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Unified Engineering
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Problem Set #12
Solutions

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UNIFIED ENGINEERING

Problem Set #12 -- SOLUTIONS

M14. Condition A: $\sigma_{11} = -p$ $\sigma_{12} = 0$
$\sigma_{22} = -p$ $\sigma_{13} = 0$
$\sigma_{33} = -p$ $\sigma_{23} = 0$

Condition B: $\sigma_{11} = 0.5p$ $\sigma_{12} = 0$
$\sigma_{22} = p$ $\sigma_{13} = 0$
$\sigma_{33} = 2p$ $\sigma_{23} = 0$

Condition C: $\sigma_{11} = 2p$ $\sigma_{12} = 0$
$\sigma_{22} = -p$ $\sigma_{13} = 0$
$\sigma_{33} = 0.5p$ $\sigma_{23} = 0$

Condition D: $\sigma_{11} = p$ $\sigma_{12} = 0$
$\sigma_{22} = 4p$ $\sigma_{13} = 0$
$\sigma_{33} = 0.5p$ $\sigma_{23} = 0$

(a) Application of the Tresca condition requires knowledge of the principal stresses.

For Conditions A, B, and C, there are no applied shear stresses, so the applied normal stresses are the principal stresses.

Put these in appropriate order based on magnitude:

Condition A	Condition B	Condition C
$\sigma_I = \sigma_{11} = -p$	$\sigma_I = \sigma_{33} = 2p$	$\sigma_I = \sigma_{11} = 2p$
$\sigma_{II} = \sigma_{22} = -p$	$\sigma_{II} = \sigma_{22} = p$	$\sigma_{II} = \sigma_{22} = -p$
$\sigma_{III} = \sigma_{33} = -p$	$\sigma_{III} = \sigma_{11} = 0.5p$	$\sigma_{III} = \sigma_{33} = 0.5p$

For condition D, there is no applied stress in the 3-~~plane~~ since $\sigma_{13} = \sigma_{23} = 0$, so σ_{33} is a principal stress. However, σ_{12} is nonzero, so the principal stresses in the 1-2 plane need to be determined.
We'll call: $\sigma_{33} = \sigma_{III}$ and label the two in the 1-2 plane as σ_I and σ_{II} (we'll get them in order of form) from last term for planar stress:

principal stresses are roots of equation:

$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

For condition D $\Rightarrow$

$$\tau^2 - \tau(p + 4p) + ([p][4p] - [2p]^2) = 0$$

$$\Rightarrow \tau^2 - 5p\tau + (4p^2 - 4p^2) = 0$$

$$\Rightarrow \tau(\tau - 5p) = 0$$

$$\Rightarrow \tau = \sigma_I = 5p$$

$$\tau = \sigma_{II} = 0$$

Finally in order for

Condition D

$\sigma_I = 5p$

$\sigma_{II} = 0.5p$

$\sigma_{III} = 0$

Now apply the Tresca criterion where yield occurs if:

$$|\sigma_I - \sigma_{II}| = \sigma_Y$$

or

$$|\sigma_{II} - \sigma_{III}| = \sigma_Y$$

or

$$|\sigma_{III} - \sigma_I| = \sigma_Y$$

in addition, the directionality associated with this is that yielding occurs via shear on the plane of maximum shear stress corresponding to the difference in those two principal stresses

Here: $\sigma_Y = 1500$ MPa

Apply each condition...

Condition A: hydrostatic stress

$\Rightarrow$ All differences $= 0$ $\Rightarrow$ $\boxed{\text{No yielding}}$ (A)

Condition B: $|\sigma_I - \sigma_{III}| = |2p - p| = p = \sigma_Y$

$\Rightarrow p = 1500$ MPa

$|\sigma_{II} - \sigma_{III}| = |p - 0.5p| = 0.5p = \sigma_Y$

$\Rightarrow p = 3000$ MPa

$|\sigma_{III} - \sigma_I| = |0.5p - 2p| = 1.5p = \sigma_Y$

$\Rightarrow p = 1000$ MPa

critical case is last one.

