Massachusetts Institute of Technology
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Cambridge, MA 02139

Unified Engineering
Spring 2005

Problem Set #13
Solutions

Unified Engineering II **Spring 2005**

Problem S17 Solution (Signals and Systems)

Problem Statement: Consider the *quadrature modulation/demodulation* system shown below. The purpose of the system is to transmit two signals, $x_1(t)$ and $x_2(t)$, over the same frequency band simultaneously. $x_1(t)$ and $x_2(t)$ are bandlimited signals, with bandwidth W. That is, their Fourier transforms $X_1(f)$ and $X_2(f)$ satisfy

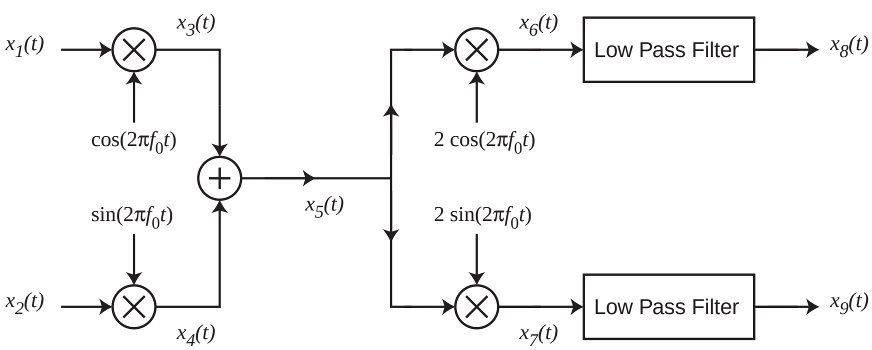
$$X_1(f) = 0, \quad |f| \geq W$$

$$X_2(f) = 0, \quad |f| \geq W$$

The bandwidth is much less than the modulation frequency, f_0. The lowpass filters shown in the diagram are ideal, with transfer function

$$L(f) = \begin{cases} 1, & |f| < W \\ 0, & |f| > W \end{cases}$$

Find the Fourier transforms of the signals $x_3(t)$, $x_4(t)$, $x_5(t)$, $x_6(t)$, $x_7(t)$, $x_8(t)$, and $x_9(t)$ in terms of $X_1(f)$ and $X_2(f)$.



Solution: Define

$$\begin{aligned}
w_1(t) &= \cos(2\pi f_0 t) \\
w_2(t) &= \sin(2\pi f_0 t) \\
w_3(t) &= 2\,\cos(2\pi f_0 t) \\
w_4(t) &= 2\,\sin(2\pi f_0 t)
\end{aligned}$$

The FTs are

$$\begin{aligned}
W_1(f) &= \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0) \\
W_2(f) &= \frac{-j}{2}\delta(f - f_0) + \frac{j}{2}\delta(f + f_0) \\
W_3(f) &= \delta(f - f_0) + \delta(f + f_0) \\
W_4(f) &= -j\delta(f - f_0) + j\delta(f + f_0)
\end{aligned}$$

Therefore,

$$\begin{aligned}X_3(f) &= X_1(f) * W_1(f) \\ &= \frac{1}{2}X_1(f-f_0) + \frac{1}{2}X_1(f+f_0)\end{aligned}$$

$$\begin{aligned}X_4(f) &= X_2(f) * W_2(f) \\ &= \frac{-j}{2}X_2(f-f_0) + \frac{j}{2}X_2(f+f_0)\end{aligned}$$

$$\begin{aligned}X_5(f) &= X_3(f) + X_4(f) \\ &= \frac{1}{2}X_1(f-f_0) + \frac{1}{2}X_1(f+f_0) + \frac{-j}{2}X_2(f-f_0) + \frac{j}{2}X_2(f+f_0)\end{aligned}$$

$$\begin{aligned}X_6(f) &= X_5(f) * W_3(f) \\ &= X_5(f-f_0) + X_5(f+f_0) \\ &= \frac{1}{2}X_1(f-2f_0) + \frac{1}{2}X_1(f) + \frac{-j}{2}X_2(f-2f_0) + \frac{j}{2}X_2(f) \\ &\quad + \frac{1}{2}X_1(f) + \frac{1}{2}X_1(f+2f_0) + \frac{-j}{2}X_2(f) + \frac{j}{2}X_2(f+2f_0) \\ &= X_1(f) + \frac{1}{2}X_1(f-2f_0) + \frac{1}{2}X_1(f+2f_0) \\ &\quad + \frac{-j}{2}X_2(f-2f_0) + \frac{j}{2}X_2(f+2f_0)\end{aligned}$$

$$\begin{aligned}X_7(f) &= X_5(f) * W_4(f) \\ &= -jX_5(f-f_0) + jX_5(f+f_0) \\ &= \frac{-j}{2}X_1(f-2f_0) + \frac{-j}{2}X_1(f) - \frac{1}{2}X_2(f-2f_0) + \frac{1}{2}X_2(f) \\ &\quad + \frac{j}{2}X_1(f) + \frac{j}{2}X_1(f+2f_0) + \frac{1}{2}X_2(f) - \frac{1}{2}X_2(f+2f_0) \\ &= X_2(f) - \frac{1}{2}X_2(f-2f_0) - \frac{1}{2}X_2(f+2f_0) \\ &\quad + \frac{-j}{2}X_1(f-2f_0) + \frac{j}{2}X_1(f+2f_0)\end{aligned}$$

Low-pass filtering $x_6(t)$ and $x_7(t)$ eliminates all but the low-freqiency terms, so that

$$\begin{aligned}X_8(f) &= X_1(f) \\ X_9(f) &= X_2(f)\end{aligned}$$

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Problem S18 Solution (Signals and Systems)

Do problem 8.26 from Openheim and Willksy, *Signals and Systems*, reprinted below:

8.26. In Section 8.2.2, we discussed the use of an envelope detector for asynchronous demodulation of an AM signal of the form $y(t) = [x(t) + A]\cos(\omega_c t + \theta_c)$. An alternative demodulation system, which also does not require phase synchronization, but does require frequency synchronization, is shown in block diagram form in Figure P8.26. The lowpass filters both have a cutoff frequency of ω_c. The signal $y(t) = [x(t) + A]\cos(\omega_c t + \theta_c)$, with θ_c constant but unknown. The signal $x(t)$ is band limited with $X(j\omega) = 0$, $|\omega| > \omega_M$, and with $\omega_M < \omega_c$. As we required for the use of the envelope detector, $x(t) + A > 0$ for all t.

