9:00 - 10: 00
35-225

MIT ID# (last four digits) SOLUTIONS -PAL

Unified Quiz 2M

March 4, 2005

- Put your MIT ID# (last four digits) on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the actual answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units. Intermediate answers and final answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators, handwritten "crib sheets", and Unified Handout #M-5 allowed.***

EXAM SCORING

#1M (30%)	
#2M (40%)	
#3M (30%)	
FINAL SCORE	

Unified Quiz Materials and Structures 2 (Q2M)
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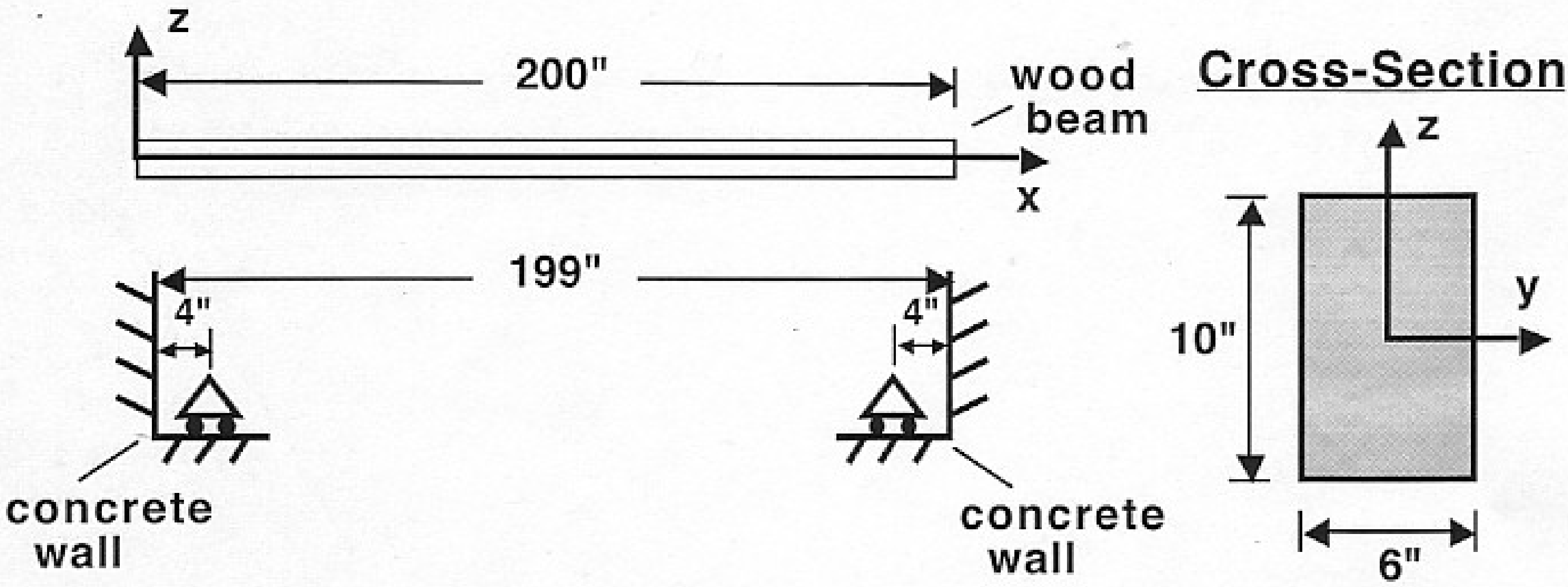
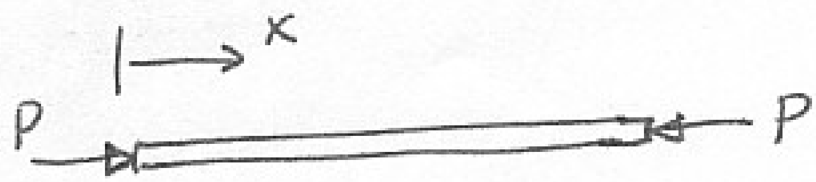
PROBLEM #1M (30%)

A 200-inch long wood (E = 3.5 Msi, ν = 0.3) beam is to be used as the major load-carrying component in the construction of a home. The beam has a rectangular cross-section 6 inches across and 10 inches deep. The beam is to rest on the poured concrete walls of the foundation and the ends are to fit between the extended concrete walls. This can be modeled as two roller joints, 4 inches inboard of the outer wall, as shown, with the end walls providing resistance so that the wood beam cannot expand, but can contract. When the concrete hardens and the forms are removed, it is found that the distance between the end-walls is 199 inches.



