**Massachusetts Institute of Technology**
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# Unified Engineering
# Spring 2005

## Problem Set #11

Due Date: Tuesday, April 26, 2005 at 5pm

| | Time Spent (minutes) |
|---|---|
| S14 | |
| S15 | |
| C13 | |
| M13 | |
| Study Time | |

**Name: ________________**

**Unified Engineering II** **Spring 2005**

**Problem S14 (Signals and Systems)**

For each of the following Laplace transforms, find the inverse Laplace transform. Note that you can use the results of Problem S10 to help with the partial fraction expansions, but you will need to use the region of convergence to determine the inverse transform.

1. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad -2 < \mathrm{Re}[s] < 2$
2. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad \mathrm{Re}[s] < -2$
3. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad -2 < \mathrm{Re}[s] < -1$
4. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad -3 < \mathrm{Re}[s] < -2$
5. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad -2 < \mathrm{Re}[s] < -1$
6. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad \mathrm{Re}[s] < -2$
7. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad -1 < \mathrm{Re}[s] < 0$
8. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad \mathrm{Re}[s] < -1$
9. $G(s) = \dfrac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)}, \quad \mathrm{Re}[s] < 0$

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## Problem S15 (Signals and Systems)

In class, you learned about a *smoother*, with transfer function

$$G_1(s) = \frac{-a^2}{(s-a)(s+a)}$$

The smoother is an example of a *low-pass filter*, which means that it tends to attenuate high-frequency sine waves, but "pass" low-frequency sine waves. Unfortunately, the smoother is non-causal, which means that it can't be implemented in real time. A similar causal low-pass filter is

$$G_2(s) = \frac{a^2}{(s+a)^2}$$

In this problem, you will compare these two low-pass filters, to see how they affect sinusoidal inputs. Consider an input signal

$$u(t) = \cos \omega t$$

1. Find the transfer function, $G_1(j\omega)$, as a function of frequency, $\omega$.

2. Since the transfer function is complex, it can be represented as

   $$G_1(j\omega) = A_1(\omega)e^{j\phi_1(\omega)}$$

   where the amplitude of the transfer function is $A_1(\omega)$, and the phase of the transfer function is $\phi_1(\omega)$. Find $A_1(\omega)$ and $\phi_1(\omega)$.

3. Find the transfer function, $G_2(j\omega)$, as a function of frequency, $\omega$, as well as $A_2(\omega)$ and $\phi_2(\omega)$.

4. For the input $u(t)$ above, show that the output of the system $G_1$ is

   $$y_1(t) = A_1(\omega)\cos(\omega t + \phi_1(\omega))$$

   and do likewise for system $G_2$.

5. $A_1$ and $A_2$ determine how much the magnitude of the input cosine wave is affected by each filter. Ideally, $A_1$ and $A_2$ would be 1, meaning that the filters don't change the magnitude of the input sine at all. Which filter (if either) changes the magnitude the least?

6. $\phi_1$ and $\phi_2$ determine how much the phase of the input cosine wave is affected by each filter. Non-zero values of $\phi$ correspond to a shifting left or right (i.e., advancing or delaying) the sine wave. Ideally, $\phi_1$ and $\phi_2$ would be zero, meaning that the filters don't change the phase of the input sine at all. Which filter (if either) produces the least phase shift?

7. Explain why the non-causal filter is preferred in signal processing applications where it can be applied.

**Problem C13. Impact of Software on Aerospace (covers lecture C13)**

a. Select an aerospace program of your choice (other than the F-16). Briefly explain (1-2 sentences) why you selected the given program.

   **Note**: The program can be any aerospace program that is currently underway either in the US or elsewhere in the world.

b. Document with some text and a table (i.e. Lecture C13, Slide 9) the increase/decrease/evolution of software components in the program. Write a short paragraph interpreting/summarizing the information presented in the table.

c. Document all references (papers / articles / web links / …) you used to research the evolution.

   **Deliverables**:
   Turn in hardcopies of (a) 'why you selected the given program,' (b) the table and the table interpretation, and (c) the references.

**Unified Engineering Problem Set #11** **Unit M5.1**
**Spring, 2005**

**M13** A spring is basically a device for storing energy. Therefore, one key consideration in the design of a spring is the maximum energy that can be stored. A second consideration is that the spring should return to its original configuration when unloaded. Thus, the material cannot yield. Consider the simplest model of a spring -- a rod in uniaxial tension.

(a) Express the total energy stored per unit volume in the spring as a function of the applied stress and the modulus of the material.

(b) Identify the combination of material properties that maximize energy storage capacity (without yielding) for:
 (i) a given volume of material;
 (ii) a given mass of material;
 (iii) a given cost of material

(c) Choose amongst the following materials for each of the three criteria listed in part (b). Comment as appropriate.

| Material | Modulus<br>E [GPa} | Yield Stress<br>$\sigma_y$ [MPa] | Density<br>$\rho$ [Mg/m$^3$] | Price<br>c [$/kg] |
|---|---|---|---|---|
| Al alloy | 70.0 | 500 | 2.7 | 2.0 |
| Spring Steel | 210 | 2400 | 8.0 | 3.0 |
| Rubber | 0.05 | 30 | 0.9 | 1.3 |
| Titanium | 116 | 1400 | 4.5 | 10.1 |
| Nickel | 214 | 2000 | 8.9 | 4.3 |
| Graphite/Epoxy | 100 | 650 | 1.5 | 200 |