Show that the system in Figure P8.26 can be used to recover $x(t)$ from $y(t)$ without knowledge of the modulator phase θ_c.

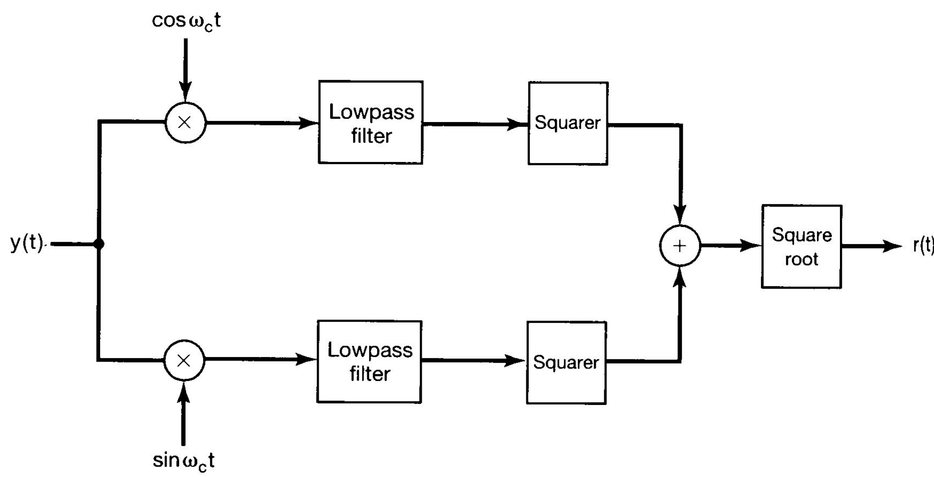
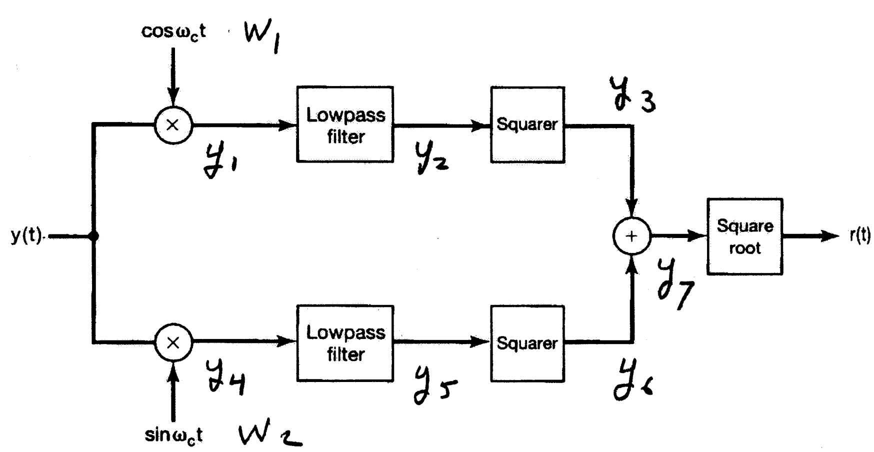


Figure P8.26

To begin, label the signals as shown below



From the problem statement,

$$y(t) = [x(t) + A]\cos(2\pi f_c t + \theta_c)$$

Define

$$\begin{aligned} z(t) &= x(t) + A \\ w(t) &= \cos(2\pi f_c t + \theta_c) \end{aligned}$$

The factor $w(t)$ can be expanded as

$$w(t) = \cos(2\pi f_c t + \theta_c) = \cos\theta_c \, \cos 2\pi f_c t - \sin\theta_c \, \sin 2\pi f_c t$$

The Fourier transform of $w(t)$ is then given by

$$\begin{aligned} W(f) &= \mathcal{F}[\cos(2\pi f_c t + \theta_c)] \\ &= \frac{1}{2}\cos\theta_c \, [\delta(f - f_c) + \delta(f + f_c)] - \frac{1}{2}\sin\theta_c \, [-j\delta(f - f_c) + j\delta(f + f_c)] \\ &= \frac{1}{2}(\cos\theta_c + j\sin\theta_c)\,\delta(f - f_c) + \frac{1}{2}(\cos\theta_c - j\sin\theta_c)\,\delta(f + f_c) \end{aligned}$$

The Fourier transform of $z(t) = x(t) + A$ is given by

$$Z(f) = \mathcal{F}[z(t)] = X(f) + A\delta(f)$$

$Z(f)$ is bandlimited, because $X(f)$ is, and of course the impulse function is bandlimited. So the FT of $y(t)$ is given by the convolution

$$\begin{aligned} Y(w) &= Z(f) * W(f) \\ &= \frac{1}{2}[(\cos\theta_c + j\sin\theta_c)\,Z(f - f_c) + (\cos\theta_c - j\sin\theta_c)\,Z(f + f_c)] \end{aligned}$$

Next, compute the spectra of $y_1(t)$ and $y_2(t)$. To do so, we need the spectra of $w_1(t)$ and $w_2(t)$:

$$\begin{aligned} W_1(f) = \mathcal{F}[w_1(t)] &= \mathcal{F}[\cos 2\pi f_c t] \\ &= \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ W_2(f) = \mathcal{F}[w_2(t)] &= \mathcal{F}[\sin 2\pi f_c t] \\ &= \frac{1}{2}[-j\delta(f - f_c) + j\delta(f + f_c)] \end{aligned}$$

Then

$$\begin{aligned} Y_1(f) &= W_1(f) * Y(f) \\ &= \frac{1}{2}[Y(f - f_c) + Y(f - f_c)] \\ &= \frac{1}{4}[(\cos\theta_c + j\sin\theta_c)\,Z(f - 2f_c) + (\cos\theta_c - j\sin\theta_c)\,Z(f)] \\ &\quad + \frac{1}{4}[(\cos\theta_c + j\sin\theta_c)\,Z(f) + (\cos\theta_c - j\sin\theta_c)\,Z(f + 2f_c)] \\ &= \frac{1}{2}\cos\theta_c\,Z(f) \\ &\quad + \frac{1}{4}[(\cos\theta_c + j\sin\theta_c)\,Z(f - 2f_c) + (\cos\theta_c - j\sin\theta_c)\,Z(f + 2f_c)] \end{aligned}$$