(a) Using the roller-model shown above, determine the end load that would need to be applied to the wood beam so that it will fit between the end-walls.

First, although this is referred to as "a beam", the applicable model for this issue (compressing the structure) is that of a rod as this is to apply axial load to make a piece of original length 200" to fit into a 199" space:



The strain will be: $\epsilon_x = \frac{\Delta L}{L} = \frac{199'' - 200''}{200''} = -\frac{1}{200}$

$$\Rightarrow \epsilon_x = -0.005$$

Since only axial stress is applied, we know:

$$\sigma_x = E_x \epsilon_x$$

PROBLEM #1M (continued)

$$\Rightarrow \sigma_x = (3.5 \times 10^6 \ lb/in^2)(-0.005) = -17{,}500 \ lbs/in^2$$

And for a rod:

$$\sigma_x = \frac{P}{A}$$

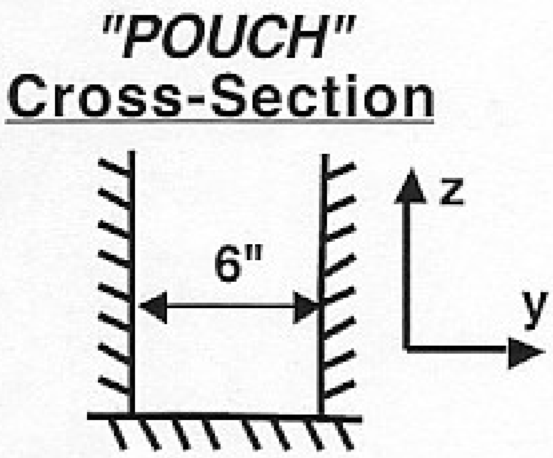
The cross-sectional area is:

$$10'' \times 6'' = 60 \ in^2$$

$$\Rightarrow P = (60 \ in^2)(-17{,}500 \ lbs/in^2)$$

$$\Rightarrow \boxed{P = -1.05 \times 10^6 \ lbs}$$

(b) The actual poured concrete configuration has a "pouch" in the wall that measures exactly 6 inches in width into which the beam is to fit. Does this present any additional issues/difficulties that must be taken into consideration? Be as clear and as quantitative as possible.



The beam is 6" in width originally

Due to Poisson's ratio:

$$\epsilon_y = -\frac{\nu}{E}\sigma_x \quad (\text{for } \sigma_x \text{ axial loading only})$$

$$\Rightarrow \epsilon_y = \frac{-0.3}{(3.5 \times 10^6 \ lb/in^2)}(-17{,}500 \ lb/in^2) = 0.0015$$

(NOTE: same as $-\nu \epsilon_x$)

The total displacement in the y-direction is:

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PROBLEM #1M (continued)

$$v = \int_0^{6''} \epsilon_y \, dy = 0.0015(6'') = 0.009''$$

The beam expands by 0.009'' and becomes 6.009'' in width

$\Rightarrow$ $\boxed{\text{it will not fit}}$

(c) Recommend what should be done in this situation given your analysis and any further ideas you may have.

- It will not be feasible to compress this since it will take a million pounds! (and the concrete wall may crack in trying to hold this)
- It may be difficult and costly to get a new beam
- $\boxed{\text{Hand machine the end to fit.}}$ It will still be able to act as a beam as it needs to since the lip of the sill is further in. Just don't machine it too much!