B

$\Rightarrow$ $\boxed{\begin{array}{l}\text{yielding at } p = 1000 \text{ MPa} \\ \text{on plane at } 45^\circ \text{ between } \sigma_{11} \text{ and } \sigma_{33}\end{array}}$

<u>Condition C</u>: $|\sigma_I - \sigma_{II}| = |2p - (-p)| = \sigma_Y$

$\Rightarrow 3p = 1500 \text{ MPa}$

$\Rightarrow p = 500 \text{ MPa}$

$|\sigma_{II} - \sigma_{III}| = |-p - 0.5p| = \sigma_Y$

$\Rightarrow 1.5p = 1500 \text{ MPa}$

$\Rightarrow p = 1000 \text{ MPa}$

$|\sigma_{III} - \sigma_I| = |0.5p - 2p| = \sigma_Y$

$\Rightarrow 1.5p = 1500 \text{ MPa}$

$\Rightarrow p = 1000 \text{ MPa}$

critical case is first C

$\Rightarrow$ $\boxed{\begin{array}{l}\text{yielding at } p = 500 \text{ MPa} \\ \text{on plane at } 45^\circ \text{ between } \sigma_{11} \text{ and } \sigma_{22}\end{array}}$

<u>Condition D</u>: $|\sigma_I - \sigma_{II}| = |5p - 0| = \sigma_Y$

$\Rightarrow 5p = 1500 \text{ MPa}$

$\Rightarrow p = 300 \text{ MPa}$

$|\sigma_{II} - \sigma_{III}| = |0 - 0.5p| = \sigma_Y$

$\Rightarrow 0.5p = 1500 \text{ MPa}$

$\Rightarrow p = 3000 \text{ MPa}$

$|\sigma_{III} - \sigma_I| = |0.5p - 5p| = \sigma_Y$

$\Rightarrow 4.5p = 1500 \text{ MPa}$

$\Rightarrow p = 333 \text{ MPa}$

critical case is first (D)

$\Rightarrow$ $\boxed{\text{Yielding at } p = 300 \text{ MPa on plane at } 45^\circ \text{ to direction of principal stress* in 1-2 plane and } \sigma_{33}}$

* find this angle in 1-2 plane by using:

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{4p}{p-(-4p)}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{4}{5}\right)$$

$$\theta_p = \frac{1}{2}(38.7^\circ)$$

$$\Rightarrow \theta_p = 19.3^\circ$$

(b) the von Mises criterion is:

$$(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = 2\sigma_Y^2$$

Look at each condition again.

Condition A: hydrostatic stress

All ~~diff~~ differences are zero, so again (A)

$\boxed{\text{NO yielding}}$

Condition B:

$$(2p - p)^2 + (p - 0.5p)^2 + (0.5p - 2p)^2 = 2\sigma_Y^2$$

$$\Rightarrow p^2 + 0.25p^2 + 2.25p^2 = 2\sigma_Y^2$$

$$\Rightarrow 3.5p^2 = (1500\ \text{MPa})^2 \times 2$$

$$\Rightarrow p = \sqrt{\frac{2}{3.5}}\,(1500\ \text{MPa}) = 1134\ \text{MPa}$$

$$\text{for } B: \boxed{p = 1134\ \text{MPa}}$$

Condition C:

$$(2p-(-p))^2 + (-p-0.5p)^2 + (0.5p-2p)^2 = 2\sigma_Y^2$$

$$\Rightarrow 9p^2 + 2.25p^2 + 2.25p^2 = 2\sigma_Y^2$$

$$\Rightarrow p = \sqrt{\frac{2}{13.5}}\,\sigma_Y$$

$$\Rightarrow \text{for } C: \boxed{p = 577\ \text{MPa}}$$

Condition D:

$$(5p-0.5p)^2 + (0.5p-0)^2 + (0-5p)^2 = 2\sigma_Y^2$$

$$\Rightarrow 20.25p^2 + 0.25p^2 + 25p^2 = 2\sigma_Y^2$$

$$\Rightarrow 45.5p^2 = 2\sigma_Y^2$$

$$\Rightarrow p = \sqrt{\frac{2}{45.5}}\,\sigma_Y^2$$

$$\Rightarrow \text{for } D: \boxed{p = 314\ \text{MPa}}$$

(c) For each of the conditions, the Tresca Criterion gives a more conservative estimate of the yielding load characteristic, p (See summary table that follows). The one case where this is not true is Condition A which is a hydrostatic case and both criteria predict no yielding as this is a fundamental basis for each.