Similarly,

$$
\begin{aligned}
Y_4(f) &= W_2(f) * Y(f) \\
&= \frac{1}{2}\left[-jY(f-f_c) + jY(f-f_c)\right] \\
&= \frac{-j}{4}\left[(\cos\theta_c + j\sin\theta_c)\, Z(f-2f_c) + (\cos\theta_c - j\sin\theta_c)\, Z(f)\right] \\
&\quad + \frac{j}{4}\left[(\cos\theta_c + j\sin\theta_c)\, Z(f) + (\cos\theta_c - j\sin\theta_c)\, Z(f+2f_c)\right] \\
&= -\frac{1}{2}\sin\theta_c\, Z(f) \\
&\quad + \frac{1}{4}\left[(-j\cos\theta_c + \sin\theta_c)\, Z(f-2f_c) + (j\cos\theta_c + \sin\theta_c)\, Z(f+2f_c)\right]
\end{aligned}
$$

Now, when $y_1(t)$ and $y_4(t)$ are passed through the lowpass filters, the $Z(f-2f_c)$ and $Z(f+2f_c)$ terms are eliminated, and the $Z(f)$ terms are passed. Therefore,

$$
\begin{aligned}
Y_2(f) &= \frac{1}{2}\cos\theta_c\, Z(f) \\
Y_5(f) &= -\frac{1}{2}\sin\theta_c\, Z(f)
\end{aligned}
$$

and

$$
\begin{aligned}
y_2(t) &= \frac{1}{2}\cos\theta_c\, z(t) \\
y_5(t) &= -\frac{1}{2}\sin\theta_c\, z(t)
\end{aligned}
$$

After passing these signals through the squarers, we have

$$
\begin{aligned}
y_3(t) &= \frac{1}{4}\cos^2\theta_c\, z^2(t) \\
y_6(t) &= \frac{1}{4}\sin^2\theta_c\, z^2(t)
\end{aligned}
$$

\$y_{7}(t)\$ is the sum of these, so that

$$
\begin{aligned}
y_7(t) &= y_3(t) + y_7(t) \\
&= \frac{1}{4}\left[\cos^2\theta_c\, z^2(t) + \sin^2\theta_c\, z^2(t)\right] \\
&= \frac{1}{4} z^2(t)
\end{aligned}
$$

Finally, $r(t)$ is obtained by passing taking the square root of $y_7(t)$, so that

$$
\begin{aligned}
r(t) &= \sqrt{z^2(t)/4} \\
&= \frac{|z(t)|}{2}
\end{aligned}
$$

if the positive root is always taken. But $z(t) = x(t) + A$ is always positive, according to the problem statement. Therefore,

$$
x(t) = 2r(t) - A
$$

Problem S19 (Signals and Systems)

Do problem 8.8 from Openheim and Willksy, *Signals and Systems*, reprinted below. Note that this system implements a type of single sideband amplitude modulation. In addition to parts (a) and (b) below, do part (c).

8.8. Consider the modulation system shown in Figure P8.8. The input signal $x(t)$ has a Fourier transform $X(j\omega)$ that is zero for $|\omega| > \omega_M$. Assuming that $\omega_c > \omega_M$, answer the following questions:

(a) Is $y(t)$ guaranteed to be real if $x(t)$ is real?

(b) Can $x(t)$ be recovered from $y(t)$?

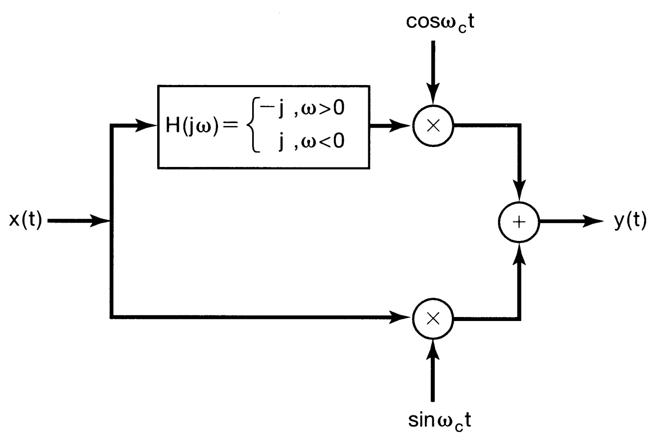


Figure P8.8

(c) For a representative spectrum $X(f)$, plot the resulting spectrum $Y(f)$. You may assume that the spectrum $X(f)$ is real, but plot both the real and imaginary parts of $Y(f)$. Does this system in fact produce single sideband modulation?

Solution: Label the signals in the problem as below:

8.8. Consider the modulation system shown in Figure P8.8. The input signal $x(t)$ has a Fourier transform $X(j\omega)$ that is zero for $|\omega| > \omega_M$. Assuming that $\omega_c > \omega_M$, answer the following questions:

(a) Is $y(t)$ guaranteed to be real if $x(t)$ is real?

(b) Can $x(t)$ be recovered from $y(t)$?

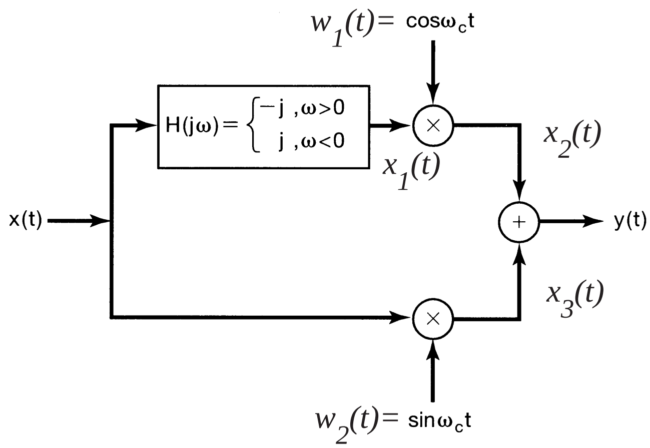


Figure P8.8

The Fourier transform of $x(t)$ is given by $X(f)$. Then the FT of $x_1(t)$ is given by

$$X_1(f) = H(f)X(f) = \begin{cases} -jX(f), & 0 < f < f_M \\ +jX(f), & -f_M < f < 0 \\ 0, & |f| > f_M \end{cases}$$