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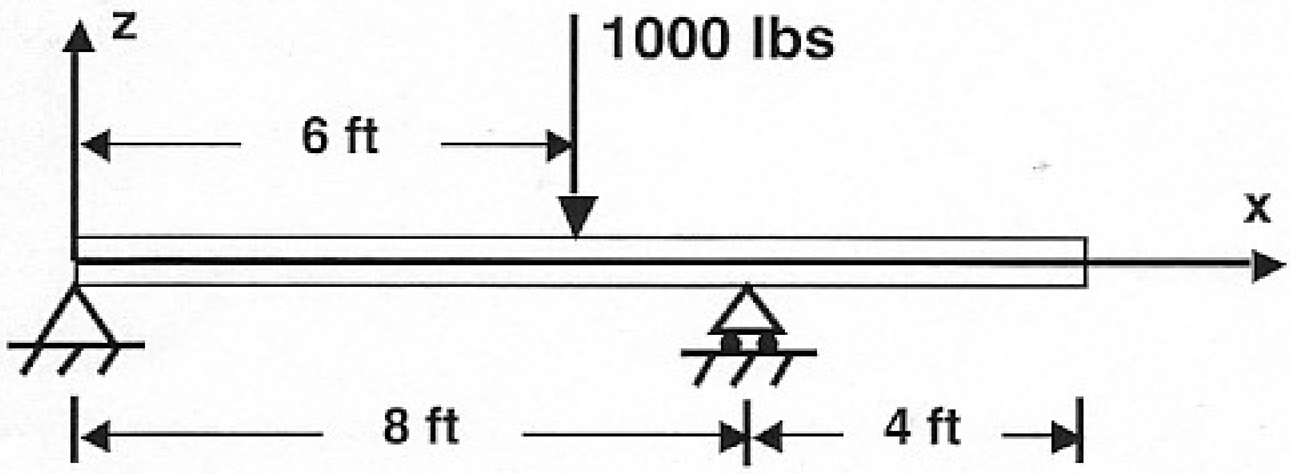
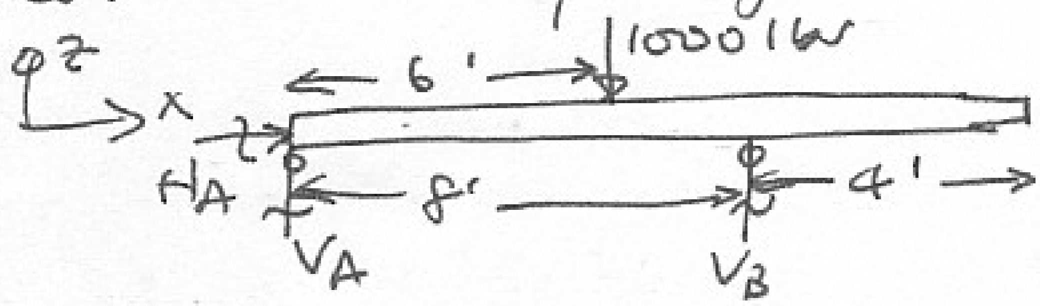
PROBLEM #2M (40%)

An aluminum beam (E = 10 Msi, ν = 0.3) is supported by a pin at one end and by a roller joint at its two-thirds span point. The beam is a total of 12 feet long and has a solid square cross-section with sides of 6 inches. The beam has a downward point load of 1000 pounds at the midspan.



(a) Sketch the shear force and bending moment resultant distributions as a function of position along the beam. Be sure to note the key values of each and their locations.

→ Draw the Free Body Diagram:



→ Do equilibrium:

$$\Sigma F_x = 0 \;\; \overset{+}{\rightarrow} \;\; \Rightarrow H_A = 0$$

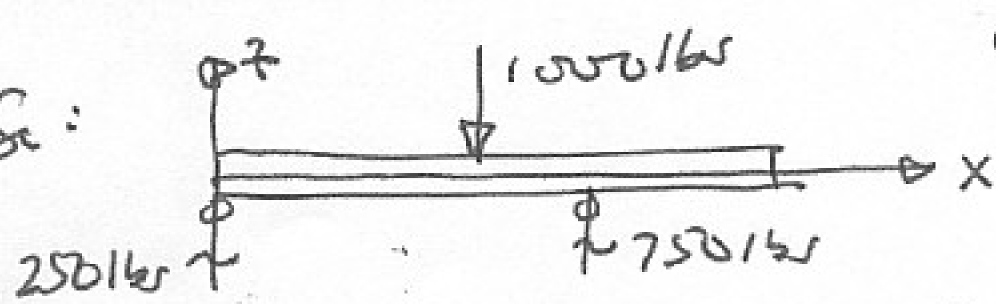
$$\Sigma F_z = 0 \;\; \uparrow + \;\; \Rightarrow V_A + V_B = 1000 \text{ lbs}$$

$$\Sigma M_A = 0 \;\; \circlearrowright + \;\; \Rightarrow (6')(1000 \text{ lbs}) - V_B(8') = 0$$

$$\Rightarrow V_B = 750 \text{ lbs}$$

gives $V_A = 250 \text{ lbs}$

So:

