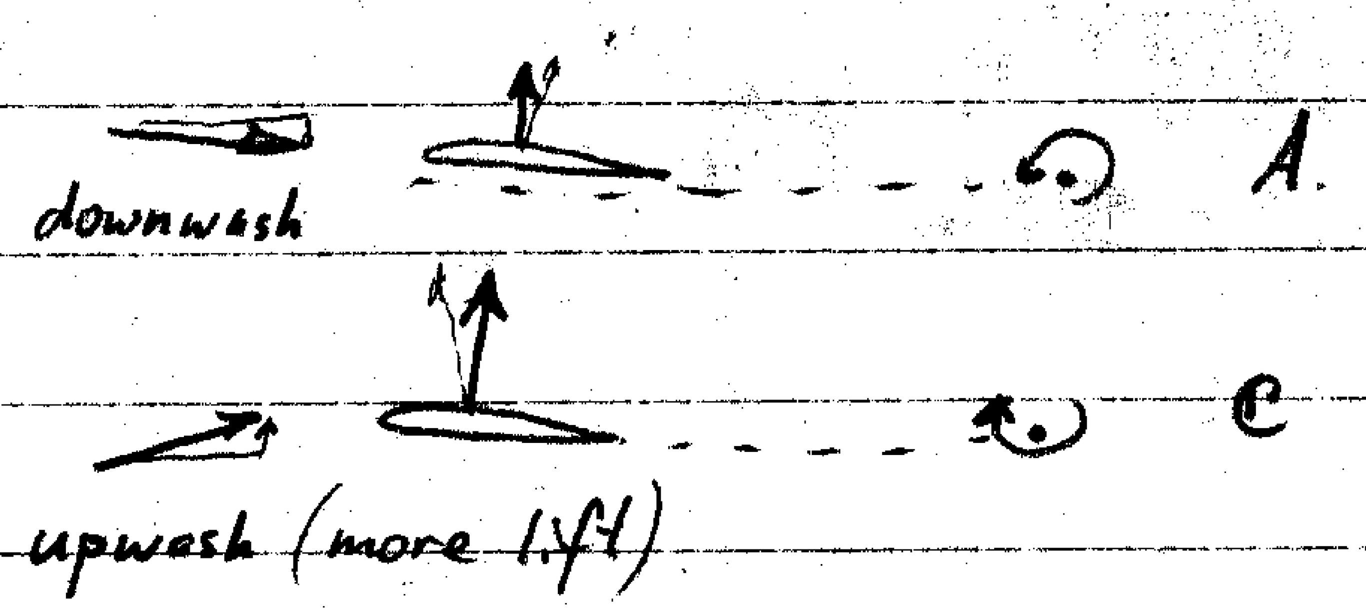
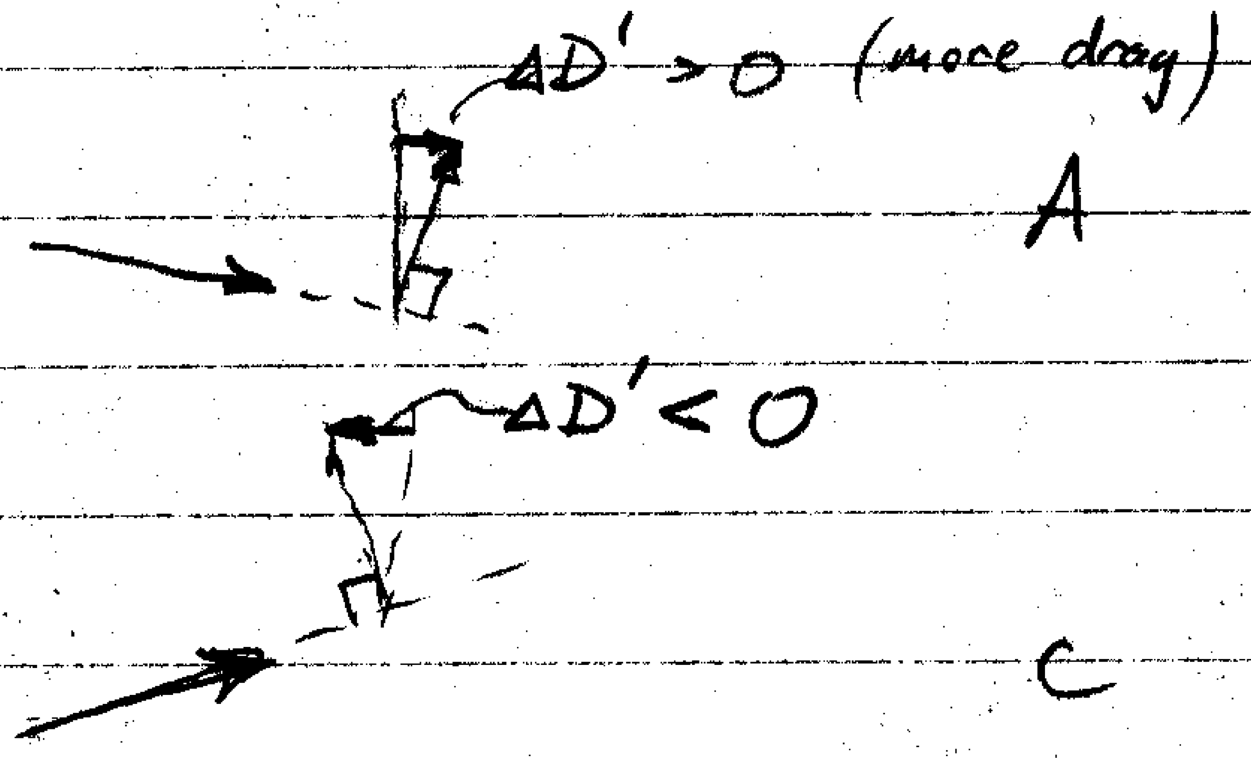
UE Fluids Quiz 1 Solution Spring '05