The signal $x_2(t)$ is given by

$$x_2(t) = w_1(t)x_1(t)$$

where $w_1(t) = \cos 2\pi f_c t$. The FT of $w_1(t)$ is

$$W_1(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

The FT of $x_2(t)$ is then

$$\begin{aligned} X_2(f) &= X_1(f) * W_1(f) \\ &= \frac{1}{2}[X_1(f - f_c) + X_1(f + f_c)] \\ &= \begin{cases} -\frac{j}{2}X(f - f_c), & f_c < f < f_c + f_M \\ +\frac{j}{2}X(f - f_c), & f_c - f_M < f < f_c \\ -\frac{j}{2}X(f + f_c), & -f_c < f < -f_c + f_M \\ +\frac{j}{2}X(f + f_c), & -f_c - f_M < f < -f_c \\ 0, & \text{else} \end{cases} \end{aligned}$$

The signal $x_3(t)$ is given by

$$x_3(t) = w_2(t)x(t)$$

where $w_2(t) = \sin 2\pi f_c t$. The FT of $w_2(t)$ is

$$W_1(f) = \frac{1}{2}[-j\delta(f - f_c) + j\delta(f + f_c)]$$

The FT of $x_3(t)$ is then

$$\begin{aligned}
X_3(f) &= X(f) * W_2(f) \\
&= \frac{1}{2}[-jX(f - f_c) + jX(f + f_c)] \\
&= \begin{cases} -\frac{j}{2}X(f - f_c), & f_c < f < f_c + f_M \\ -\frac{j}{2}X(f - f_c), & f_c - f_M < f < f_c \\ +\frac{j}{2}X(f + f_c), & -f_c < f < -f_c + f_M \\ +\frac{j}{2}X(f + f_c), & -f_c - f_M < f < -f_c \\ 0, & \text{else} \end{cases}
\end{aligned}$$

Finally, the FT of $y(t)$ is given by

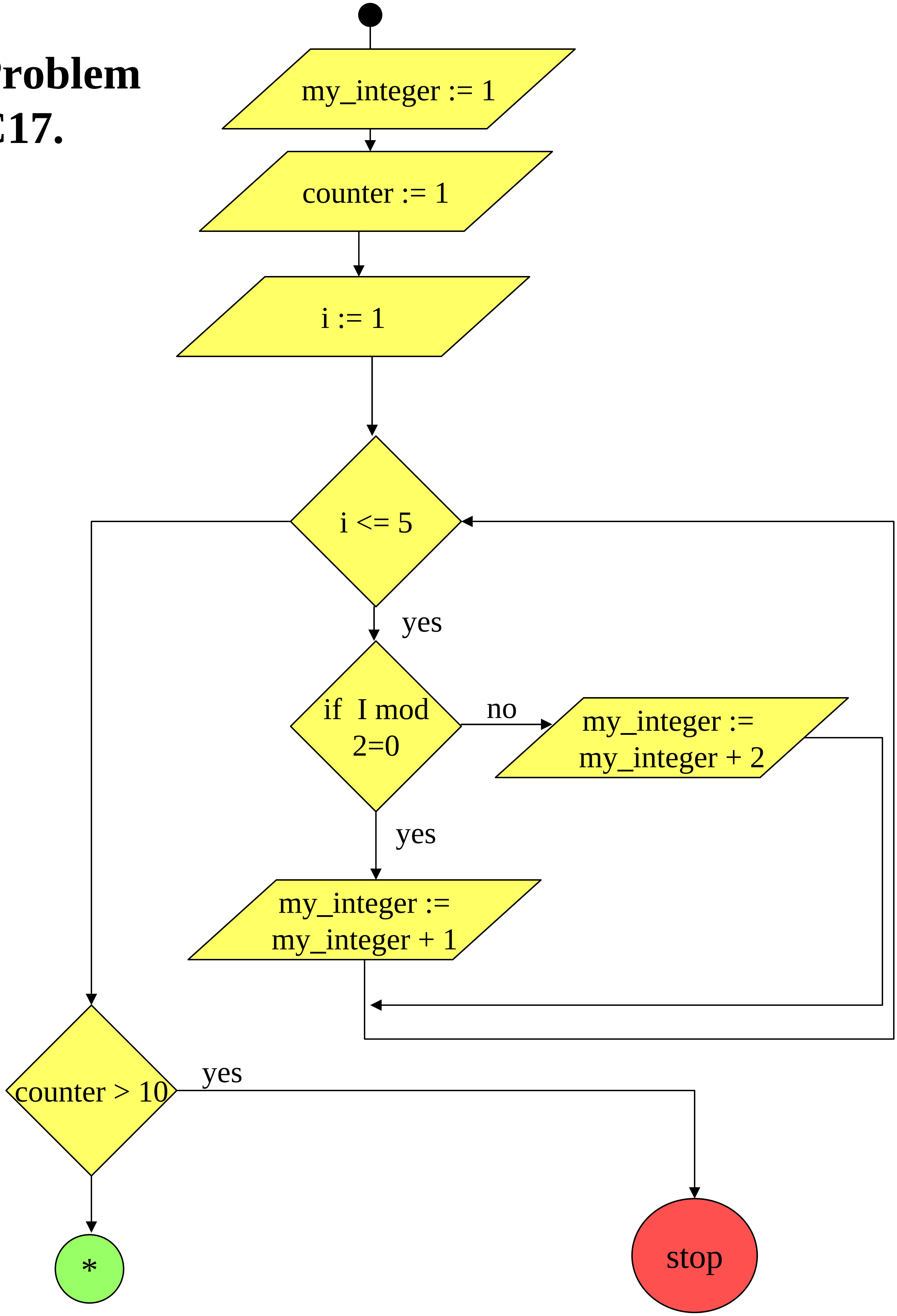
$$\begin{aligned}
Y(f) &= X_2(f) + X_3(f) \\
&= \begin{cases} -jX(f - f_c), & f_c < f < f_c + f_M \\ 0, & f_c - f_M < f < f_c \\ 0, & -f_c < f < -f_c + f_M \\ +jX(f + f_c), & -f_c - f_M < f < -f_c \\ 0, & \text{else} \end{cases} \\
&= \begin{cases} -jX(f - f_c), & f_c < f < f_c + f_M \\ +jX(f + f_c), & -f_c - f_M < f < -f_c \\ 0, & \text{else} \end{cases}
\end{aligned}$$