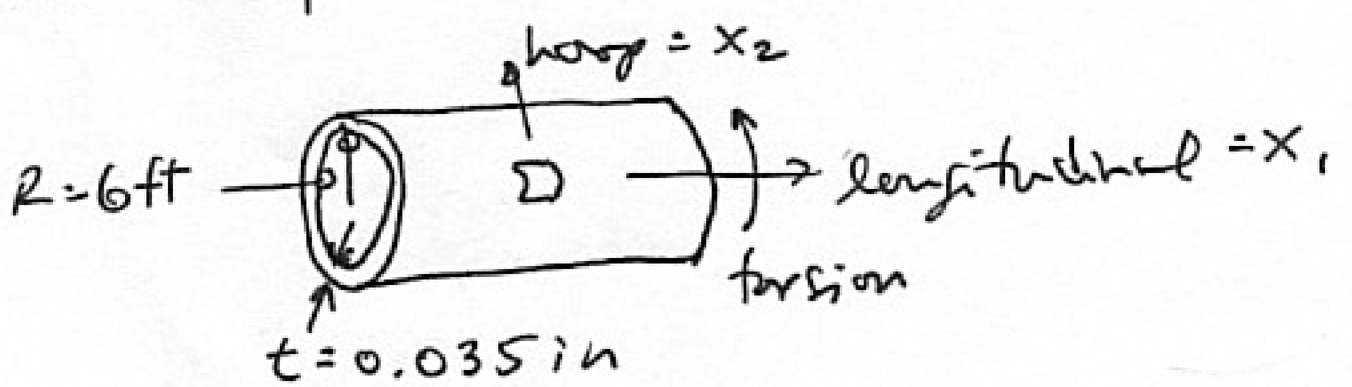
Summary:

Critical p . [MPa]

Condition	Tresca	von Mises
A	—	—
B	1000	1134
C	500	577
D	300	314

The Tresca criterion considers yielding on a single plane and thus the two principal stresses acting on that plane. In contrast, the von Mises criterion involves and interacts all the applied stresses and thus slightly higher values.

Mrs. Airplane fuselage



At limit, $p = 10$ psi (pressure differential)

$$\sigma_{hoop} = \sigma_{22} = \frac{pR}{t}$$

$$\sigma_{long} = \sigma_{11} = \frac{pR}{2t}$$

(a) stress of material is sum of stress due to pressure and stress ~~from~~ empennage loads accounting for 50% load carrying factor of skin

so:

$$\sigma_{11} = \frac{1}{2}\left(\sigma_{11\,(\text{due to } p)} + \sigma_{11\,(\text{applied wing})}\right) \quad (1)$$

$$\sigma_{22} = \frac{1}{2}\left(\sigma_{22\,(\text{due to } p)}\right) \quad (2)$$

$$\sigma_{12} = \frac{1}{2}\left(\sigma_{12\,(\text{applied torsion})}\right) \quad (3)$$

using the pressure equations. At this limit condition:

$$\sigma_{11\,(\text{due to } p)} = \frac{10\,psi\,(6ft)(12\,in/ft)}{2\,(0.035\,in)} = 10{,}286\,psi$$

$$\sigma_{22\,(\text{due to } p)} = \frac{10\,psi\,(6ft)(12\,in/ft)}{0.035\,in} = 20{,}572\,psi$$

using in the above:

$$\sigma_{11} = 5143 + \sigma_{11\,(\text{applied wing})} \;[psi] \quad (1')$$

$$\sigma_{22} = 10{,}286\,psi \quad (2')$$

$$\sigma_{12} = \left(\sigma_{12\,(\text{applied torsion})}\right) \;[psi] \quad (3')$$

recognizing that $\sigma_{11A.L.}$ and $\sigma_{12A.T.}$ are half of the applied load condition. Now use the Tresca condition. As before:

$$|\sigma_I - \sigma_{II}| = \sigma_Y \;\text{ or }\; |\sigma_{II} - \sigma_{III}| = \sigma_Y \;\text{ or }\; |\sigma_{III} - \sigma_I| = \sigma_Y$$

Here we have plane stress with $\sigma_{III} = 0$, so this becomes:

$$|\sigma_I - \sigma_{II}| = \sigma_Y$$

$$|\sigma_{II}| = \sigma_Y \qquad |\sigma_{III}| = \sigma_Y$$

with $\sigma_Y = 50$ ksi

It is necessary to find the principal stresses for the planar stress case. Again (as in first problem) use:

$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

and find roots. Do so in this form. Use quadratic solution for:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \sigma_I, \sigma_{II} = \frac{1}{2}\left\{(\sigma_{11} + \sigma_{22}) \pm \left[(\sigma_{11} + \sigma_{22})^2 - 4(\sigma_{11}\sigma_{22} - \sigma_{12})^2\right]^{1/2}\right\}$$

write out:

$$\Rightarrow = \frac{1}{2}\left\{(\sigma_{11} + \sigma_{22}) \pm \left[\sigma_{11}^2 + 2\sigma_{11}\sigma_{22} + \sigma_{22}^2 - 4\sigma_{11}\sigma_{22} + 4\sigma_{12}^2\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left\{(\sigma_{11} + \sigma_{22}) \pm \left[\sigma_{11}^2 - 2\sigma_{11}\sigma_{22} + \sigma_{22}^2 + 4\sigma_{12}^2\right]^{1/2}\right\}$$