1.

i) $L'_A < L'_B < L'_C$



iii) $D'_A > D'_B > D'_C$



$$\frac{x}{c} = \frac{1}{2}(1-\cos\theta)$$

$$1-2\frac{x}{c} = \cos\theta$$

2. $$\frac{dz_A}{dx} = 4\frac{h}{c}\left(1-2\frac{x}{c}\right) = 4\frac{h}{c}\cos\theta$$

$$\frac{dz_B}{dx} = -2\frac{h}{c}\frac{x}{c} = \frac{h}{c}(\cos\theta-1)$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \left(\cancel{\alpha}^{\,0} - \frac{dz_A}{dx}\right)d\theta \qquad A_0 = \frac{1}{\pi}\int_0^\pi \left(\cancel{\alpha}^{\,0} - \frac{dz_B}{dx}\right)d\theta$$

$$A_0 = \frac{1}{\pi}\int_0^\pi -4\frac{h}{c}\cos\theta\, d\theta = 0 \qquad A_0 = \frac{1}{\pi}\int_0^\pi \frac{h}{c}(1-\cos\theta)d\theta = \frac{h}{c}$$

$$A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz_A}{dx}\cos\theta\, d\theta \qquad A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz_B}{dx}\cos\theta\, d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^\pi 4\frac{h}{c}\cos^2\theta\, d\theta = 4\frac{h}{c} \qquad A_1 = \frac{2}{\pi}\int_0^\pi \frac{h}{c}(\cos^2\theta - \cos\theta)\,d\theta = \frac{h}{c}$$

$$A_2 = 0 \qquad A_2 = 0$$

$$A_3 = 0 \qquad A_3 = 0$$

$$\vdots \qquad \vdots$$

a) $$C_l = \pi(2A_0 + A_1) = 4\pi\frac{h}{c} \qquad C_l = \pi(2A_0 + A_1) = 3\pi\frac{h}{c}$$