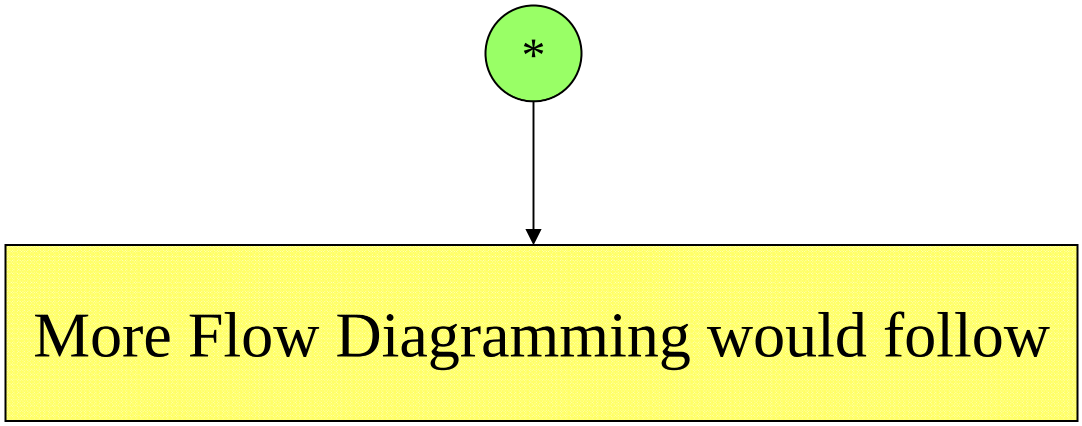
First, $y(t)$ is guaranteed to be real if $x(t)$ is real, because if $x(t)$ real, $X(f)$ has conjugate symmetry, and then $Y(f)$ has conjugate symmetry, which implies $y(t)$ real. Second, $x(t)$ can be recovered from $y(t)$ as follows. If $y(t)$ is modulated by $2\sin 2\pi f_c t$, the resulting signal is $z(t) = 2y(t)\sin 2\pi f_c t$, which has FT

$$\begin{aligned}
Z(f) &= -jY(f - f_c) + jY(f + f_c) \\
&= \begin{cases} -X(f - 2f_c), & 2f_c < f < 2f_c + f_M \\ +X(f), & -f_M < f < 0 \\ +X(f), & 0 < f < f_M \\ -X(f + 2f_c), & -2f_c - f_M < f < -2f_c \\ 0, & \text{else} \end{cases}
\end{aligned}$$

If $z(t)$ is then passed through a lowpass filter, with cutoff at $f = \pm f_M$, then the resulting signal is identical to $x(t)$.

Problem C17.


my_integer := 1
counter := 1
i := 1
i <= 5
yes
if I mod 2=0
no
my_integer := my_integer + 2
yes
my_integer := my_integer + 1
counter > 10
yes
*
stop



There are many ways in which to model the flow of data through a program. The short example above is one way.

The important thing to understand is that even though code may be in hundreds of different packages and procedures, these are just ways of dividing the code into smaller understandable segments. If you took all the files from one program, it would be possible to make one large data flow diagram.

```
package My_Test_Package is
  (i) My_Integer : Integer := 1;

procedure My_Integer_Increment;

procedure My_Integer_Double_Increment;

procedure Manipulate_Integer(
current_integer_value : out integer);

end My_Test_Package;
```

```
package body My_Test_Package is

  procedure My_Integer_Increment is
  begin
    (A) My_Integer := My_Integer + 1;
    end My_Integer_Increment;

  procedure My_Integer_Double_Increment is
  begin

 (B)  My_Integer := My_Integer + 2;
  end My_Integer_Double_Increment;


  procedure Manipulate_Integer(
Current_Integer_Value : out Integer) is
  begin
 (C) if My_Integer mod 2  = 0 then

      (D)   Current_Integer_Value := My_Integer/2;
    else

 (E)  Current_Integer_Value := My_Integer+1;
    end if;
  end Manipulate_Integer;

end My_Test_Package;
```

```
with My_Test_Package;

with Ada.Text_Io;

procedure Demo_Data_Flow is
   Counter : Integer :=1;
   My_Test_Integer : Integer;

begin
   for I in 1 .. 5 loop
      if I mod 2 = 0 then
         My_Test_Package.My_Integer_Increment;
      else
         My_Test_Package.My_Integer_Double_Increment;
      end if;
   end loop;

   loop
      exit when Counter > 10;
      My_test_Package.Manipulate_Integer(My_Test_Integer)
      if My_Test_Integer mod 2 = 0 then
         Counter := Counter +2;
         My_Test_Package.My_Integer_Increment;
      else
         Counter := Counter +1;
         My_Test_Package.My_Integer_Double_Increment;
      end if;
      Ada.Text_Io.Put("My Test Integer Value is");
      Ada.Text_Io.Put(Integer'Image(My_Test_Integer));
      Ada.Text_IO.New_Line;
      Ada.Text_Io.Put(Integer'Image(Counter));
      Ada.Text_Io.New_Line;
   end loop;
   Ada.Text_Io.Put_Line("Exited Loop");
end Demo_Data_Flow;
```

- Defs on the following lines

  1. def My_Integer
  6. def Counter
  12, (A). def My_Integer
  14, (B). def My_Integer
  20 (D or E). def My_Test_Integer
  22. def Counter
  23, (A). def My_Integer
  25. def Counter
  26, (B). def My_Integer

