changes in altitude is

$$G(s) = \frac{2g}{mv_0} \frac{1}{s \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

where the *natural frequency* of the phugoid mode is

$$\omega_n = \sqrt{2}\frac{g}{v_0}$$

the *damping ratio* is

$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0)}$$

and $g = 9.82$ m/s is the acceleration due to gravity. The transfer function can be normalized by the constant factor $\frac{2g}{mv_0}$, so that

$$\bar{G}(s) = \frac{1}{s \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

is the normalized transfer function, corresponding to normalized input

$$u(t) = \frac{2g}{mv_0}\delta T$$

1. Find and plot the impulse response corresponding to the transfer function $\bar{G}(s)$, using partial fraction expansion and inverse Laplace techniques. Hint: The poles of the system are complex, so you will have to do complex arithmetic.

2. Suppose we try to control the altitude through a feedback loop, as shown below
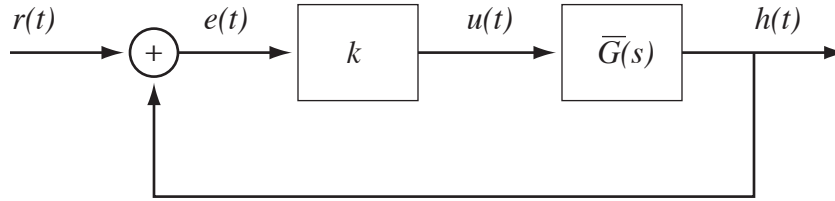
That is, the control input $u(t)$ (normalized throttle) is a gain k times the error, $e(t)$, which is the difference between the altitude $h(t)$ and the altitude reference $r(t)$. The transfer function from $r(t)$ to $h(t)$ can be shown to be

$$\frac{H(s)}{R(s)} = \frac{k\bar{G}(s)}{1 + k\bar{G}(s)}$$

For the gain k in the range $[0, 0.1]$, plot the poles of the system in the complex plane. You should find that for any positive k, the complex poles are made less stable. What gain k makes the complex poles unstable, i.e., for what gain is the damping ratio zero?

3. For the gain k in the range $[-0.1, 0]$, plot the poles of the system in the complex plane. You should find that for any negative k, the real pole is unstable.

Note that neither positive gain or negative gain makes the system more stable than without feedback control. It is possible to do better with a dynamic gain, but this problem should give you an idea of why the phugoid dynamics are so hard to control with throttle only.

Problem S13 (Signals and Systems)

For each signal below, find the bilateral Laplace transform (including the region of convergence) by directly evaluating the Laplace transform integral. If the signal does not have a transform, say so.

1. $g(t) = \cos(at)\sigma(-t)$

2. $g(t) = te^{at}\sigma(-t)$

3. $g(t) = \sin(\omega_0 t)\, e^{-a|t|}$, for all t

The problems in this problem set cover lectures C11, C12, and C13.

Download and unzip the following file from the C&P class webpage:

<div align="center">

`C&P_PSet4_Files.zip`

</div>

Some answers (C11a, C11b, C12) will need to have their answers zipped and uploaded to:

<div align="center">

https://spacestation.mit.edu/unified/

</div>

Four *modified* files are expected:

<div align="center">

`heap_sort_pset.ads,` `heap_sort_pset.adb,`
`robust_array_spec.ads` and `robust_array_body.adb`

</div>

Please zip/compress/store these three .adb files. Name the file:

<div align="center">

"C1112_Lastname_Firstname.zip"

</div>

Problem C11. Heap Sort

Download the following files from the class website

- ❖ `binary_tree.ads`
- ❖ `binary_tree.adb`
- ❖ `heap_sort_pset.ads`
- ❖ `heap_sort_pset.adb`
- ❖ `heap_test.adb`

The Heapify procedure currently implemented in `heap_sort_pset.adb` operates on an array of integers.

a. Modify `heap_sort_pset.ads` to specify
- ❖ An array of node pointers called node_pointer_array

- ❖ Modify the specification of the Heap_Sort subprogram/procedure to use the array of node pointers (instead of an array of integers)

- ❖ Modify the specification of the Heapify procedure to use an array of node pointers.

- ❖ Specify a subprogram called create_links that uses the array of node_pointers as inputs and creates the links across all the nodes in the heap.

b. Modify `heap_sort_pset.adb` to
- ❖ Modify the implementation of the heapify procedure to use an array of node pointers.

- ❖ Implement the subprogram/procedure called create_links that uses the array of node_pointers as inputs and creates the links across all the nodes in the heap.

- ❖ Modify the implementation of the heap_sort procedure to use an array of node pointers

Verify the behavior of your program using `Heap_Test.adb`.

Turn in hard copies of the listings of `heap_sort_pset.ads` and `heap_sort_pset.adb`. Submit both modified files electronically, online.