$$\frac{dC_l}{d(h/c)} = 4\pi \qquad \frac{dC_l}{d(h/c)} = 3\pi$$

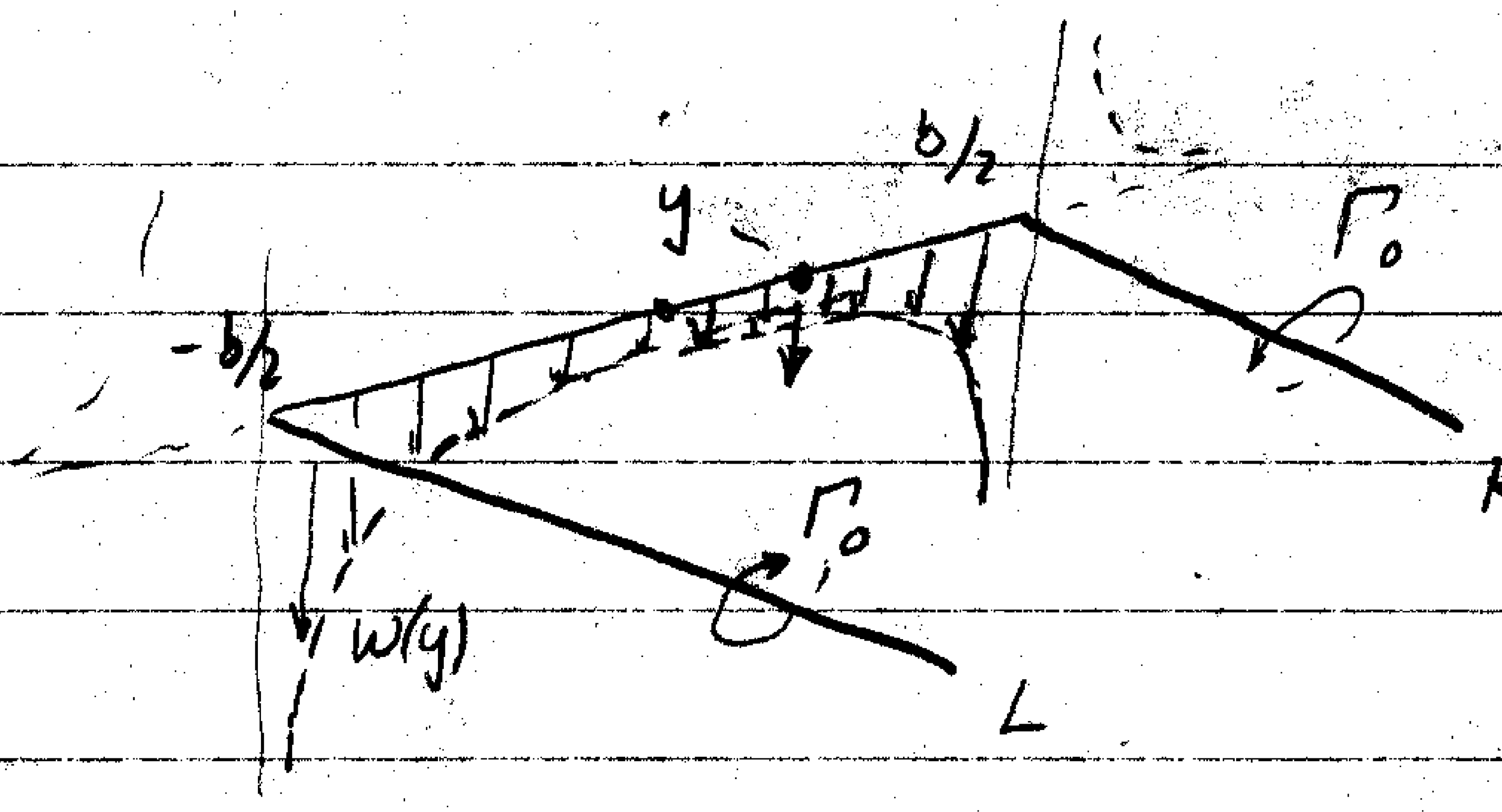
b) $$C_{m,c/4} = \frac{\pi}{4}(A_2 - A_1) = -\pi\frac{h}{c} \qquad C_{m,c/4} = \frac{\pi}{4}(A_2 - A_1) = -\frac{\pi}{4}\frac{h}{c}$$

$$\frac{dC_{m,c/4}}{d(h/c)} = -\pi \qquad \frac{dC_{m,c/4}}{d(h/c)} = -\frac{\pi}{4}$$

A. has $\frac{4}{3}\times$ larger control power, but $4\times$ larger torque.

3.