- All defs, some use

1,20	12,20
14,20	23,20
26,20	6,19
22,19	25,19
20,29	

- Uses on the following lines

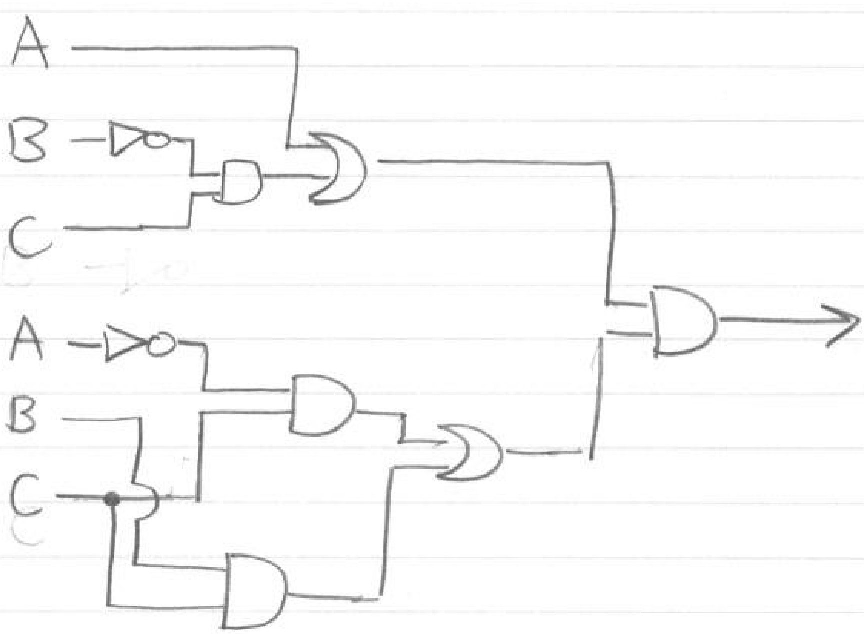
  12, (A). C-use My_Integer
  14, (B). C-use My_Integer
  19. P-use Counter
  20. P-use My_Integer
  21. P-use My_Test_Integer **and**
  C-use My_Test_Integer (D or E)
  22. C-use Counter
  23, (A). C-Use My_Integer
  25. C-use of Counter
  26, (B). C-use My_Integer
  29. C-use My_Test_Integer
  31. C-use Counter

C18)

a) $(A+\bar{B}C)\cdot(\bar{A}C+B\bar{C})$.

A	B	C	$X = A+\bar{B}C$	$Y = \bar{A}C+B\bar{C}$	$X\cdot Y$
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

b)



There are several different solutions...

C19

a)

Given: $\left[\left((A+\bar{B}C)\cdot(\bar{A}C+B\bar{C})\right)+\left((A\bar{B}+\bar{C})\cdot(\bar{A}B+\bar{B}C)\right)\right]$

Expanded: $A\bar{A}C + AB\bar{C} + \bar{B}\bar{A}C + A\bar{B}\bar{A}B + A\bar{B}C +$

$\bar{C}\bar{A}B + \bar{C}\bar{B}C$

$X\bar{X}$ removed: $AB\bar{C} + \bar{B}\bar{A}C + A\bar{B}C + \bar{C}\bar{A}B$

rewrite: $AB\bar{C} + A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

simplify $\boxed{B\bar{C} + \bar{B}C}$

b)

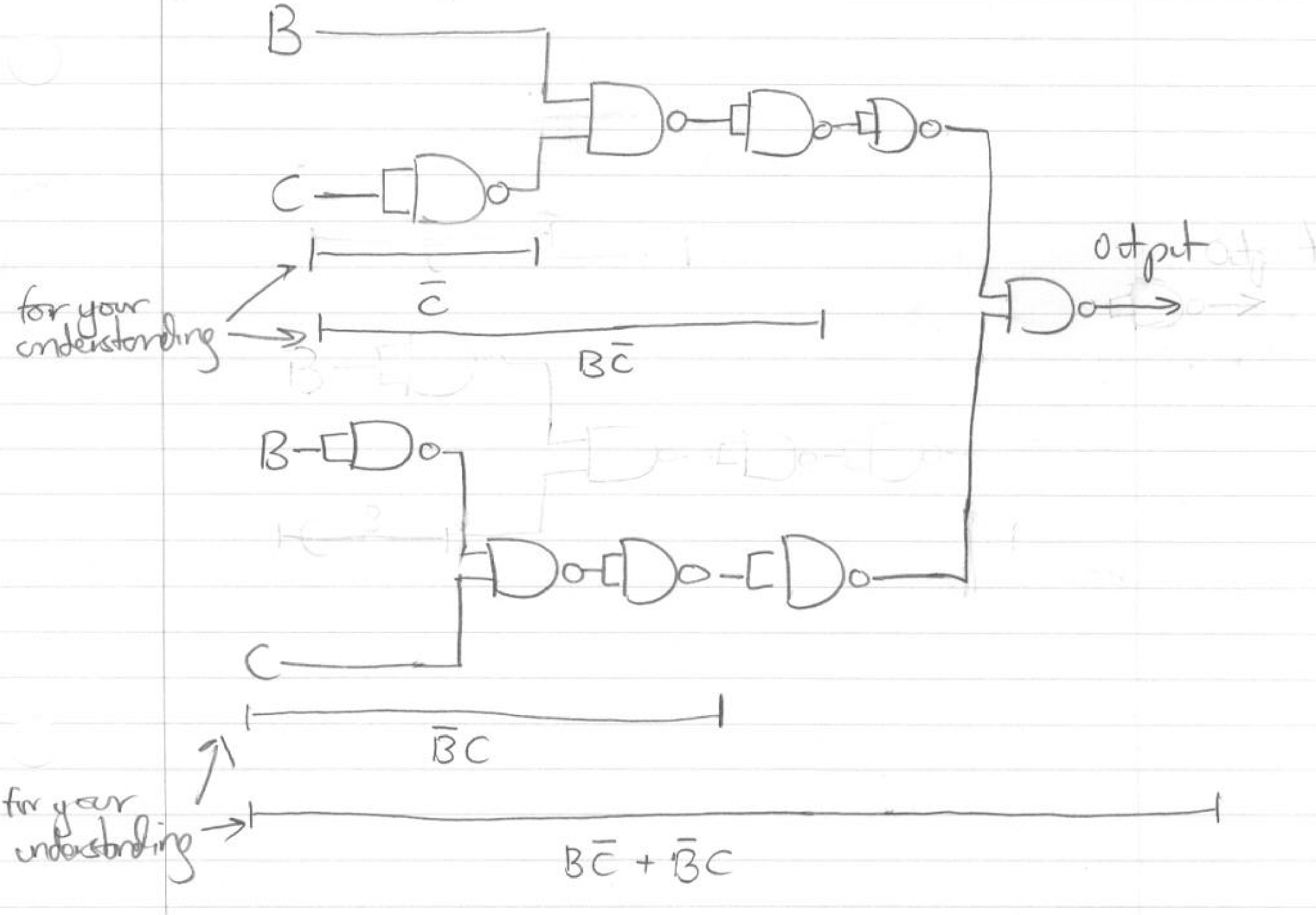
ABC	W: $A+\bar{B}C$	X: $\bar{A}C+B\bar{C}$	Y: $A\bar{B}+\bar{C}$	Z: $\bar{A}B+\bar{B}C$	WX	YZ	
0 0 0	0	0	1	0	0	0	0
0 0 1	1	1	0	1	1	0	1
0 1 0	0	1	1	1	0	1	1
0 1 1	0	1	0	1	0	0	0
1 0 0	1	0	1	0	0	0	0
1 0 1	1	0	1	1	0	1	1
1 1 0	1	1	1	0	1	0	1
1 1 1	1	0	0	0	0	0	0