So:

$$\sigma_I = \left(\frac{\sigma_{11} + \sigma_{22}}{2}\right) + \sqrt{\sigma_{12}^2 + \left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2}$$

$$\sigma_{II} = \left(\frac{\sigma_{11} + \sigma_{22}}{2}\right) - \sqrt{\sigma_{12}^2 + \left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2}$$

The Tresca condition can be rewritten using these expressions:

$$50\,ksi = |\sigma_{I} - \sigma_{II}| = \left| 2\sqrt{\sigma_{12}^{2} + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^{2}} \right| \quad (4)$$

$$50\,ksi = |\sigma_{I}| = \left| \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) + \sqrt{\sigma_{12}^{2} + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^{2}} \right| \quad (5)$$

$$50\,ksi = |\sigma_{II}| = \left| \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) - \sqrt{\sigma_{12}^{2} + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^{2}} \right| \quad (6)$$

Now rewrite these in terms of the total stresses from equations (1'), (2') and (3'):

from (4)

$$50\,ksi = 2\sqrt{\sigma_{12_{A.T.}}^{2} + \left(\frac{5143 + \sigma_{11_{A.L.}} - 10,286}{2}\right)^{2}}$$

$$\Rightarrow \boxed{50\,ksi = 2\sqrt{\sigma_{12_{A.T.}}^{2} + \left(\frac{\sigma_{11_{A.L.}} - 5143}{2}\right)^{2}}} \quad (4)'$$

from (5)

$$50\,ksi = \left| \left(\frac{5143 + \sigma_{11_{A.L.}} + 10,286}{2}\right) + \sqrt{\sigma_{12_{A.T.}}^{2} + \left(\frac{\sigma_{11_{A.L.}} - 5143}{2}\right)^{2}} \right|$$

$$\Rightarrow \boxed{50\,ksi = \left| \left(\frac{15,429 + \sigma_{11_{A.L.}}}{2}\right) + \sqrt{\sigma_{12_{A.T.}}^{2} + \left(\frac{\sigma_{11_{A.L.}} - 5143}{2}\right)^{2}} \right|} \quad (5')$$

and from (6)

$$\boxed{50\,ksi = \left| \left(\frac{15,429 + \sigma_{11_{A.L.}}}{2}\right) - \sqrt{\sigma_{12_{A.T.}}^{2} + \left(\frac{\sigma_{11_{A.L.}} - 5143}{2}\right)^{2}} \right|} \quad (6')$$

Now apply different values of σ_{11} for each case and determine the value of σ_{12} that causes failure, then plot them.

all in [ksi]

from equation (4'):

$\sigma_{11\,A.C.}$	$\sigma_{12\,A.T.} = \sqrt{(50)^2 - \left(\frac{\sigma_{11}}{2} - 5.14\right)^2}$
0	±24.9
+10, -10	±24.9, ±23.8
+20, -20	±23.9, ±21.6
+30, -30	±21.7, ±17.8
+40, -40	±17.9, ±10.7
+50.1, -44.8	0

from equation (5'):