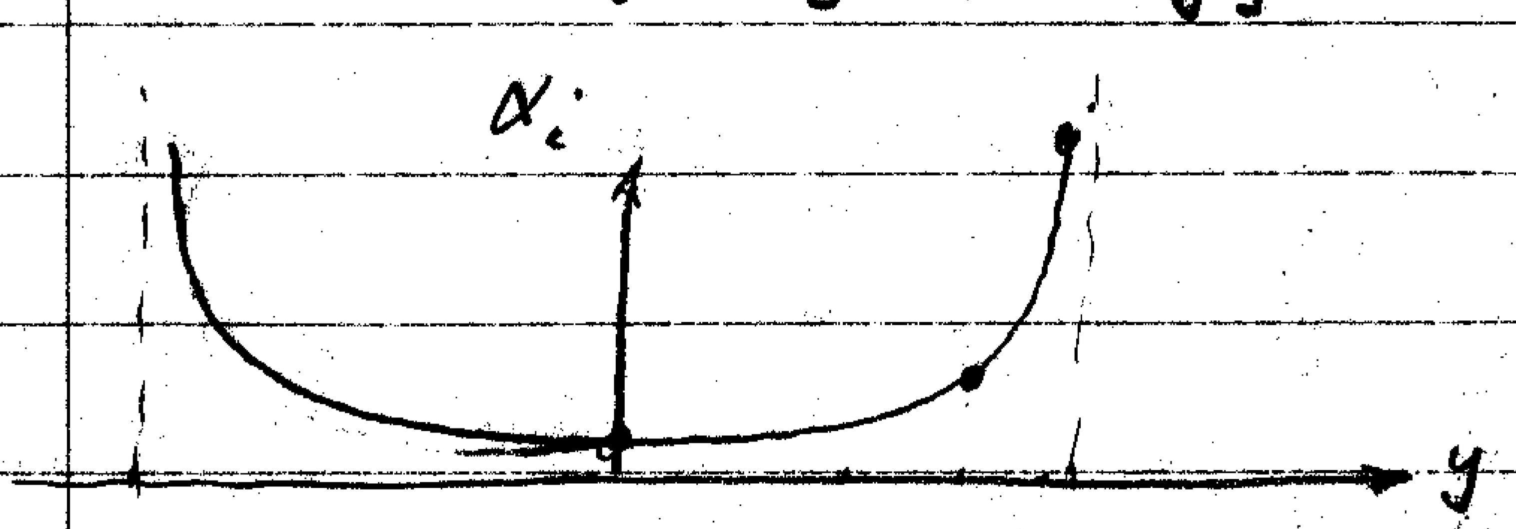
a) $w(y)$ is from trailing legs of the horseshoe vortex



$$w(y) = \underbrace{\frac{-\Gamma_0}{4\pi}\frac{1}{b/2-y}}_{R} - \underbrace{\frac{\Gamma_0}{4\pi}\frac{1}{b/2+y}}_{L}$$

$$w(y) = \frac{-\Gamma_0}{4\pi}\left[\frac{1}{b/2-y} + \frac{1}{b/2+y}\right]$$

y	w
0	$-\frac{0.5}{4\pi}\left[1+1\right] \cong \frac{-1}{4\pi} = -0.08$
0.75	$-\frac{0.5}{4\pi}\left[\frac{1}{0.25}+\frac{1}{1.75}\right] = \frac{-2.3}{4\pi} = -0.18$
0.95	$-\frac{0.5}{4\pi}\left[\frac{1}{0.05}+\frac{1}{1.95}\right] = \frac{-10.2}{4\pi} = -0.82$



$$\alpha_i(y) = -w(y)/V_\infty$$

$\alpha_i(0) = 0.9^\circ$

$\alpha_i(0.75) = 2.1^\circ$

$\alpha_i(0.95) = 9.3^\circ$

b) $\Gamma = \frac{1}{2}V_\infty c\, C_\ell$, $\Gamma = \Gamma_0 = 0.5\ m/s$, $c = c_0 = 0.2\ m$

$$\rightarrow C_\ell = \frac{2\Gamma}{V_\infty c} = \frac{2\cdot 0.5}{5\cdot 0.2} = 1.0 \quad \text{constant } C_\ell$$

$$C_\ell = 2\pi\left[\cancel{\alpha}^{0} + \alpha_{geom} - \cancel{\alpha_{L=0}}^{0} - \alpha_i\right]$$

$$\alpha_{geom}(y) = \frac{C_\ell}{2\pi} + \alpha_i = \frac{1}{2\pi} + \frac{\Gamma_0}{4\pi V_\infty}\left[\frac{1}{b/2-y} + \frac{1}{b/2+y}\right] , \quad \frac{\Gamma_0}{2V_\infty} = 0.05\ m$$

$$\rightarrow \alpha_{geom}(y) = \frac{1}{2\pi}\left\{1 + 0.05\left[\frac{1}{1-y} + \frac{1}{1+y}\right]\right\} \quad \text{radians}$$

c) $\alpha_{geom}(y)$ gets very large (infinite!) as we approach each tip at $y = \pm b/2$. Wing requires lots of washin.