Use this for K-Map

C	0	1
A/B		
0 0	0	1
0 1	1	0
1 1	1	0
1 0	0	1

$\boxed{B\bar{C} + \bar{B}C}$

c) If you wanted to be very literal in this translation:



A simpler representation can be found using DeMorgan's laws to convert the expression:

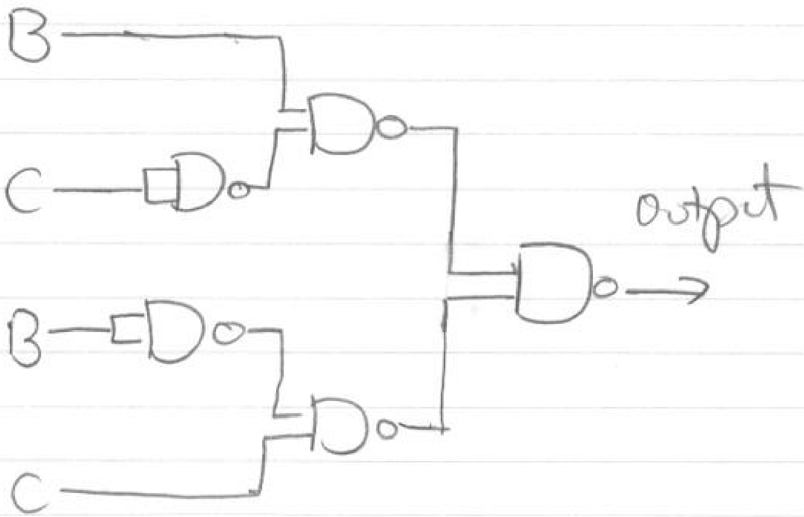
$$\overline{AB} \rightarrow \bar{A} + \bar{B}$$

$$\overline{A+B} \rightarrow \bar{A}\,\bar{B}$$

Given: $B\bar{C} + \bar{B}C$

Invert twice: $\overline{\overline{B\bar{C} + \bar{B}C}}$

apply DeMorgan: $\overline{\overline{B\bar{C}} \cdot \overline{\bar{B}C}}$



C20

a)

Given: $(A+\bar{B})\cdot(\bar{A}+B)\cdot(\bar{B}+\bar{C})\cdot(\bar{A}+C)$

expand: $(A\bar{A}+AB+\bar{B}\bar{A}+B\bar{B})\cdot(\bar{B}\bar{A}+\bar{B}C+\bar{C}\bar{A}+\bar{C}C)$

expand:

$$\boxed{\begin{aligned}&A\bar{A}\bar{B}\bar{A} + A\bar{A}\bar{B}C + A\bar{A}\bar{C}\bar{A} + A\bar{A}\bar{C}C +\\ &AB\bar{B}\bar{A} + AB\bar{B}C + AB\bar{C}\bar{A} + AB\bar{C}C +\\ &\bar{B}\bar{A}\bar{B}\bar{A} + \bar{B}\bar{A}\bar{B}C + \bar{B}\bar{A}\bar{C}\bar{A} + \bar{B}\bar{A}\bar{C}C +\\ &B\bar{B}\bar{B}\bar{A} + B\bar{B}\bar{B}C + B\bar{B}\bar{C}\bar{A} + B\bar{B}\bar{C}C\end{aligned}}$$

remove $X\bar{X}$ because (true and false) = false always!

b)

$$\begin{aligned}&\cancel{A\bar{A}\bar{B}\bar{A}} + \cancel{A\bar{A}\bar{B}C} + \cancel{A\bar{A}\bar{C}\bar{A}} + \cancel{A\bar{A}\bar{C}C} +\\ &\cancel{AB\bar{B}\bar{A}} + \cancel{AB\bar{B}C} + \cancel{AB\bar{C}\bar{A}} + \cancel{AB\bar{C}C} +\\ &\bar{B}\bar{A}\bar{B}\bar{A} + \bar{B}\bar{A}\bar{B}C + \bar{B}\bar{A}\bar{C}\bar{A} + \cancel{\bar{B}\bar{A}\bar{C}C} +\\ &\cancel{B\bar{B}\bar{B}\bar{A}} + \cancel{B\bar{B}\bar{B}C} + \cancel{B\bar{B}\bar{C}\bar{A}} + \cancel{B\bar{B}\bar{C}C}\end{aligned}$$

result: $\bar{A}\bar{B} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

simplify: $\bar{A}\bar{B} + \bar{A}\bar{B}$

$\bar{A}\bar{B}$

c) $(A+\bar{B})\cdot(\bar{A}+B)\cdot(\bar{B}+\bar{C})\cdot(\bar{A}+C)$

			W	X	Y	Z	
A	B	C	$A+\bar{B}$	$\bar{A}+B$	$\bar{B}+\bar{C}$	$\bar{A}+C$	WXYZ
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	1	0
0	1	1	0	1	0	1	0
1	0	0	1	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	1	1	1	0	0
1	1	1	1	1	0	1	0