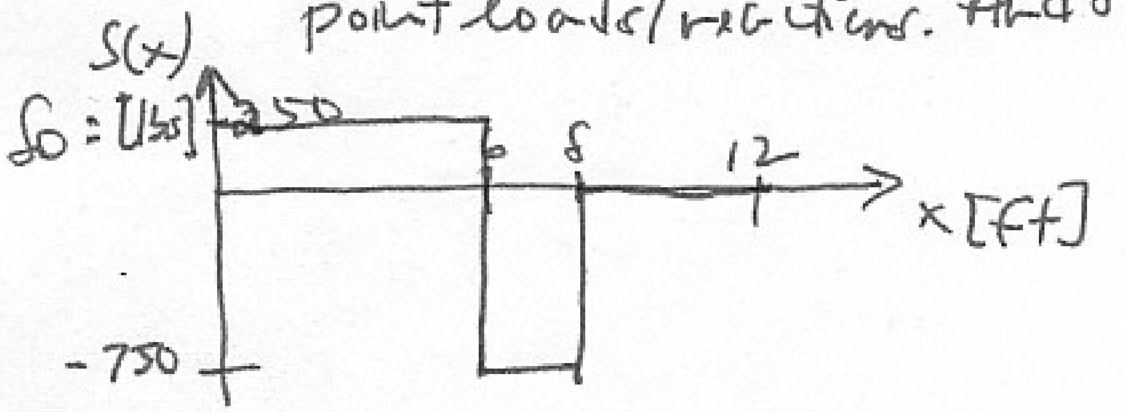


→ in between loads and reactions,

$q(x) = 0$

PROBLEM #2M (continued)

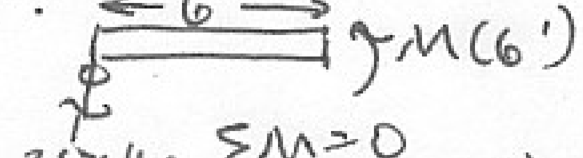
→ since $\frac{dS(x)}{dx} = q(x) \Rightarrow S(x)$ is a constant between the loads and reactions with changes equal to the point loads/reactions. And $S(x)$ starts at V_A:



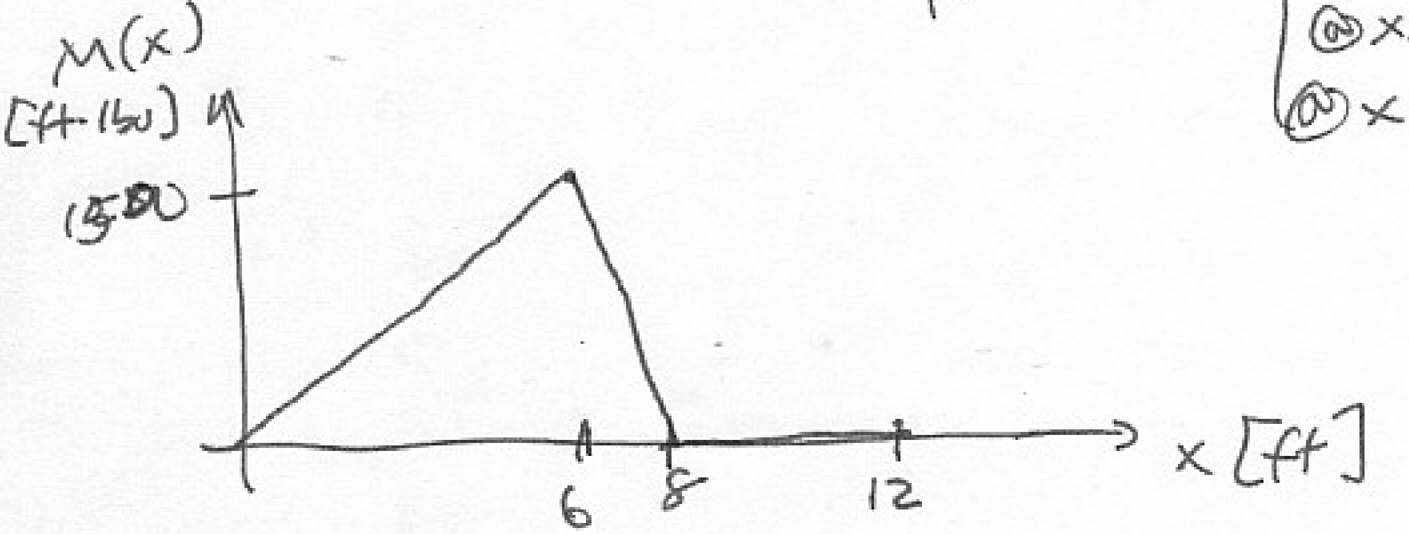
$\Sigma F_z = 0$
$\Rightarrow S(0) = V_A$