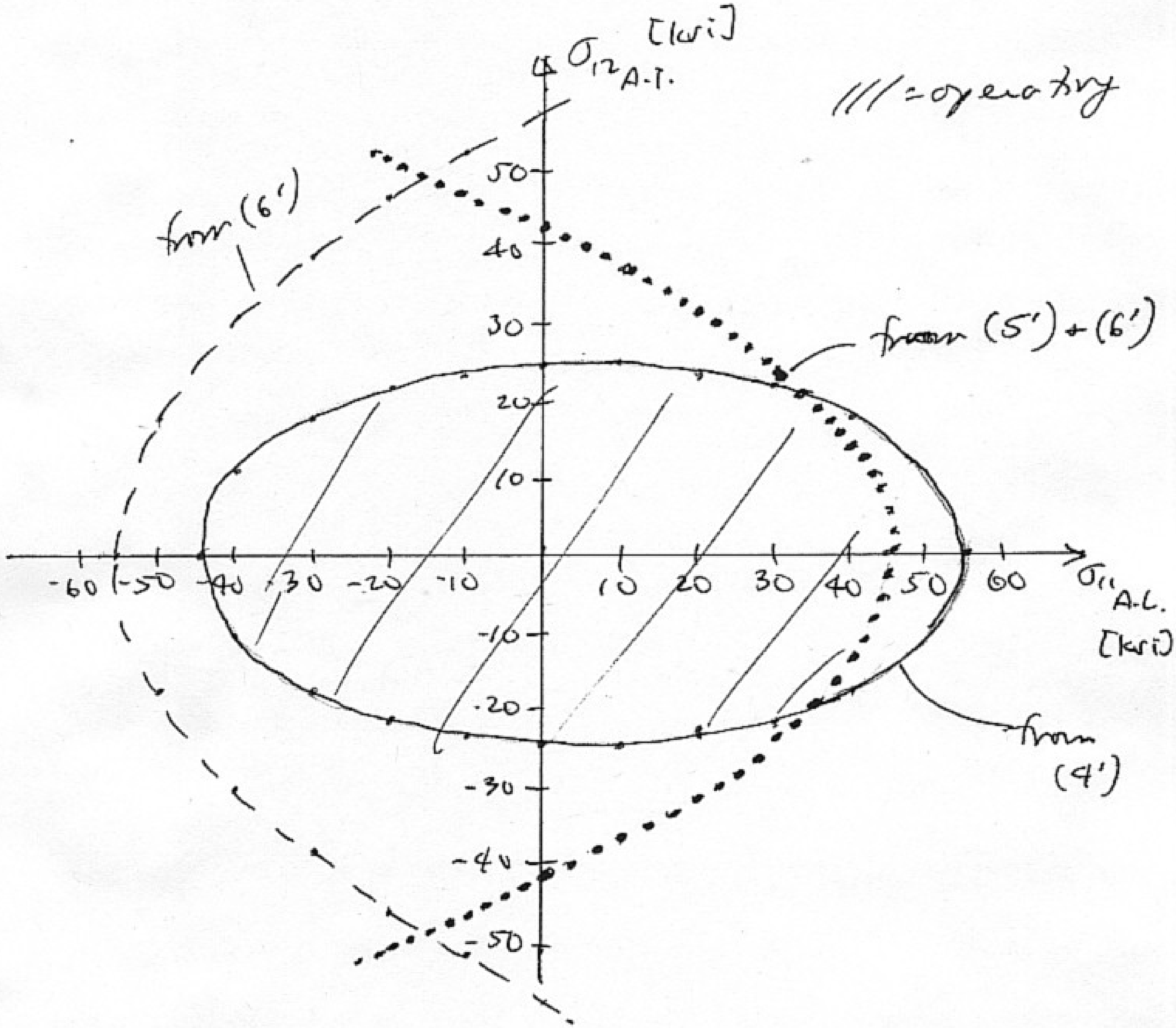
$\sigma_{11\,A.C.}$	$\sigma_{12\,A.T.} = \sqrt{\left(\frac{84.6 - \sigma_{11}}{2}\right)^2 - \left(\frac{\sigma_{11} - 5.14}{2}\right)^2}$	$\sigma_{12\,A.T.} = \sqrt{\left(\frac{\sigma_{11} - 84.6}{2}\right)^2 - \left(\frac{\sigma_{11} - 5.14}{2}\right)^2}$
0	±42.2	±42.2
+10, -10	±37.2, ±46.7	±37.2
+20, -20	±31.4, ±50.8	±31.4
+30, -30	±24.3, ±54.5	same as
+40, -40	±13.9, ±58.1	←
+50, -50	–, ±61.9	
+44.8	0	0

from equation (6'):

$\sigma_{11\,A.C.}$	$\sigma_{12\,A.T.} = \sqrt{\left(\frac{\sigma_{11} - 84.6}{2}\right)^2 - \left(\frac{\sigma_{11} - 5.14}{2}\right)^2}$	$\sigma_{12\,A.T.} = \sqrt{\left(\frac{115.4 + \sigma_{11}}{2}\right)^2 - \left(\frac{\sigma_{11} - 5.14}{2}\right)^2}$
0	±42.2	±57.7
+10, -10	±37.2	±62.7, ±52.2
+20, -20	±31.4	±67.3, ±46.0
+30, -30	±24.3	±71.6, ±38.9
+40, -40	±13.9	±75.7, ±30.2
+50, -50	outside of	±79.6, ±17.1
+60, -60	operating	±83.3, –
-55.1	limits	0

Plot the limiting lines

"Operating stress envelope" for fuselage material via Tresca condition with limit pressure already accounted for