K-Map:

A/B \ C	0	1
0 0	1	1
0 1	0	0
1 1	0	0
1 0	0	0

$\boxed{\bar{A}\bar{B}}$

d) Many students understood

① $\overline{ABCD}$ as not $(ABCD)$

many other students:

② $\overline{ABCD}$ as $\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$

Since this was unclear, both will be accepted as a solution. The answers below will represent ① then ②

Answer for ①

A	B	C	D	$\overline{ABCD}$	$\bar{A}B\bar{C}$	$A\overline{BCD}$	$A\bar{B}C\bar{D}$	$AB\bar{C}D$	$\overline{AB}C\bar{D}$	$\bar{A}BCD$	
0	0	0	0	1							1
0	0	0	1	1							
0	0	1	0	1							
0	0	1	1	1							
0	1	0	0	1							
0	1	0	1	1							
0	1	1	0	1							
0	1	1	1	1							
1	0	0	0	1							
1	0	0	1	1							
1	0	1	0	1							
1	0	1	1	1							
1	1	0	0	1							
1	1	0	1	1							
1	1	1	0	1							
1	1	1	1	0	0	0	0	0	0	0	0

Since we are taking the 'or' of all of these terms... the fact that $\overline{ABCD}$ is true for so many of the cases means we only need to verify the last case.

Using a K-Map for this case is trivial:

A/B \ C/D	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

This is how you would circle if you wanted SOP form ... (but we don't want that)

Method 1:

but for POS form...

A/B \ C/D	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	$\boxed{0}$	1
10	1	1	1	1

when using a Kmap for POS form things are a little different:

you invert each variable and add them up...

so...

$$\begin{matrix} A & B & C & D \\ 1 & 1 & 1 & 1 \end{matrix} \Rightarrow \begin{matrix} \bar{A} & \bar{B} & \bar{C} & \bar{D} \\ 0 & 0 & 0 & 0 \end{matrix} \Rightarrow \boxed{\begin{matrix} \bar{A}+\bar{B}+\bar{C}+\bar{D} \\ \hline 0+0+0+0 \end{matrix}}$$

Method 2:

This is the way Professor Lundquist taught you, this is the method you are responsible to know.

① Write the truth table

ABCD	F	F	$\overline{F}$
0000	See the earlier truth table for intermediate steps.	1	0
0001		1	0
0010		1	0
0011		1	0
0100		1	0
0101		1	0
0110		1	0
0111		1	0
1000		1	0
1001		1	0
1010		1	0
1011		1	0
1100		1	0
1101		1	0
1110		1	0
1111		0	1

② Invert the output result

③ Create the K-Map based on $\overline{F}$

A/B \ C/D	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	0
10	0	0	0	0

④ Write the expression:

Note $\overline{F}$ $\rightarrow$ $\overline{F} = ABCD$

⑤ Convert w/ DeMorgan's Law

$F = \overline{ABCD}$

$$\boxed{F = \overline{A} + \overline{B} + \overline{C} + \overline{D}}$$

Answer for (2)

A	B	C	D	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}$	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}C\bar{D}$	$AB\bar{C}D$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BCD$	
0	0	0	0	1	0	0					1
0	0	0	1	0	0	0					0
0	0	1	0	0	0	0			1		1
0	0	1	1	0	0	0					0
0	1	0	0	0	1	0					1
0	1	0	1	0	1	0					1
0	1	1	0	0	0	0					0
0	1	1	1	0	0	0				1	1
1	0	0	0	0	0	1					1
1	0	0	1	0	0	0					0
1	0	1	0	0	0	0	1				1
1	0	1	1	0	0	0					0
1	1	0	0	0	0	0					0
1	1	0	1	0	0	0		1			1
1	1	1	0	0	0	0					0
1	1	1	1	0	0	0					0

Got tired of writing zeroes...

K-Map:

C/D A/B	00	01	11	10
00	1	0	0	1
01	1	1	1	0
11	0	1	0	0
10	1	0	0	1

expression:

$$\bar{B}\bar{D} + \bar{A}B\bar{C} + \bar{A}BD + B\bar{C}D$$

But... converting this guy is a pain!

$\boxed{\text{Method 1}}$

Let's do a POS K-Map:

C/D A/B	00	01	11	10
00	1	0	0	1
01	1	1	1	0
11	0	1	0	0
10	1	0	0	1

$$\boxed{(B + \bar{D}) \cdot (\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{B} + \bar{C} + D) \cdot (\bar{A} + \bar{B} + D)}$$

Method 2

This is the method you are responsible for knowing!

① Write the truth table

ABCD	See the earlier truth table for intermediate steps.	F	$\overline{F}$
0000		1	0
0001		0	1
0010		1	0
0011		0	1
0100		1	0
0101		1	0
0110		0	1
0111		1	0
1000		1	0
1001		0	1
1010		1	0
1011		0	1
1100		0	1
1101		1	0
1110		0	1
1111		0	1

② Invert the output result

③ Create K-map based on $\overline{F}$

A/B \ C/D	00	01	11	10
00	0	1	1	0
01	0	0	0	1
11	1	0	1	1
10	0	1	1	0

④ Write the expression:

Note $\overline{F}$ $\longrightarrow$ $\overline{F} = \overline{B}D + ABC + BC\overline{D} + AB\overline{D}$

⑤ Convert w/ <u>DeMorgan's</u> Laws

$$F = \overline{\overline{B}D + ABC + BC\overline{D} + AB\overline{D}}$$

$$F = \overline{\overline{B}D} \bullet \overline{ABC} \bullet \overline{BC\overline{D}} \bullet \overline{AB\overline{D}}$$

$$F = (\overline{\overline{B}} + \overline{D}) \bullet (\overline{A} + \overline{B} + \overline{C}) \bullet (\overline{B} + \overline{C} + \overline{\overline{D}}) \bullet (\overline{A} + \overline{B} + \overline{\overline{D}})$$

$$\boxed{F = (B + \overline{D}) \bullet (\overline{A} + \overline{B} + \overline{C}) \bullet (\overline{B} + \overline{C} + D) \bullet (\overline{A} + \overline{B} + D)}$$