→ since $\frac{dM(x)}{dx} = S(x) \Rightarrow M(x)$ goes up linearly between load points.

Key points:
- @ $x = 0$, $M = 0$ (no moment for pin)
- @ $x > 8'$ $M = 0$ (no moment at free end)
- @ $x = 6'$:



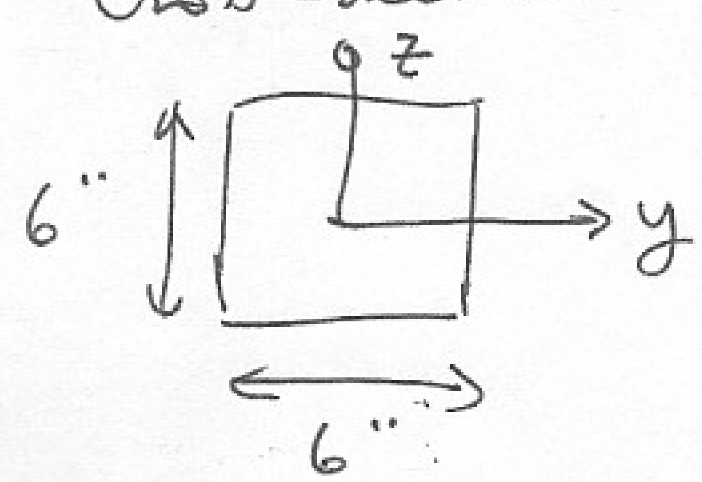
$\Sigma M = 0$
$\Rightarrow M(6') = (6')(250)\text{lbs/ft}$
$= 1500$ lbs.ft



(b) Determine the location of the maximum axial stress (i.e. σ_{xx}).

$\sigma_{xx} = -\frac{Mz}{I} \Rightarrow$ maximum $|\sigma_{xx}|$ at maximum $|M|$ and $|z|$

Cross-Section



$\Rightarrow$ $\boxed{\text{@ } x = 6 \text{ ft}, \quad z = 3 \text{ in}}$

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PROBLEM #2M (continued)

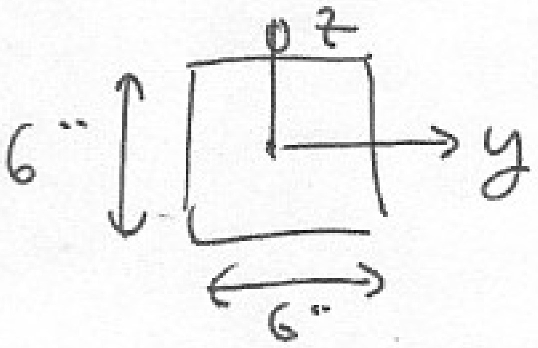
(c) Determine the location of the maximum shear stress (i.e. σ_{xz}).

$$\sigma_{xz} = -\frac{SQ}{Ib}$$

$\Rightarrow$ maximum $|\sigma_{xz}|$ at maximum $|S|$ and $|Q|$

$|S|_{max}$ occurs from $x = 6'$ to $8'$

$\rightarrow$ for a symmetric section, Q is maximum at the centerline



$\Rightarrow$ $\boxed{\text{max at } x = 6\text{ft to } 8\text{ft},\ z = 0}$

(d) How do the answers to parts (a), (b), and (c) change if steel (E = 30 Msi, ν = 0.3) is used rather than aluminum?