(b) With the "damage tolerant" approach, use the basic fracture mechanics equation:

$$\sigma_f = \frac{K_c}{\sqrt{\pi a}}$$

Here $2a = 0.25\,in \Rightarrow a = 0.125\,in$

$K_{IC} = 31\ ksi\sqrt{in}$ for the 2024 aluminum

$$\Rightarrow \sigma_f = \frac{31\ ksi\sqrt{in}}{\sqrt{\pi(0.125\,in)}}$$

$$\Rightarrow \sigma_f = 49.5\ ksi$$

Thus, if the stress perpendicular to the crack exceeds 49.5 ksi, there is failure. However, the crack could be oriented many directions, so we must find the principal stresses (i.e. the maximum extensional stresses) and then the related direction for the worst case. Consider the possible cases of the loads that are applied

① Just pressure and those resultant stresses.

Then: $\sigma_{11} = 5{,}143$ psi

$\sigma_{22} = 10{,}286$ psi (as per earlier)

Both are principal, since no shear stress is applied and both are well below the critical value of 49.5 ksi

(2) Consider pressure stresses with stresses due to longitudinal load. From before (1'):

$$\sigma_{11} = 5143\,psi + \sigma_{11_{A.C.}}$$

$$\sigma_{22} = 10{,}286\,psi$$

We see that σ_{22} is well below the critical stress, so the criterion here is:

$$\sigma_{11} = 5.143\,ksi + \sigma_{11_{A.C.}} < 49.5\,ksi$$

$$\Rightarrow \sigma_{11_{A.C.}} < 44.4\,ksi \quad (*)$$

(3) All three loads: pressure, longitudinal, and torsional. From before (1'), (2'), (3') and then the previous work to find the principal stresses for the full case:

$$\sigma_{I} = \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) + \sqrt{\sigma_{12}^2 + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2} < 49.5\,ksi \quad (7)$$

$$\sigma_{II} = \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) - \sqrt{\sigma_{12}^2 + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2} < 49.5\,ksi \quad (8)$$

All stresses in [ksi]

Note that σ_I always is larger than σ_{II}, so only (7) needs to be considered. Further note that this is only valid for tensile stresses.

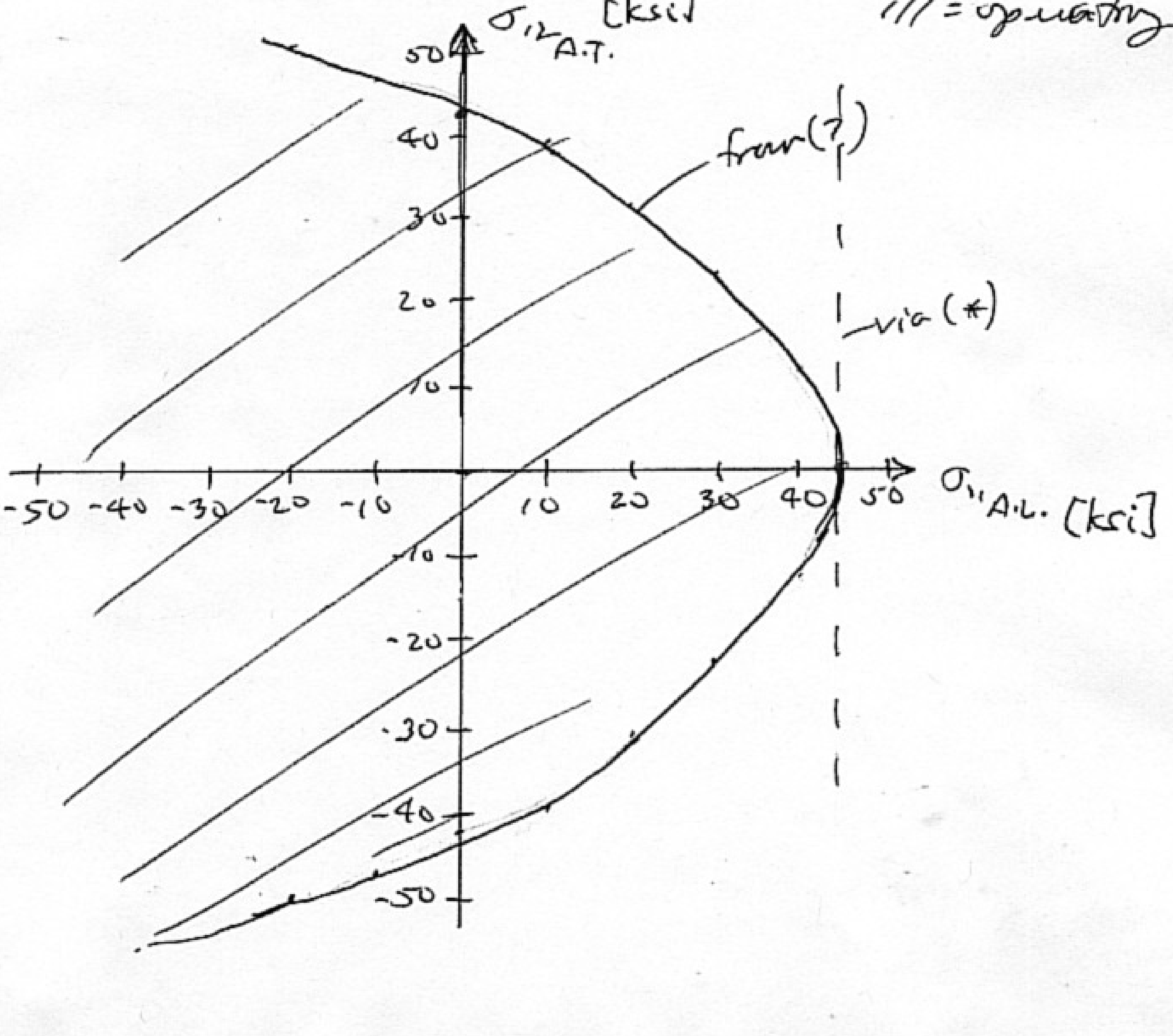
As before, assemble data for (7), and then plot these with the other condition (equation (*))