Problem C12. Robust Programming

With the following files:

- ❖ `robust_array_spec.ads`
- ❖ `robust_array_body.adb`
- ❖ `robust_array_test.adb`

Implement two exception handlers in the robust_array package to handle array overflow and array underflow.

Recall: An overflow is said to occur when an array violates its upper bound, and an underflow is said to occur when an array violates its lower bound.

Turn in hard copies of the listings of `robust_array_spec.ads` and `robust_array_body.adb`. Submit both modified files electronically, online.

M10   Determine the buckling load and mode for a column of length L clamped at
one end with a load P applied at the other end. The column has a constant
cross-section with Area A and moment of inertia I and is made of a material
with modulus of E.
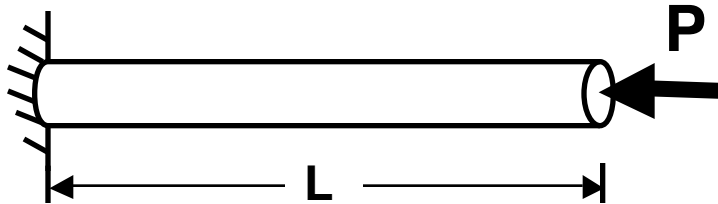
M11   A series of 5-meter long columns is to be produced using D6ac steel (modulus =
200 GPa, σ_{cu} = 1375 MPa, σ_{cy} = 1305 MPa). These columns will be simply-
supported and have circular cross-sections with various diameters. A
compressive load will be applied at the column end with the roller supports
(think of it as a ring of roller balls). You are asked to determine a design chart
showing the maximum load-carrying capability of these products as a function
of the diameter. Be sure to clearly indicate your reasoning and "important
points" on the design chart.
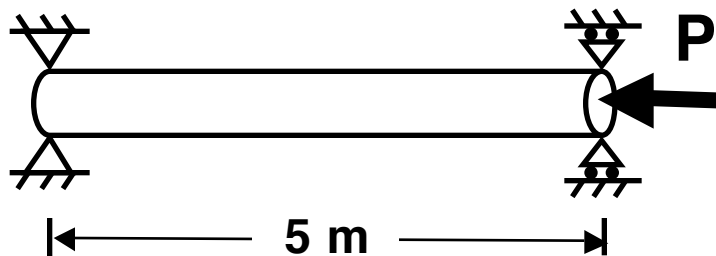
(a) Determine the buckling load for this configuration as a function of the
diameter of the steel columns.

(b) Determine the squashing load for this configuration as a function of the
diameter of the steel columns.

(c) Determine the design chart.

M12 An aluminum (E = 10.3 Msi, $\nu = 0.3$, $\sigma_{\alpha u} = 63$ ksi, $\sigma_{cy} = 55$ ksi) column is 30 inches long and has a rectangular cross-section of 3 inches by 2 inches. The column is assumed simply-supported and is used to support a compressive load of magnitude P. Assume that the load is applied off the centerline of the column by an eccentricity of value e. Consider five cases of eccentricity normalized by the total length of the column: 0, 0.01, 0.02, 0.05, 0.1.
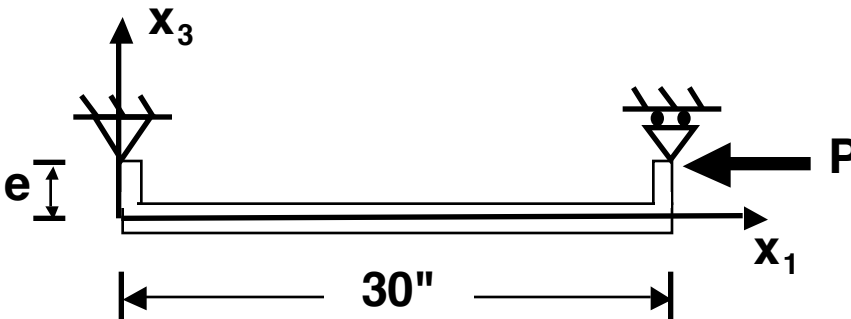
Cross- Section
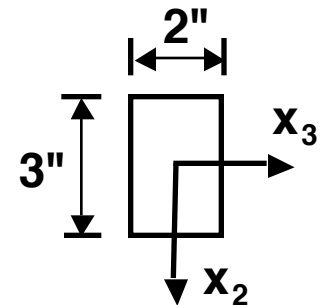
(a) For each case, determine the maximum load the column can carry.

(b) For each case, determine a normalized relationship between the applied load and the lateral deflection of the center of the column. Normalize the center deflection of the column by the specimen length and normalize the column load, P, by the critical buckling load, P_{cr}.

(c) Plot the normalized center deflection of the column versus the normalized load for the five cases of eccentricity.