Since the configuration is statically determinate the stresses are not affected by the modulus, but are only determined from equilibrium

$\Rightarrow$ $\boxed{\text{DO NOT CHANGE}}$

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PROBLEM #2M (continued)

(e) How does the maximum deflection of the beam change when the beam is made of steel rather than aluminum?

The deflection is related to the moment via:

$$\frac{d^2w}{dx^2} = \frac{M}{EI}$$

The moment does not change ($M(x)$) since the configuration is statically determinate. The boundary conditions stay the same ($w = 0$ @ $x = 0$) and ($w = 0$ @ $x = 8'$). The cross-section and thus I, is the same. However, the modulus does change and does enter into this:

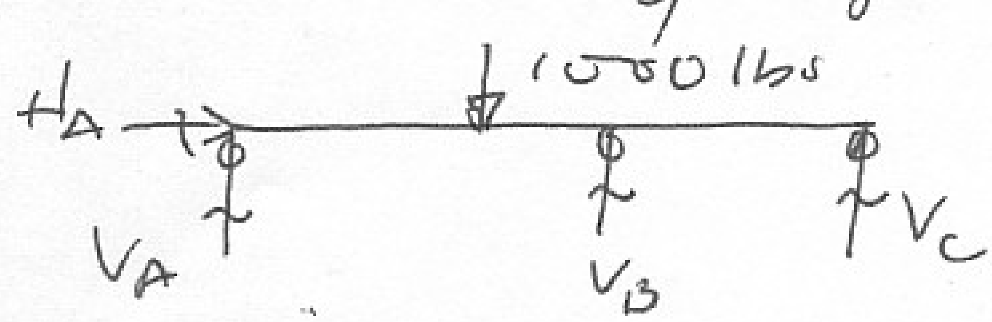
$$w \propto 1/E$$

So:
$$\frac{w_{steel}}{w_{Al}} = \frac{1/E_{steel}}{1/E_{Al}} = \frac{1/30\,Msi}{1/10\,Msi}$$

$\Rightarrow$ $\boxed{\text{deflection decreases to } 1/3 \text{ of Aluminum value}}$

(f) A third support (a pin) is added at the tip of the beam. Would the procedure for determining the answers to part (a) change? Be sure to explain ***clearly***. Use figures, ratios, etc. as appropriate.

→ Draw the Free Body Diagram:



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PROBLEM #2M (continued)

there are now more reactions than degrees of freedom and this configuration has become

⇒ Statically Indeterminate

It is now necessary to include the entire behavior of the beam and the constitutive response (deflection behavior) and solve these simultaneously to determine the reactions