via equation (7) all in [ksi]

$\sigma_{11_{A.L.}}$	$\sigma_{12_{A.T.}} \lessgtr \sqrt{\left(\frac{83.6-\sigma_{11}}{2}\right)^2-\left(\frac{\sigma_{11}-5.14}{2}\right)^2}$	$\sigma_{12_{A.T.}} \lessgtr \sqrt{\left(\frac{\sigma_{11}-83.6}{2}\right)^2-\left(\frac{\sigma_{11}-5.14}{2}\right)^2}$
0	± 41.7	
+10, −10	±39.3, ±46.2	same as ←
+20, −20	±30.9, ±50.2	
+30, −30	±23.7, ±54.0	
+40, −40	±13.1, ±57.5	
+50, −50	−, ±60.8	
+44.4	0	

Plot:

"Operating stress envelope" for fuselage material via damage-tolerant approach with limit pressure already accounted for



(c) Each of these approaches are different criteria and the plots do look substantially different as may well be expected. The Tresca condition gives the yield point while the damage tolerant approach gives the stress at which a crack will critically propagate. This latter case occurs only for tensile stresses whereas the Tresca condition considers compressive stresses as well. The damage tolerant approach gives the greatest possible operating area.

M16. 1. D

2. G

3. A

4. H

5. B

6. F

7. E

8. C

1. $$G(j\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-T)\, e^{-j\omega t}\, dt$$

$$= e^{-j\omega T} \quad \text{(Using the "sifting property")}$$

$\therefore$, $$\boxed{G(j\omega) = e^{-j\omega T}}$$

2. $$G(j\omega) = \int_{-T}^{T} 1 \cdot e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega} e^{-j\omega t} \Big|_{t=-T}^{T}$$

$$= \frac{-1}{j\omega}\left[e^{-j\omega T} - e^{+j\omega T}\right]$$

$$= \frac{1}{j\omega}\left[e^{+j\omega T} - e^{-j\omega T}\right]$$

$G(j\omega)$ can be simplified by application of Euler's formula, or by inspection. The result is

$$\boxed{G(j\omega) = \frac{2}{\omega} \sin \omega T}$$

3. $$G(j\omega) = \int_{-\infty}^{\infty} \frac{1}{t^2+T^2} e^{-j\omega t}\, dt$$

<u>But</u>, I don't know how to do this integral.
Use duality:

If $\mathcal{F}[g(t)] = f(\omega)$, then

$$\mathcal{F}[f(t)] = 2\pi\, g(-\omega)$$

$g(-\omega)$ is given by

$$g(-\omega) = \frac{1}{(-\omega)^2+T^2} = \frac{1}{\omega^2+T^2}$$

$$= \frac{1}{-s^2+T^2} = \frac{-1}{(s+T)(s-T)}$$

$$= \frac{1/2T}{s+T} - \frac{1/2T}{s-T}$$

$$= \frac{1}{2T}\left[\frac{1}{j\omega+T} - \frac{1}{j\omega-T}\right]$$

Therefore,

$$f(t) = 2\pi\, \mathcal{F}^{-1}\left[g(-\omega)\right]$$

$$= 2\pi\, \frac{1}{2T}\left[e^{-tT}\sigma(t) + e^{+tT}\sigma(-t)\right]$$

$$= \frac{\pi}{T}\, e^{-|t|T}$$

$\therefore$

$$\boxed{G(j\omega) = f(\omega) = \frac{\pi}{T} e^{-|\omega| T}}$$

4. $g(t) = \dfrac{\sin \pi t/T}{\pi t/T}$

Use duality:

$$\mathcal{F}[g(t)] = G(j\omega) = f(\omega)$$

$$\mathcal{F}[f(t)] = 2\pi\, g(-\omega)$$

In this case,

$$g(-\omega) = \frac{\sin(-\pi\omega/T)}{-\pi\omega/T} = \frac{\sin \pi\omega/T}{\pi\omega/T}$$

If we let $T' = \pi/T$, this becomes

$$g(-\omega) = \frac{\sin \omega T'}{\omega T'}$$

The inverse FT (From part 1) is

$$\mathcal{F}^{-1}[g(-\omega)] = \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega T'}\right)$$

$$= \frac{1}{2T'} \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega/2}\right)$$

$$= \begin{cases} 1/2T', & |t| \le T' \\ 0, & \text{else} \end{cases}$$

$$= f(t)/2\pi$$

Therefore,

$$f(t) = \begin{cases} \pi/T', & |t| \le T' \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow \quad G(j\omega) = f(\omega)$$

$$= \begin{cases} \pi/T', & |\omega| \le T' \\ 0, & \text{else} \end{cases}$$

$$= \begin{cases} T, & |\omega| \le \pi/T \\ 0, & \text{else} \end{cases}$$

5. Let $F(j\omega) = \dfrac{\sin \omega T}{\omega T}$

Then $G(j\omega) = [F(j\omega)]^2$

$\Rightarrow \quad g(t) = f(t) * f(t)$ (convolution property)

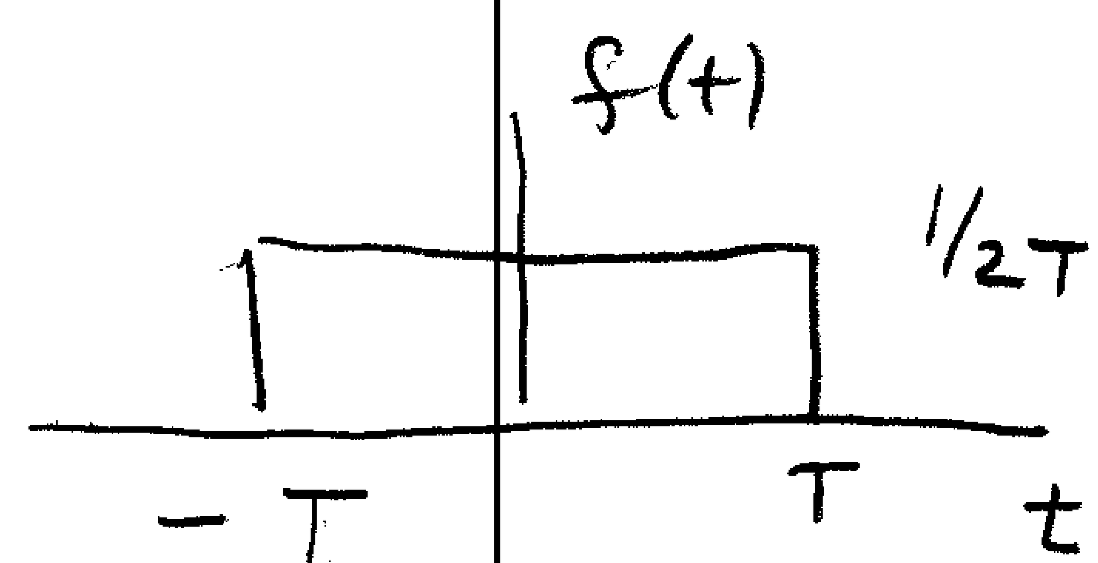
Using the results of part (1),

$$f(t) = \mathcal{F}^{-1}\left[\frac{\sin \omega T}{\omega T}\right]$$

$$= \frac{1}{2T}\,\mathcal{F}^{-1}\left[\frac{\sin \omega T}{\omega/2}\right]$$

$$= \begin{cases} 1/2T, & |t| \le T \\ 0, & \text{else} \end{cases}$$

$=$



g(t) is the convolution of f(t) with f(t), which is