⇒ Procedure does change

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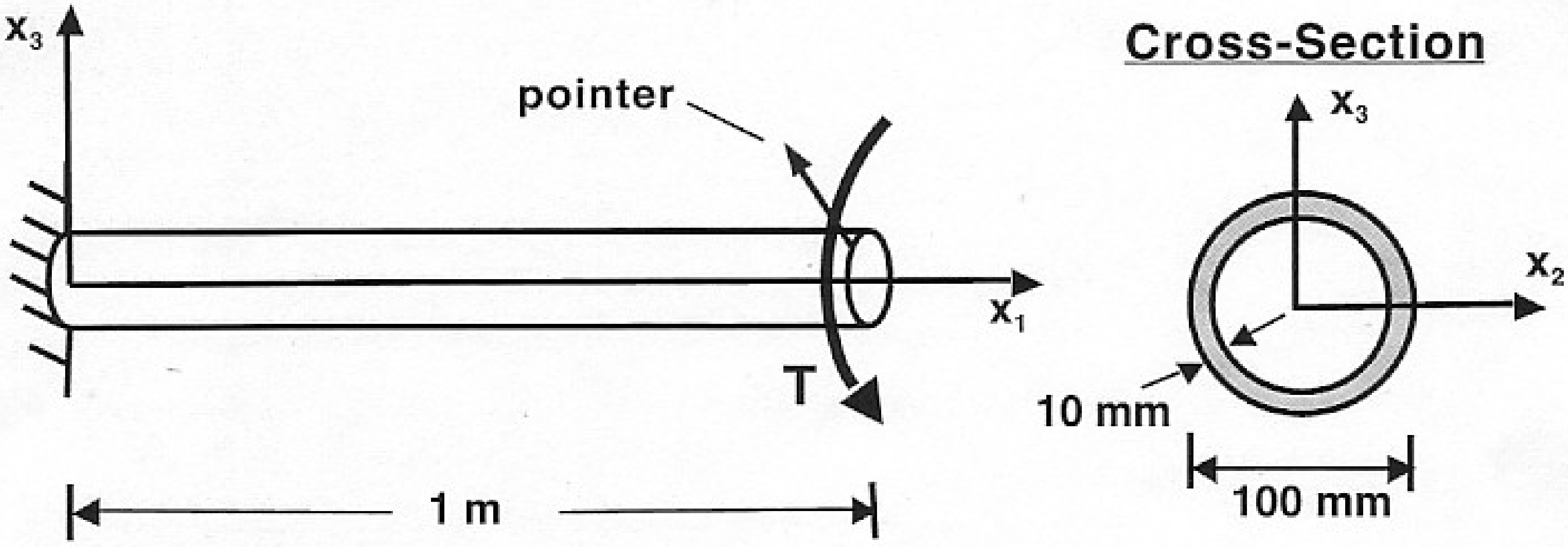
PROBLEM #3M (30%)

A shaft configuration has been chosen to be used as a metering device to determine torque applied in an overall system. The shaft is rigidly attached to a wall and is loaded at its end by the applied torque. Attached to the end of the shaft is an arrow to indicate the rotation of the shaft. The shaft is 1 meter in length and is a tube with an outer diameter of 100 mm and a wall thickness of 10 mm. Two materials are being considered for use. The first is steel which has a Young's modulus of 210 GPa, a Poisson's ratio of 0.3, and a yield stress of 345 MPa. The second is titanium which has a Young's modulus of 105 GPa, a Poisson's ratio of 0.3, and a yield stress of 1400 MPa.



(a) One of the critical cases is indicated by a rotation of 1°. Determine the ratio of the torques required for this case for the two materials under consideration.

The operative equation is "torque-twist":

$$\frac{d\phi}{dx_1} = \frac{T(x_1)}{GJ}$$

The end value of the applied torque will change, but the remaining configuration is the same. It is statically determinate and the distribution of the torque resultant will be the same in both cases except ~~the~~ be multiplied by a different end torque, T. ~~The~~ boundary condition is the same ($\phi = 0$, @ $x = 0$) and the cross-section, and thus polar moment of inertia J is the same in both cases. If the tip rotation is to be the same and:

$$\phi \propto T/GJ \quad \text{then:}$$

PROBLEM #3M (continued)

$$1 = \frac{\phi_{steel}}{\phi_{Ti}} = \frac{T_{steel}/G_{steel}}{T_{Ti}/G_{Ti}}$$

Both materials are isotropic, so $G = \frac{E}{2(1+\nu)}$ with $\nu = 0.3$ in both cases, so this becomes:

$$\frac{T_{steel}}{T_{Ti}} = \frac{E_{steel}}{E_{Ti}} = \frac{210\ GPa}{105\ GPa}$$

$$\Rightarrow \boxed{\frac{T_{steel}}{T_{Ti}} = 2}$$

(b) For the critical cases of the 1° rotation, determine the ratio of the maximum stress for the two materials under consideration.

The stress equation for the <u>resultant</u> shear stress is: $\tau_{res} = \frac{Tr}{J}$

The polar moment of area is the same in both cases; the maximum value of r is the same in both cases.

The distribution of torque is the same in both cases except it is multiplied by

PROBLEM #3M (continued)

a different end value. The ratio of maximum stresses will thus be the ratio of the torques:

$$\frac{\tau_{max_{steel}}}{\tau_{max_{Ti}}} = \frac{T_{steel}}{T_{Ti}}$$

$$\Rightarrow \boxed{\frac{\tau_{max_{steel}}}{\tau_{max_{Ti}}} = 2}$$